

The Robust Optimization in Centralized Supply Chain

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Abstract: Considering the inaccurate demand forecasting in supply chain, we introduce robust optimization to reduce uncertainty. The method is mainly to modify the probability distribution of the demand, in order to obtain a more accurate demand. A classical model and a corresponding robust model are established in the context of a fixed number of products offered by the supplier. As to calculation, we also propose the fast Fourier transform approach which greatly reduces the amount of computation. Finally, the process of robust optimization and improved algorithm are interpreted by numerical examples. The results show that the expected revenue of the robust model is lower. Because the method is conservative and robust.

Keywords: Supply Chain, Robust Optimization, Demand Forecasting Uncertainty, Fast Fourier Transform Approach

1. Introduction

Demand forecasting arises for lowering the effect of bullwhip at first. As a key to supply chain management, it can not only reduce the decision-making errors but increase revenue.

Generally, demand forecasting in supply chain is not so accurate. Researchers have pointed out its disadvantages. Jukka Hallikas et. al regarded the demand forecasting uncertainty as a type of risk. In his word, the inaccurate forecast will increase the difficulty of material control, and even affect the long-term production planning and capacity investment decision [1]. Huang Xiaoyuan and Yan Nina thought the deviation in demand forecasting as a kind of supply chain uncertainty, which stems from customer uncertainty [2]. It will cause information asymmetry between enterprises in supply chain, thus affect the stability of the whole supply chain.

The significance of accurate demand forecasting has been stressed by many scholars.

In order to get more accurate demand data, scholars have focused on improving the methods of demand forecasting. Thomas R. Willemain et. al has addressed the irregular demand prediction problem in supply chain. The paper shows that compared with the exponential smoothing and Croston method, bootstrapping method can obtain more accurate demand distribution based on the fixed lead time [3].

Considering various demand forecasting methods all have advantages and disadvantages, Luis Aburto, Richard Weber put forward the hybrid system which combines various techniques to integrate the advantages of all methods [4]. In this way, the accuracy of demand forecasting in supply chain has increased greatly. While Fang, F., Wong, T. N. constructed a Bayesian model in the paper and the results showed the model effectively reduces the cost [5]. They think accurate and fast prediction information can help manufacturers make optimal decisions. Moreover Kou Yukun et. al established the collaboration demand model of supply chain to improve the prediction accuracy for the perspective of Petri and Agent [6].

Meanwhile, some researchers begin to introduce the robust optimization method into supply chain. It aims to reduce the uncertainty of demand and improve the prediction accuracy too.

Cheng-Liang Chen, Wen-Cheng Lee modeled the uncertain market demand situation by using the discrete multiple known probability and use fuzzy set to reflect the compatibility of price preference between the sellers and buyers [7]. It shows that the robustness measures as part into the objective function can greatly reduce the fluctuation of the value of the objective function. D. Bertsimas et. al mainly concerned optimally controlling a supply chain with stochastic demand [8]. Considering the discrete demand is not evenly distributed over time, the author adopted robust

optimization and constructed the equivalent model to fix demand sequence. This method is better than dynamic programming. Not only the calculation of large scale supply chain, it can also solve the dimension problem when using dynamic programming for network supply chain. At the same time, Perakis G, Sood A explored the multi-period oligopoly market and used the robust optimization to solve dynamic pricing for a single perishable product with a fixed inventory [9]. Realizing the fact that historical data can only help obtain the demand interval, Yan Nina et. al used a variety of robust optimization methods to study the problem of competition between multiple retailers under uncertain demand environment from different perspectives [10]. Considering the operation management of the closed loop supply chain, the corresponding robust model was constructed by Xu Jiawang based on the uncertainty of customer demand [11]. While, Feng Pan, Rakesh Nagi thought the design of supply chain in the emerging market cannot accurately predict the customer's demand, so the authors used the robust optimization method to solve the problem of uncertain demand in the supply chain [12]. Taking into account demand and other uncertain factors, Li C, Liu S. proposed robust optimization to reduce the bullwhip effect in supply chain [13]. While, Aouam T, Brahimi N. used robust optimization to formulate a new model to integrate production planning and order acceptance under demand uncertainty [14]. Ait-Alla A et. al also investigated the robust production planning in the fashion industry [15]. With uncertain demand, Melamed M et. al apply the adjustable robust optimization to solving production planning problem in order to get the maximum profit [16]. Carrizosa E et. al used robust optimization to cope with autoaggressive demand in the newsvendor problem [17].

Based on above literature, we introduce robust optimization to construct a new model of centralized supply chain. Moreover, an efficient algorithm is used to reduce computation. In section 4 we give some numerical simulations and section 5 is a concluding section.

2. Basic Assumptions and Notation

Suppose there are one supplier and multiple retailers in the market, and the retailer is independent of each other. We use D_j to denote the real demand the retailer j ($j=1,2,\dots,m$) faces with and Q to denote the total quantity the supplier can offer. The demand faced by the retailer is a random variable and belongs to the Poisson distribution. Moreover, the retailer j orders a quantity Q_j from the supplier and sells at the price Pr_j . We also choose v to represent unit salvage value contained by the surplus products and c to represent the unit product cost. Meanwhile, we assume that Pr_j is bigger than c , while v has the smallest value compared with c and Pr_j .

It means

$$Pr_j > c > v$$

Assuming that the cost isn't concerned, we can figure out the revenue each retailer get as following:

$$\pi^{r_j}(Q_j) = Pr_j \cdot E \min(Q_j, D_j) + v \cdot E(Q_j - D_j)^+$$

The supplier's residual value:

$$v \left(Q - \sum_{j=1}^m Q_j \right)$$

Then, we can have the revenue function in centralized supply chain:

$$\begin{aligned} \pi^c(Q_j) &= v \left(Q - \sum_{j=1}^m Q_j \right) + \sum_{j=1}^m [Pr_j \cdot E \min(Q_j, D_j) + v \cdot E(Q_j - D_j)^+] - cQ \\ &= (v - c)Q + \sum_{j=1}^m [(Pr_j - v) \cdot E \min(Q_j, D_j)] \end{aligned}$$

3. Models

If Q is fixed, the model is formulated as follows:

$$\begin{aligned} \max_{Q_j} \pi^c(Q_j) &= (v - c)Q + \sum_{j=1}^m [(Pr_j - v) \cdot E \min(Q_j, D_j)] \\ s.t. \quad &\sum_{j=1}^m Q_j \leq Q \\ &Q_j \in \mathbb{Z}^+ \end{aligned} \quad (1)$$

We use Q_j^* as the optimal solution.

It's obvious that the key to get the maximal expected revenue $\max \pi^c(Q_j)$ is to figure out

$\max \left\{ \sum_{j=1}^m [(Pr_j - v) \cdot E \min(Q_j, D_j)] \right\}$ firstly. It is sorted as a standard separable problem and can be calculated by dynamic programming. While the computational complexity of this method is of the order of $O(mQ^2)$, it's necessary to reduce the calculation. Consequently, to apply this approach we need an efficient algorithm to compute the function values $E[\min\{Q_j, D_j\}]$. The improved algorithm is called fast Fourier transform (FFT). See references [18, 19].

3.1. An Efficient Algorithm

As said above, we need an efficient algorithm to compute the function values $E[\min\{Q_j, D_j\}]$

The first step is to introduce the function $F_j: \mathbb{Z} \rightarrow \mathbb{R}$

$$F_j(Q_j) = E[\min\{Q_j, D_j\}]$$

For given $Q_j \in \mathbb{Z}^+$, that

$$F_j(n) = \sum_{t=1}^n P\{D_j \geq t\} \quad (2)$$

Derivation process:

$$\begin{aligned} F_j(n) &= E\min(n, D_j) = D_j P(D_j \leq n) + nP(D_j > n) \\ &= D_j [P(D_j = 1) + P(D_j = 2) + \dots + P(D_j = n)] + n[P(D_j = n+1) + \dots] \\ &= P(D_j = 1) + 2 \cdot P(D_j = 2) + \dots + n \cdot P(D_j = n) + n[P(D_j = n+1) + \dots] \\ &= [P(D_j = 1) + P(D_j = 2) + \dots] + [P(D_j = 2) + P(D_j = 3) + \dots] \\ &\quad + \dots + [P(D_j = n) + P(D_j = n+1) + \dots] \\ &= \sum_{t=1}^n P(D_j \geq t) \end{aligned}$$

By equation (2), we know that:

$$F_j(n) - F_j(n-1) = P\{D_j \geq n\} \geq 0$$

$$F_j(n-1) - F_j(n-2) = P\{D_j \geq n-1\}$$

Then,

$$\begin{aligned} &[F_j(n) - F_j(n-1)] - [F_j(n-1) - F_j(n-2)] \\ &= P\{D_j \geq n\} - P\{D_j \geq n-1\} \\ &= -P(D_j = n-1) \leq 0 \end{aligned}$$

Clearly, $F_j(n) - F_j(n-1)$ is non-increasing in n . So, F_j is a discrete concave function.

Thus, solving problem (1) equals solving the following model:

$$\begin{aligned} \max &\left\{ \sum_{j=1}^m (Pr_j - v) F_j(Q_j) \right\} \\ \text{s.t.} &\quad \sum_{j=1}^m Q_j \leq Q \\ &\quad Q_j \in Z^+ \end{aligned} \quad (3)$$

Obviously, $Q_j \leq Q$, now introduce for $1 \leq h \leq Q$, the values

$$\alpha_{jh} = F_j(h) - F_j(h-1) = \sum_{k=h}^{\infty} p_{jk}$$

where $p_{jk} = P\{D_j = k\}$

Therefore, $(Pr_j - v)\alpha_{jh}$ gives the marginal value of increasing Q_j from $h-1$ to h .

The core of this improved algorithm is to introduce a $m \times Q$ marginal revenue matrix.

$$\begin{bmatrix} (Pr_1 - v)\alpha_{11} & (Pr_1 - v)\alpha_{12} & \dots & (Pr_1 - v)\alpha_{1Q} \\ (Pr_2 - v)\alpha_{21} & (Pr_2 - v)\alpha_{22} & \dots & (Pr_2 - v)\alpha_{2Q} \\ \vdots & \vdots & \dots & \vdots \\ (Pr_m - v)\alpha_{m1} & (Pr_m - v)\alpha_{m2} & \dots & (Pr_m - v)\alpha_{mQ} \end{bmatrix}$$

Sorting $(Pr_j - v)\alpha_{jh}$ values from the big one to the small one and adding up the first Q terms are the only things we have to do to get the optimal value of function (3). Meanwhile, the optimal solution Q_j^* equals the number of times index j appears among these Q terms. As discrete concave, the marginal value in each row j are in descending order, ie: $(Pr_j - v)\alpha_{j1} \geq (Pr_j - v)\alpha_{j2} \geq \dots \geq (Pr_j - v)\alpha_{jQ}$. Thus, every time we have to do is to compare those values on the left side which haven't been selected for comparison last time. In this way, we just need to compare total m values each time. Consequently, the computational complexity of the proposed approach reduces to the order of $O(mQ)$.

3.2. A Robust Optimization Approach

In the process of modeling, we also need to know the probability distribution of demand. Such probabilities are always estimated by analyzing the historical data, and hence they are prone to inaccuracies. In order to reduce the impact of inaccurate demand estimation, we propose the robust optimization approach. References [20, 21, 22]

Assume the random variable D_j , representing the actual demand faced by the retailer j , is concentrated on $\{0, \dots, K\}$, and this demand has an estimated probability vector $\hat{p}_j = (\hat{p}_{j0}, \dots, \hat{p}_{jK})^T$. Each \hat{p}_{jk} is assumed to be positive. To compensate for possible estimation errors, we consider $1 \leq j \leq m$ the probability vector \hat{p}_j belonging to the uncertain set P_j given by:

$$P_j = \left\{ p_j \in \mathbb{R}^{K+1} \mid p_j \in \hat{p}_j + \Delta_j, p_j^T e = 1 \right\}$$

Where

$$\Delta_j = \left\{ b_j = (b_{j0}, \dots, b_{jK})^T \in \mathbb{R}^{K+1} \mid \sum_{k=0}^K \left(\frac{b_{jk}}{\hat{p}_{jk}} \right)^2 \leq \delta_j^2 \right\}$$

with $\delta_j \in [0, 1]$

Clearly, the demand depends on its distribution probability p_j , hence we can denote this random variable by $D_j(p_j)$. Now we have the robust counterpart of problem (1):

$$\begin{aligned} \max_{Q_j} \min_{p_j \in P_j} \pi^c(Q_j) &= (v-c)Q + \sum_{j=1}^m \left[(Pr_j - v) \cdot E \left[\min(Q_j, D_j(p_j)) \right] \right] \\ \text{s.t.} \quad &\sum_{j=1}^m Q_j \leq Q \\ &Q_j \in \mathbb{Z}^+ \end{aligned} \quad (4)$$

Same as the chapter 3.1, we introduce $G_j: \mathbb{Z}^+ \rightarrow \mathbb{R}$ given by

$$G_j(n) = \min_{p_j \in P_j} E \left[\min(n, D_j(p_j)) \right] \quad (5)$$

For every $p_j \in P_j$, the function

$$n \rightarrow E \left[\min \{n, D_j(p_j)\} \right]$$

is discrete concave on \mathbb{Z}^+ . Because the point-wise infimum of a collection of concave functions G_j is again concave, the function is also concave on \mathbb{Z}^+ .

The problem (4) can be rewritten as

$$\begin{aligned} \max \quad &\sum_{j=1}^m (Pr_j - v) G_j(Q_j) \\ \text{s.t.} \quad &\sum_{j=1}^m Q_j \leq Q \\ &Q_j \in \mathbb{Z}^+ \end{aligned} \quad (6)$$

For given $p_j \in P_j$,

$$E \left[\min \{Q_j, D_j(p_j)\} \right] = \sum_{k=0}^{Q_j-1} k p_{jk} + Q_j \sum_{k=Q_j}^K p_{jk} = c(Q_j)^T p_j$$

where

$$\begin{aligned} c(Q_j)^T &= (c_0(Q_j), c_1(Q_j), \dots, c_K(Q_j)) \\ &= (0, 1, \dots, Q_j-1, Q_j, Q_j, \dots, Q_j) \end{aligned}$$

By equation (5), we have

$$\begin{aligned} G_j(Q_j) &= \min \left\{ c(Q_j)^T p_j \mid p_j \in P_j \right\} \\ &= c(Q_j)^T \hat{p}_j + \min \left\{ c(Q_j)^T b_j \mid b_j \in \Delta_j, b_j^T e = 0 \right\} \end{aligned} \quad (7)$$

Using standard nonlinear programming techniques [21, 22], it's easy to show

$$\begin{aligned} \min \left\{ c^T y \mid y^T A y \leq \delta^2, e^T y = 0 \right\} \\ = -\delta \sqrt{c^T A^{-1} c - \frac{(e^T A^{-1} c)^2}{e^T A^{-1} e}} \end{aligned} \quad (8)$$

where A is symmetric and positive definite.

It shows that the last term in relation (7) has an analytic

expression.

Using $c_0(Q_j) = 0$, we have

$$\begin{aligned} G_j(Q_j) &= c(Q_j)^T \hat{p}_j - \delta_j \sqrt{\sum_{k=1}^K p_{jk} c_k^2(Q_j) - \frac{\left(\sum_{k=1}^K p_{jk} c_k(Q_j) \right)^2}{\sum_{k=0}^K p_{jk}}} \\ &\quad (Q_j \leq Q) \end{aligned}$$

Same as the chapter 3.1, we introduce β_{jh} , for $1 \leq h \leq Q$,

$$\beta_{jh} = G_j(h) - G_j(h-1)$$

At the same time, we can get a $m \times Q$ marginal value matrix

$$\begin{bmatrix} (Pr_1 - v) \beta_{11} & (Pr_1 - v) \beta_{12} & \cdots & (Pr_1 - v) \beta_{1Q} \\ (Pr_2 - v) \beta_{21} & (Pr_2 - v) \beta_{22} & \cdots & (Pr_2 - v) \beta_{2Q} \\ \vdots & \vdots & \cdots & \vdots \\ (Pr_m - v) \beta_{m1} & (Pr_m - v) \beta_{m2} & \cdots & (Pr_m - v) \beta_{mQ} \end{bmatrix}$$

Because G_j is discrete concave, the he marginal value in each row j are in descending order, ie: $(Pr_j - v) \beta_{j1} \geq (Pr_j - v) \beta_{j2} \geq \cdots \geq (Pr_j - v) \beta_{jQ}$. Certainly, we can also use FFT to get the optimal objective function value.

4. Numerical Simulation

4.1. Classical Model

Firstly, we give a certain value of some variables.

We set,

$$Pr_1 = 30, Pr_2 = 20, Pr_3 = 25, Pr_4 = 35, v=3, c=10$$

We also suppose that there exist four retailers in the market and the quantity the supplier can offer is ten.

It means $m=4, Q=10$.

At the same time, we assume

$$\lambda_1=2.9, \lambda_2=3.2, \lambda_3=3.0, \lambda_4=3.5$$

Because the demand variable belongs to Poisson distribution, we use Excel to randomly generate a 4×10 matrix about α_{jh} ($j=1,2,3,4; h=1,2,\dots,10$):

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \alpha_{15} & \alpha_{16} & \alpha_{17} & \alpha_{18} & \alpha_{19} & \alpha_{110} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \alpha_{25} & \alpha_{26} & \alpha_{27} & \alpha_{28} & \alpha_{29} & \alpha_{210} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \alpha_{35} & \alpha_{36} & \alpha_{37} & \alpha_{38} & \alpha_{39} & \alpha_{310} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \alpha_{45} & \alpha_{46} & \alpha_{47} & \alpha_{48} & \alpha_{49} & \alpha_{410} \end{bmatrix}$$

=

0.945	0.785	0.554	0.330	0.168	0.074	0.028	0.010	0.003	0.001
0.950	0.801	0.577	0.352	0.184	0.084	0.033	0.012	0.004	0.001
0.959	0.828	0.620	0.397	0.219	0.105	0.044	0.016	0.005	0.001
0.969	0.863	0.678	0.462	0.274	0.141	0.064	0.026	0.009	0.002

Then a 4×10 marginal value matrix can be obtained:

25.508	21.200	14.953	8.914	4.536	1.997	0.769	0.261	0.077	0.017
16.149	13.610	9.801	5.992	3.136	1.422	0.565	0.197	0.060	0.014
21.092	18.223	13.631	8.734	4.816	2.308	0.971	0.359	0.115	0.028
31.001	27.619	21.700	14.795	8.753	4.524	2.057	0.823	0.283	0.073

Now we can apply FFT to finger out the first ten maximum values. Below are Main steps in FFT:

(1) Find the first maximum. In the first column, $31.001 > 25.508 > 21.092 > 16.149$, easily we know the value of first maximum is 31.001;

(2) Find the second maximum. Because $27.619 > 25.508 > 21.092 > 16.149$, the value of second maximum is 27.619;

(3) Find the third maximum. As $25.508 > 21.700 > 21.092 > 16.149$, we can get the value of third maximum is 25.508;

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(8) Find the eighth maximum. For $16.149 > 14.953 > 14.795 > 13.631$, the value of eighth maximum is 16.149;

(9) Find the ninth maximum. $14.953 > 14.795 > 13.631 > 13.610$, so the value of ninth maximum is 14.953;

(10) Find the tenth maximum. $14.795 > 13.631 > 13.610 > 8.914$, so the value of tenth maximum is 14.795.

For now, we have found the first ten maximum values as follows:

(31.001, 27.619, 25.508, 21.700, 21.200, 21.092, 18.233, 16.149, 14.953, 14.795)

$$\therefore Q_1^*=3, Q_2^*=1, Q_3^*=2, Q_4^*=4,$$

$$\max_{Q_1, Q_2, Q_3, Q_4} \pi^c(Q_1, Q_2, Q_3, Q_4) = 142.250$$

It means, the quantity the first retailer ordered is three; the quantity the second retailer ordered is one; the quantity the third retailer ordered is two; the quantity the fourth retailer ordered is four. And the maximum revenue the whole supply chain can get is 142.250.

4.2. Robust Model

Set $K=10$, $\sigma_1=0.2$, $\sigma_2=0.4$, $\sigma_3=0.6$, $\sigma_4=0.8$

$$\hat{p}_1 = (0.055, 0.160, 0.231, 0.224, 0.162, 0.094, 0.045, 0.019, 0.007, 0.002, 0.001)^T$$

$$\hat{p}_2 = (0.050, 0.149, 0.224, 0.224, 0.168, 0.101, 0.050, 0.022, 0.008, 0.003, 0.001)^T$$

$$\hat{p}_3 = (0.041, 0.130, 0.209, 0.223, 0.178, 0.114, 0.061, 0.028, 0.011, 0.004, 0.001)^T$$

$$\hat{p}_4 = (0.030, 0.106, 0.185, 0.216, 0.189, 0.132, 0.077, 0.039, 0.017, 0.007, 0.002)^T$$

Then, we can get a 4×10 matrix about β_{jh} ($j=1, 2, 3, 4; h=1, 2, \dots, 10$):

0.864	0.717	0.511	0.312	0.163	0.073	0.028	0.010	0.003	0.001
0.789	0.662	0.487	0.312	0.171	0.081	0.033	0.012	0.004	0.001
0.720	0.615	0.472	0.323	0.193	0.098	0.043	0.016	0.005	0.001
0.657	0.574	0.459	0.338	0.222	0.126	0.061	0.025	0.009	0.002

Thus the marginal value matrix can be obtained:

23.324	19.349	13.804	8.421	4.388	1.964	0.764	0.260	0.077	0.017
13.414	11.246	8.274	5.301	2.915	1.370	0.556	0.196	0.060	0.014
15.849	13.539	10.380	7.112	4.237	2.156	0.941	0.355	0.114	0.028
21.023	18.370	14.685	10.820	7.106	4.017	1.938	0.801	0.280	0.073

Same as the chapter 4.1, the first ten maximum values can be found by using FFT:

(23.324, 21.023, 19.349, 18.370, 15.849, 14.685, 13.804, 13.539, 13.414, 11.246)

$$\therefore Q_1^*=3, Q_2^*=2, Q_3^*=2, Q_4^*=3,$$

$$\max_{Q_1, Q_2, Q_3, Q_4} \pi^c(Q_1, Q_2, Q_3, Q_4) = 94.603$$

It means, the quantity the first retailer ordered is three; the quantity the second retailer ordered is two; the quantity the third retailer ordered is two; the quantity the fourth retailer ordered is three. And the maximum revenue the whole supply chain can get is 94.603.

When the quantity of the products the supplier offered is ten, the revenue of the whole supply chain can get is 142.250 in the classical model and 94.603 in the robust model. It seems that the expected revenue under the robust model is smaller than the classical model. We may attribute this result to the conservatism and robustness of the robust optimization. It means that we can get the optimal solution of the robust model no matter what the external conditions are like.

5. Conclusion

This paper mainly focuses on the problem of inaccuracy demand forecasting in supply chain. We construct a robust model with one supplier and multiple retailers. Robust optimization is used to reduce the uncertainty of demand forecasting. Furthermore, numerical simulations show the method is applicable and can be implemented. Owing to its conservatism and robustness, we may get a smaller expected revenue in the numerical simulation. While, in realization, the scenarios the retailers faced with are not exactly the same as the scenario in the classical model. It means that the optimal solution of the robust model can also be achieved regardless of the outer conditions. The significance of robust optimization is to take into account the worst environment in order to obtain the optimal solution that can adapt to various scenarios, and also reflects the decision maker are risk averse.

As to the improved algorithm, we can see its convenience and efficiency. For solving the discrete problem, it reflects its unique advantages in terms of reducing computational complexity. Compare with the dynamic programming, it can get the result faster and have lower calculation.

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