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The Analytical Solution of Some Partial Differential Equations by the SBA Method

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Abstract: Many phenomena in nature, especially in the current context of climate change, are modeled by nonlinear partial differential equations. Numerical methods exist to solve these equations numerically. But, the search for exact solutions, when they exist, is always necessary, in order to better explain the modeled phenomenon. The interest of the search for the exact solution results in the advantage of avoiding to analyze again the margins of errors, which sometimes, require the minimization. Thus, several methods are implemented to search for possible exact solutions. Despite the existence of various methods, difficulties have always surfaced. The case considered is that of strongly nonlinear partial differential equations. Thus, in the literature approached a new method called, the SBA method. In this paper, is used an analytical method called SOME-BLAISE-ABBO method (SBA method), to solve nonlinear partial differential equations. It is a method that is exclusively presented for the solution of exclusively nonlinear partial differential equations. The fundamental objective of this work is to show the effectiveness of the method for nonlinear problems. To this end, a reaction-convection-diffusion problem, a biological population model and a system of coupled Burgers' equations are chosen to demonstrate the effectiveness, accuracy and efficiency of the said method. The easy obtaining of the exact solutions of these three chosen nonlinear problems allowed us to affirm the effectiveness of the method.

Keywords: Nonlinear PDEs, Reaction-Convection-Diffusion Problem, Biological Population Model, Coupled Burgers' Equations, SBA Method

1. Introduction

The specific objective of this work is to demonstrate the effectiveness of the SBA method for finding exact solutions, when they exist, of some rather complex problems, due to their non-linearity. In spite of the efforts of the researchers for the improvement of the methods of resolution of the equations or systems of nonlinear partial differential equations, the obtaining of the exact solutions, when they exist has been several times rather difficult, sometimes inaccessible or almost impossible. Depending on the nature of the non-linearity of the equation, especially for partial differential equations, the use classical, semi-analytical or approximation methods for the search for solutions does not always lead to the exact solutions sought, when they exist. In order to obtain satisfactory results, algorithms of some classical methods have sometimes been improved or modified to solve some of these complex problems. With these algorithms, although modified or improved, the access to the exact solution some sometimes presented considerable difficulties, to the point of being satisfied with an approximate solution. Always with the aim of arriving at the exact solution, the researchers did not cross their arms. They have sometimes opted for the combination of methods, which has sometimes led to satisfactory results. When will refer to some cases such as: Laplace-Adomian method, Laplace-VIM method, and many other [1, 3-7, 15-17].

In the review of the literature that we have gone through, we were interested in the SBA method [11-13]. We were impressed by the efficiency of this method to solve nonlinear PDEs [4].

To experiment with this SBA method, there are three specific problems chosen. a reaction-convection-diffusion problem, a biological population model and a system of coupled Burger's equations have been chosen.

The reaction-diffusion-convection problems are very useful mathematical models in applied sciences such as biology modeling, physics, chemistry, astrophysics, hydrology, medicine and engineering [8, 10]. The second problem also expresses a diffusion phenomenon. This is the biological population model [6, 15, 16]. The biological population model is a spatial diffusion model, subject to certain initial conditions. It is none other than the diffusion of a biological species in a domain D is described by the nonlinear degenerate parabolic partial differential equation. In recent years, coupled partial differential equations have been employed in various fields of engineering and applied sciences. Coupled Burgers' equations are coupled partial differential equations (PDEs), and describe the approximation theory of flow through a shock wave traveling in a viscous fluid. these coupled models have also been used to study poly-dispersive sedimentation phenomena. [2, 3, 9, 14, 17].

2. Description of the Method

The SBA method is very powerful because it avoids the computation of Adomian polynomials. The SBA method is based on the Adomian decomposition method, the successive approximation method and the Picard principle [4, 11-13].

Let us consider the following functional equation:

$$Au = f \tag{1}$$

Where $A: H \rightarrow H$, is an operator not necessarily linear, and H is a Hilbert space adequately chosen given the operator A, f is given function and u the unknown function. Let:

$$A = L + R + N \tag{2}$$

Where L is n inversible operator in the Adomian sense, R the linear remainder and N an nonlinear operator. The equation (1) therefore becomes:

$$Au = Lu + Ru + Nu \tag{3}$$

The reverse gives us:

$$u = \theta + L^{-1}(f) - L^{-1}(Ru) - L^{-1}(Nu) \tag{4}$$

Where θ is such that $L(\theta) = 0$. The equation (4) is the Adomian canonical form, using the successive approximations. We get:

$$u^k = \theta + L^{-1}(u^k) - L^{-1}(Ru^k) - L^{-1}(f);\ k \geq 1 \tag{5}$$

This yields the following Adomian algorithm [4, 5, 11-13]:

$$\begin{cases} u_0^k = \theta + L^{-1}(f) - L^{-1}(Nu^{k-1});\ k \geq 1 \\ u_{n+1}^k = -L^{-1}(Ru_n^k);\ n \geq 0 \end{cases} \tag{6}$$

The Picard principle is then applied to the equation (6); let u^0 be such that $N(u^0) = 0$; for $k = 1$, we get:

$$\begin{cases} u_0^1 = \theta + L^{-1}(f) - L^{-1}(Nu^0) \\ u_{n+1}^1 = -L^{-1}(Ru_n^1);\ n \geq 0 \end{cases} \tag{7}$$

If the series

$$\left(\sum_{n=0}^{\infty} u_n^1\right)$$

Converges, then

$$u^1 = \sum_{n=0}^{\infty} u_n^1(x,t) \tag{8}$$

For $k = 2$, we get

$$\begin{cases} u_0^2 = \theta + L^{-1}(f) - L^{-1}(Nu^1) \\ u_{n+1}^2 = -L^{-1}(Ru_n^2);\ n \geq 0 \end{cases} \tag{9}$$

If the series

$$\left(\sum_{n=0}^{\infty} u_n^2\right)$$

Converges, then

$$u^2 = \sum_{n=0}^{\infty} u_n^2(x,t) \tag{10}$$

This process is repeated to k. That is to say, it 's proceeded in the same way for $k \geq 2$. If the series

$$\left(\sum_{n=0}^{\infty} u_n^k\right)$$

Converges, then

$$u^k = \sum_{n=0}^{\infty} u_n^k(x,t)$$

Therefore
$u = \lim_{k\to\infty} u^k$ is the solution of the problem.
With the following hypothesize: at the step k,

$$N(u^k) = 0, \forall\, k \geq 1.$$

3. Applications

This section is devoted to the application of the SBA method. From the results that will obtain, the robustness and the effectiveness of our method will be affirm.

3.1. Example 1

Consider the following nonlinear convection-diffusion-reaction initial value problem [8].

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) + 3u\frac{\partial u}{\partial x} + 2u - 2u^2 \tag{11}$$

With $u(x,0) = 2\sqrt{e^x - e^{-4x}}, x \in \mathbb{R}$
Where $u = u(x,t)$.
From the expression of the equation, can identify the non-linear terms given by the following relation

$$Nu = \frac{\partial}{\partial x}\left(u\frac{\partial u}{\partial x}\right) + 3u\frac{\partial u}{\partial x} - 2u^2$$

The principle of the application of the SBA method requires to identify first to all the nonlinear part. The equation is then written

$$\frac{\partial u}{\partial t} = Nu + 2u \tag{12}$$

Integrating the equation (12) with respect to t, gives the following canonical form

$$u(x,t) = u(x,0) + 2\int_0^t u(x,s)ds + \int_0^t Nu(x,s)ds \tag{13}$$

Let us applay to (13) the method of the successive approximations, one obtains

$$u^k(x,t) = u(x,0) + 2\int_0^t u^k(x,s)ds + \int_0^t Nu^{k-1}(x,s)ds \tag{14}$$

Knowing that

$$u^k(x,t) = \sum_{n=0}^{\infty} u_n^k(x,t)$$

The relation (14) is written

$$\sum_{n=0}^{\infty} u_n^k(x,t) = u(x,0) + 2\int_0^t u^k(x,s)ds + \int_0^t Nu^{k-1}(x,s)ds$$

It is from the application of the method of successive approximations that the following algorithm of the SBA method is derived:

$$\begin{cases} u_0^1(x,t) = u(x,0) + \int_0^s Nu^0(x,s)ds \\ u_{n+1}^1(x,t) = 2\int_0^t u_n^1(x,s)\,ds \end{cases} \tag{15}$$

Applying Picard's principle, assume that there exists
$u^0 \in H\ (of\ Hilbert)$ such that $Nu^0 = 0$.
The system (15) becomes:

$$\begin{cases} u_0^1(x,t) = u(x,0) \\ u_{n+1}^1(x,t) = 2\int_0^t u_n^1(x,s)\,ds \end{cases} \tag{16}$$

$$u_0^1(x,t) = 2\sqrt{e^x - e^{-4x}}.$$

For $n = 0$,

$$u_1^1(x,t) = 2\int_0^t u_0^1(x,s)\,ds = 2\int_0^t 2\sqrt{e^x + e^{-4x}}\,ds = 2\left(2\sqrt{e^x + e^{-4x}}\int_0^t ds\right)$$

$$u_1^1(x,t) = 2\left(2t\sqrt{e^x - e^{-4x}}\right)$$

For $n = 1$

$$u_2^1(x,t) = 2\int_0^t u_1^1(x,s)\,ds = 4\left(2\sqrt{e^x + e^{-4x}}\int_0^t tds\right)$$

$$u_2^1(x,t) = 4\left(2t\sqrt{e^x - e^{-4x}}\right)\frac{t^2}{2}$$

The same calculation procedure leads us to the following other results:

$$u_3^1(x,t) = 8\left(2t\sqrt{e^x - e^{-4x}}\right)\frac{t^3}{6}$$

$$u_4^1(x,t) = 16\left(2t\sqrt{e^x - e^{-4x}}\right)\frac{t^4}{24}$$

And so on, therefore, the following sum deduced:

$$u^1(x,t) = 2\sqrt{e^x + e^{-4x}} + \left(2\sqrt{e^x + e^{-4x}}\right)(2t) + \left(2\sqrt{e^x + e^{-4x}}\right)\frac{(2t)^2}{2!} + +\left(2\sqrt{e^x + e^{-4x}}\right)\frac{(2t)^3}{3!} + \left(2\sqrt{e^x + e^{-4x}}\right)\frac{(2t)^4}{4!} + \cdots$$

At step $k = 1$, we have:

$$u^1(x,t) = 2\sqrt{e^x + e^{-4x}}\left[1 + 2t + \frac{(2t)^2}{2!} + \frac{(2t)^3}{3!} + \frac{(2t)^4}{4!} + \cdots\right]$$

$$u^1(x,t) = \left(2\sqrt{e^x + e^{-4x}}\right)e^{2t} \tag{17}$$

At step $k = 2$, there is:

$$\begin{cases} u_0^2(x,t) = u(x,0) + \int_0^s Nu^1(x,s)ds \\ u_{n+1}^2(x,t) = 2\int_0^t u_n^2(x,s)\, ds \end{cases} \tag{18}$$

$$Nu^1 = \frac{\partial}{\partial x}\left(u^1 \frac{\partial u^1}{\partial x}\right) + 3u^1 \frac{\partial u^1}{\partial x} - 2(u^1)^2$$

After all calculations, $Nu^1 = 0$.
Therefore:

$$\begin{cases} u_0^2(x,t) = u(x,0) \\ u_{n+1}^2(x,t) = 2\int_0^t u_n^2(x,s)\, ds \end{cases}$$

Recursively, way, the same result as is obtained as in step 1.
So:

$$u^1(x,t) = u^2(x,t) = u^3(x,t) = u^4(x,t) = \cdots = u^k(x,t)$$

Thus the exact solution of our problem is:

$$u(x,t) = \sum_{n=0}^{\infty} u_n^k(x,t) = 2\sqrt{e^x + e^{-4x}} \sum_{n=0}^{\infty} \frac{(2t)^n}{n!}$$

Either:

$$u(x,t) = \sum_{n=0}^{\infty} u_n^k(x,t) = 2\sqrt{e^x + e^{-4x}} \cdot e^{2t} \tag{19}$$

3.2. Example 2

Consider the biological population model in the form [15]:

$$u_t(x,y,t) = \frac{\partial^2 u^2(x,y,t)}{\partial x^2} + \frac{\partial^2 u^2(x,y,t)}{\partial y^2} - u(x,y,t)\left(1 + \frac{8}{9}u(x,y,t)\right) \tag{20}$$

With the initial condition

$$u(x,y,0) = exp\left(\frac{1}{3}(x+y)\right) \tag{21}$$

By applying all the different steps of the SBA method as presented and used in the previous example, there will be:

$$\frac{\partial u}{\partial t} = Nu - u \tag{22}$$

With

$$Nu = \frac{\partial^2 u^2}{\partial x^2} + \frac{\partial^2 u^2}{\partial y^2} - \frac{8}{9}u^2$$

The integration with respect to t of the equation (22) gives:

$$u(x,y,t) = u(x,y,0) - \int_0^t u(x,y,s)ds + \int_0^t (Nu)\, ds$$

From the use of the principle of successive approximations as in the Adomian Decomposition Method, follows:

$$u^k(x,y,t) = u(x,y,0) - \int_0^t u^k(x,y,s)\, ds + \int_0^t Nu^k\, ds \tag{23}$$

Since the solution is in the form of the following series

$$u^k(x,y,t) = \sum_{n=0}^{\infty} u_n^k(x,y,t)$$

The expression (23) becomes:

$$\sum_{n=0}^{\infty} u_n^k(x,y,t) = u(x,y,0) - \int_0^t u_n^k(x,y,s)\, ds + \int_0^t Nu^{k-1}\, ds \tag{24}$$

From the expression (24), the following SBA algorithm is derived

$$\begin{cases} u_0^k(x,y,t) = u(x,y,0) + \int_0^t Nu^{k-1}\, ds \\ u_{n+1}^k(x,y,t) = -\int_0^t u_n^k(x,y,s)\, ds \end{cases} \tag{25}$$

Step $k = 1$

$$\begin{cases} u_0^1(x,y,t) = u(x,y,0) + \int_0^t Nu^{0}\, ds \\ u_{n+1}^1(x,y,t) = -\int_0^t u_n^1(x,y,s)\, ds \end{cases} \tag{26}$$

Hypothesis: Picard's principle, $Nu^0 = 0$. Thus, we will have:

$$u_0^1(x,y,0) = u(x,y,0) = exp\left(\frac{1}{3}(x+y)\right)$$

$$u_{n+1}^k(x,y,t) = -\int_0^t u_n^k(x,y,s)\, ds$$

For $n = 0$

$$u_1^1(x,y,t) = -\int_0^t u_0^1(x,y,s)\, ds = -t.exp\left(\frac{1}{3}(x+y)\right)$$

For $n = 1$

$$u_2^1(x,y,t) = -\int_0^t u_1^1(x,y,s)\, ds = \frac{t^2}{2}\cdot exp\left(\frac{1}{3}(x+y)\right)$$

For $n = 2$

$$u_3^1(x,y,t) = -\int_0^t u_2^1(x,y,s)\, ds = -\frac{t^3}{3!}\cdot exp\left(\frac{1}{3}(x+y)\right)$$

For $n = 3$

$$u_4^1(x,y,t) = -\int_0^t u_3^1(x,y,s)\, ds = \frac{t^4}{4!}\cdot exp\left(\frac{1}{3}(x+y)\right)$$

By the same process the other terms are calculated; thus:

$$\sum_{n=0}^{\infty} u_n^1(x,y,t) = exp\left(\frac{1}{3}(x+y)\right)\sum_{n=0}^{\infty}\frac{(-t)^n}{n!} = exp\left(\frac{1}{3}(x+y)\right)exp(-t)$$

$$\sum_{n=0}^{\infty} u_n^1(x,y,t) = exp\left(\frac{1}{3}(x+y) - t\right) \tag{27}$$

For the step $k = 2$

$$\begin{cases} u_0^2(x,y,t) = u(x,y,0) + \int_0^t Nu^{1}\, ds \\ u_{n+1}^2(x,y,t) = -\int_0^t u_n^2(x,y,s)\, ds \end{cases} \tag{28}$$

$$Nu^1 = \frac{\partial^2 (u^1)^2}{\partial x^2} + \frac{\partial^2 (u^1)^2}{\partial y^2} - \frac{8}{9}(u^1)^2 = \frac{8}{9}(u^1)^2 - \frac{8}{9}(u^1)^2 = 0$$

Proceeding in the same way in this step 2 as in step 1, we find the same data obtained in step 1. Therefore, step by step, we get the same results. So, the exact solution is given by:

$$u^k(x,y,t) = \sum_{n=0}^{\infty} u_n^k(x,y,t)$$

The exact solution is obtained by passing to the following limit

$$u(x,y,t) = exp\left(\frac{1}{3}(x+y) - t\right)$$

3.3. Example 3

Consider the nonlinear coupled Burgers equations [3]:

$$\begin{cases} u_t = u_{xx} + 2uu_x - (uv)_x \ (a) \\ v_t = v_{xx} + 2vv_x - (uv)_x \ (b) \end{cases} \tag{29}$$

With the initial condition $u(x,0) = v(x,0) = \cos x$
The nonlinear terms are denoted for equation (29a) and equation (29b) respectively by:

$$\begin{cases} N_1(u,v) = 2uu_x - (uv)_x \\ N_2(u,v) = 2vv_x - (uv)_x \end{cases} \tag{30}$$

The system is then written:

$$\begin{cases} u_t = u_{xx} + N_1(u,v) \\ v_t = v_{xx} + N_2(u,v) \end{cases} \tag{31}$$

The integration with respect to t of equation (31) gives:

$$\begin{cases} u(x,t) = u(x,0) + \int_0^t \frac{\partial^2 u(x,s)}{\partial x^2} ds + \int_0^t N_1(u,v)\, ds \\ v(x,t) = v(x,0) + \int_0^t \frac{\partial^2 v(x,s)}{\partial x^2} ds + \int_0^t N_2(u,v)\, ds \end{cases} \tag{32}$$

By the same principle of the successive approximations, it follows:

$$u^k(x,t) = u(x,0) + \int_0^t \frac{\partial^2 u^k(x,s)}{\partial x^2} ds + \int_0^t N_1(u^{k-1}, v^{k-1})\, ds \tag{33}$$

$$v^k(x,t) = v(x,0) + \int_0^t \frac{\partial^2 v^k(x,s)}{\partial x^2} ds + \int_0^t N_2(u^{k-1}, v^{k-1})\, ds \tag{34}$$

The solutions of the equations (33) and (34) are respectively written as a series

$$u^k(x,t) = \sum_{n=0}^{\infty} u_n^k(x,t);\ v^k(x,t) = \sum_{n=0}^{\infty} v_n^k(x,t)$$

Thus, the following SBA algorithm follows

$$\begin{cases} u_0^k(x,t) = u(x,0) + \int_0^t N_1(u^{k-1}, v^{k-1})\, ds \\ u_{n+1}^k(x,t) = \int_0^t \frac{\partial^2 u_n^k(x,s)}{\partial x^2} ds \end{cases} \tag{35}$$

$$\begin{cases} v_0^k(x,t) = v(x,0) + \int_0^t N_2(u^{k-1}, v^{k-1})\, ds \\ v_{n+1}^k(x,t) = \int_0^t \frac{\partial^2 v(x,s)}{\partial x^2} ds \end{cases} \tag{36}$$

For $k = 1$

$$\begin{cases} u_0^1(x,t) = u(x,0) + \int_0^t N_1(u^0, v^0)\, ds \\ u_{n+1}^1(x,t) = \int_0^t \frac{\partial^2 u_n^1(x,s)}{\partial x^2} ds \end{cases} \tag{37}$$

$$\begin{cases} v_0^1(x,t) = v(x,0) + \int_0^t N_2(u^0, v^0)\, ds \\ v_{n+1}^1(x,t) = \int_0^t \frac{\partial^2 v_n^1(x,s)}{\partial x^2} ds \end{cases} \tag{38}$$

Suppose that there are u^0 and $v^0 \in H\ (Hilbert\ space)$, such that: $N_1(u^0, v^0) = N_2(u^0, v^0) = 0$; thus

$$\begin{cases} u_0^1(x,t) = u(x,0) = cosx \\ v_0^1(x,t) = v(x,0) = cosx \end{cases}$$

$n = 0$

$$\begin{cases} u_1^1(x,t) = \int_0^t \frac{\partial^2 u_0^1(x,s)}{\partial x^2} ds \ = -tcosx \\ v_1^1(x,t) = \int_0^t \frac{\partial^2 v_0^1(x,s)}{\partial x^2} ds \ = -tcosx \end{cases}$$

$n = 1$

$$\begin{cases} u_2^1(x,t) = \int_0^t \frac{\partial^2 u_1^1(x,s)}{\partial x^2} ds = \frac{t^2}{2} cosx \\ v_2^1(x,t) = \int_0^t \frac{\partial^2 v_1^1(x,s)}{\partial x^2} ds = \frac{t^2}{2} cosx \end{cases}$$

$n = 2$

$$\begin{cases} u_3^1(x,t) = \int_0^t \frac{\partial^2 u_2^1(x,s)}{\partial x^2} ds = \frac{(-t)^3}{3!} cosx \\ v_3^1(x,t) = \int_0^t \frac{\partial^2 v_2^1(x,s)}{\partial x^2} ds = \frac{(-t)^3}{3!} cosx \end{cases}$$

$$\begin{cases} \sum_{n=0}^{\infty} u_n^1(x,t) = (cosx)\left[1 + (-t) + \frac{(-t)^2}{2!} + \frac{(-t)^3}{3!} + \cdots\right] \\ \sum_{n=0}^{\infty} u_n^1(x,t) = (cosx)\left[1 + (-t) + \frac{(-t)^2}{2!} + \frac{(-t)^3}{3!} + \cdots\right] \end{cases}$$

Either

$$\begin{cases} \sum_{n=0}^{\infty} u_n^1(x,t) = (cosx)e^{-t} \\ \sum_{n=0}^{\infty} u_n^1(x,t) = (cosx)e^{-t} \end{cases} \quad (39)$$

For $k = 2$

$$N_1(u^1, v^1) = 2u^1 u_x^1 - (u^1 v^1)_x$$

$$N_1(u^1, v^1) = -2[(cosx)(sinx)]e^{-t} + 2[(cosx)(sinx)]e^{-t}$$

Hence: $N_1(u^1, v^1) = 0$
Likewise, $N_2(u^1, v^1) = 0$. Therefore

$$u^k(x,t) = v^k(x,t) = \sum_{n=0}^{\infty} u_n^k(x,t) = \sum_{n=0}^{\infty} v_n^k(x,t)$$

The exact solution of the system of coupled Burgers equations is:

$$\begin{cases} u(x,t) = (cosx)e^{-t} \\ v(x,t) = (cosx)e^{-t} \end{cases}$$

4. Conclusion

In our work, we considered three nonlinear problems. Our objective was the exact solution of each problem, to verify the effectiveness of the SBA method for nonlinear problems. We obtained with ease all the expected exact solutions. These specifically chosen examples allowed us to demonstrate the effectiveness of the method. Moreover, the calculations are simple, clear for all its classical different steps. We can affirm that the SBA method is efficient and appropriate for nonlinear problems.

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