Pure and Applied Mathematics Journal
2021; 10(5): 107-120
http://www.sciencepublishinggroup.com/j/pamj
doi: 10.11648/j.pamj.20211005.12
ISSN: 2326-9790 (Print); ISSN: 2326-9812 (Online)

Approximation Forms of Soft Subgraph and Its Application on the Cardiovascular System

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To cite this article:
Magdy Salah El-Azab, Mohamed Shokry, Reham Emad Aly. Approximation Forms of Soft Subgraph and Its Application on the Cardiovascular System. *Pure and Applied Mathematics Journal*. Vol. 10, No. 5, 2021, pp. 107-120. doi: 10.11648/j.pamj.20211005.12

Received: September 13, 2021; **Accepted**: October 11, 2021; **Published**: November 10, 2021

Abstract: The incorporation of many various and modern mathematical tools provides efficient and successful methods to model problems with uncertainty such as graph theory, Fuzzy sets, rough sets and soft sets. This powerful incorporation of the three different concepts rough sets, soft sets and graphs is known as soft rough graphs that is introduced previously by Noor et al. The aim of this paper is to propose new concepts of linking soft set, rough set and graph theory in order to create new types of sub-graphs according to properties on the original graph by using generalized relationship through the out-link vertices or directed cycle. Our approach is based on introducing new structure for the roughness of the soft graphs by defining new types of operators by using closed paths. Then, it applies all of these concepts to the cardiovascular system in the human body in order to explain some phenomena and medical facts in a mathematical style. Finally, it discusses the comparison properties and containment relationships between various kinds of new approximation soft subgraph.

Keywords: Graph Theory, Soft Set, Soft Graph Set, Rough Set Theory, The Cardiovascular System

1. Introduction

The traditional and the outdated methods are not always absolutely influential and successful to solve many problems that diverse the uncertainty data in different fields such as engineering, economics, computers sciences, physics and social sciences. so, these cause the crisp and deterministic of the reasoning, modeling and computing. In order to tackle these uncertainties and variabilities that cannot be solved within a traditional mathematical structure. Many concepts and theories have been introduced such as fuzzy set theory, soft set theory, rough set theory, graph theory and etc.

In 1999s, Molodtsov [2] introduced a new concept of soft sets as a mathematical tool for dealing uncertainty of features. This new concept helps many working researchers in many fields of research which became more popular to use it because it has enough parameters like game theory, theories of probability, Integration, theories of measurement theory and others. Then, many models and applications were generated to solve problems in decision making and medical diagnosis.

In 1980s. Zdzislaw Pawlak [3, 31] introduced the outstanding mathematical concept that depend on the classification of uncertain or incomplete information without involving any additional information about the data was known as rough sets. The rough sets theory is based on using an equivalence relation (transitive - reflective - symmetric) that depends on dividing the data into classes with no intersections between them, but through this division he was able to find the equivalence classes that fall entirely within a definite set is the lower approximation of the set and the upper approximation of the set.

There are many advantages to the experts and researchers to utilize the concepts of the rough set -based analysis methods in their issues and researches in many and various fields [21] because these concepts depend on defining the major and fundamental attributes, extracting the features, minimizing data sets, creating a decision base and recognizing patterns. The concepts of rough set have been successfully applied on different branches such as bioinformatics, machine learning [26], economics and finance, signal and image processing, software engineering, robotics, power systems, logic [22, 23], decision making [24, 25], control engineering, data mining [15]. It is also utilized in the medical field [16], the concept of approximate sets addresses the uncertainties in medical images by dividing the image into parts based on the degree of color, of which the upper and lower approximations can be determined for any part of the image by observing tasks such as identifying features, reducing dimensions, and classifying the pattern and the image.

In 1736s, Euler [4] known as the father of graph theory who settled the problem of Konigsberg Bridge. he introduced the concepts of graph theory [11]. It is a prosperous discipline that contains a significant and enormous increase of powerful and massive theorems of wide applicability for representing the data by means of diagrams, matrices or relations in many extensive and popular branches of many fields like computer science, chemistry, science, statistical physics [28], computer science and medicine [12, 13, 27, 30, 32].

In 2011, Feng et al [5] introduced new concept is called soft rough sets which is considered as a combination of soft sets and rough sets. . There are many applications, presented by many researchers and experts in data labeling problems, information systems, knowledge-based systems, forecasting, data mining, reduction problems and description logic [6, 8, 33, 34].

In 2015, Akram et al [9] introduced new concept is known as the soft graphs and also, describing the basic operations on soft graph that based on processing the multi-attributes problems related with graph theory. Then, many generalizations of soft graphs are presented [10, 14]

In this paper, the circulatory system and its components was represented as a graph. From the resultant graph, new concepts of soft graph can be studied which consists of vital organs in the human body and the extent of their influence in the human body.

The importance of the paper problem lies in linking the circulatory system to a mathematical model that relies on the graph theory, the soft group and the approximate theory, in order to make approximate spaces to know the extent of the effect of organs on the human body through the mathematical model.

In section 2, some basic concepts are revised. Section 3, is aimed to present the new concept of cycled soft subgraph (ƇȘ - subgraph) which is strengthened with medical model by representing the circulatory system as a graph and interpreting each one of them from the standpoint of mathematics and medicine. Section 4, is about how to set the lower and upper approximations of the vertices and edges. As well as studying the properties of containments, intersections, and unions on the resulting approximations. Section 5, is concerned about some operations on rough soft graph by presenting examples to demonstrate the new concept of approximation soft graph.

2. Preliminaries

In this section, some pertinent definitions and concepts of graph theory, rough theory and soft set theory will be reviewed and will beneficial for the paper.

Definition 2.1 [17, 29] A graph G is an ordered pair (V, E), where V is a finite set and $E \subseteq \binom{V}{2}$ is a set of pairs of elements in V.

1) The set V is called the set of vertices and E is called the set of edges of G.
2) The edge e = {u,v} $\in \binom{V}{2}$ is also denoted by e = uv.
3) If e = uv ∈ E is an edge of G, then u is called adjacent to v and u is called incident to e.
4) If e_1 and e_2 are two edges of G, then e_1 and e_2 are called adjacent if $e_1 \cap e_2 \neq \Phi$, i.e., the two edges are incident to the same vertex in G.

Definition 2.2 [18] A graph G. A Walk in G is a non-empty alternating sequence $W = v_0 e_0\ v_1\ e_1 \cdots e_{k-1}\ v_k$ of vertices and edges in G such that $e_i = \{v_i, v_{i+1}\}$ for all i< k. If $v_0 = v_k$, the walk is closed. If the vertices in a walk are all distinct, it defines an obvious path in G. In general, every walk between two vertices contains a path between these vertices. A path is an open walk with no repeated vertices. A closed walk with no repeated vertices except at the initial and terminal vertices is called a cycle.

Definition 2.3 [19] A subgraph of a graph G is a graph whose vertex and edge sets are subsets of those of G. If v ⊆ V and e ⊆ E, then G' is a subgraph of G (and G a super graph of G'), written as G ' ⊆ G. An induced subgraph $G' = (V', E')$ consists of subset of vertices $V' \subseteq V$ and subset of edge $E' = \{xy \in E, x, y \in V'\}$ is denoted by G' and written as $G' \subseteq G$.

Definition 2.4 [20] An ordered pair (U, ℘) is called a soft universe where U is an initial universe set and ℘ be the universe of all possible parameters in U. Let (S, A) is a pair of a soft set over U where A is a subset of Ƥ and S is a set valued of mapping given by S: A → ℘ (u) and the power set of U is ℘ (u)

Definition 2.5 [5] Let U be the universe of discourse and assuming Ɍ be an equivalence relation Ɍ ⊆ U × U. Let Ș = (S, A) be a soft set over U. Then, the pair A = (U, Ș) is called soft approximation space.

$Ɍ_L(X) = \{u \in U: \exists\ \alpha \in A, [u \in S(\alpha)_Ɍ \subseteq X\}$ is called the soft lower approximation of X

$Ɍ^U(X) = \{u \in U: \exists\ \alpha \in A, :, [u \in S(\alpha)_Ɍ, S(\alpha)_Ɍ \cap X \neq \emptyset\}$ is called the soft upper approximation of X.

3. Medical Application of Cycled Soft Sub-Graph on the Cardiovascular System of Human Body

Authors in [9] were able to combine the concept of soft set and concepts of graph theory. they have introduced certain types of soft graphs. Also, they studied some concepts of connectedness and some of the deletion properties of the vertices and edges that caused the dis-connectivity of the graph.

In this section, a new concept is generalized by using a general relationship to obtain a soft graph through the out-link vertices or directed cycle called cycled soft subgraph is denoted by $\mathfrak{CS}$-subgraph. It is strengthened the new concept of cycled soft subgraph with vital and medical phenomena by representing the circulation of human body as a graph and interpreting each one of them from the standpoint of mathematics and medicine.

3.1. New Concept of Cycled Soft Sub-Graph

The aim of this section is to introduce the new notion of cycled soft subgraph that based on the properties of soft set.

Let U be the universe of discourse and $\wp$ be the universe of all possible parameters related to the objects in U. Parameters are considered to be attributes, characteristics or diagnoses or functions of objects in U. The power set of U is denoted by $\wp$ (U). Let A be a set of parameters that can have an arbitrary nature (human organs, symptoms of viruses, doses of treatment, types of diseases, etc.).

*Definition 3.*1 Let A pair ($\mathfrak{G}$, A) is called soft set over U, where A $\subseteq \wp$ and $\mathfrak{G}$ is a set valued mapping, $\mathfrak{G}$: A $\to \wp$ (U), The ordered pair (U, $\wp$) is known as soft universe, Where:

($\mathfrak{G}$, A) be a soft set over V with its approximation function $\mathfrak{G}$: A $\to \wp$ (V) is defined as

$\mathfrak{G}(X) = \{\, y \in V \mid xRy \iff xCy \,\}$, xCy where x and y on same closed cycle, for all x $\in A$

And

($\mathfrak{Y}$, A) be a soft set over E with its approximation function $\mathfrak{Y}$: A$\to$ $\wp(E)$ is defined as

$$\mathfrak{Y}(X) = \{uv \in E \mid u, v \subseteq \mathfrak{G}(X)\} \text{ For all } x \in A$$

Definition 3.2 A cycled soft subgraph $\mathbb{H}$(x) is said to be $\mathfrak{CS}$ – vertex induced if

$$\mathbb{H}(x)= (\mathfrak{G}(x), \mathfrak{Y}(x)) = (\mathfrak{G}(x))$$

Definition 3.3 A cycled soft graph $\mathbb{H}$(x) is said to be $\mathfrak{CS}$ – edge induced if

$$\mathbb{H}(x)= (\mathfrak{G}(x), \mathfrak{Y}(x)) = (\mathfrak{Y}(x))$$

Definition 3.4 A graph $G^* = (G, \mathfrak{G}, \mathfrak{Y}, A)$ is called cycled soft graph where $\mathfrak{G}$: A $\to \wp$ (V) be a map such that $\mathfrak{G}(X) = \{\, y \in V \mid xRy \iff xCy \,\}$ and $\mathfrak{Y}$: A$\to$ $\wp(E)$ be a map such that $\mathfrak{Y}(X) = \{uv \in E \mid u, v \subseteq \mathfrak{G}(X)\}$ For all x $\in A$ if it satisfies the following conditions:

1) G = (V;E) is a simple crisp graph.
2) A be a non-empty set of parameters.
3) $(\mathfrak{G}, A)$ is a soft set over V.
4) $(\mathfrak{Y}, A)$ is a soft set over E, defined as $\mathfrak{Y}$ (x) = { $e_{ij} \in$ E, $e_{ij} \subseteq \mathfrak{G}(x)$}.
5) $\mathbb{H}$(x)= ($\mathfrak{G}$ (x), $\mathfrak{Y}$ (x)) is a cycled soft subgraph of G for all x $\in sub$A.

Where G^*={ $\mathbb{H}$(x): x$\in$ A}

Now, we will explain in the next section the new concept of cycled soft subgraph with an example. However, we cannot find a better example than the circulation of the human body to apply on it.

3.2. Representing the Cardiovascular System by Soft Graph

The aim of this section is to link graph theory with medical application by representing the cardiovascular system of human body as a graph to apply all of above on it to know the benefit of studying and interpreting each one of them from the standpoint of mathematics and medicine.

Section 3.2.1 gives a simple survey about the cardiovascular system of the human body and its function. Also, it is mentioned the systemic circulation and the pulmonary circulation in the cardiovascular system.

The purpose of section 3.2.2 is to represent the cardiovascular system as a graph which is classified into set of vertices that represent the organs of human body which receive and send the blood through them. The edges represent the pathway of blood through the cardiovascular system.

3.2.1. The Cardiovascular System of Human Body

The cardiovascular system is the most complicated part of the body that is absolutely critical to its health and longevity. It is responsible for delivering nutrients, hormones, oxygen and cellular waste to all cells through the entire body. The heart, lungs, blood vessels and blood make up the entirety of the cardiovascular system.

The blood vessels form a vast network to help keep the flow of the blood in single direction. This network classified into three main types: the arteries that carry blood away from the heart, the veins that drains the blood back to the heart and finally, the capillaries that linked the arteries to the veins that permit oxygen, nutrients and waste products to be exchanged between tissues and the blood.

The blood vessels of the body are functionally divided into two major distinctive circulation:

1) The pulmonary circulation is the portion where the fresh oxygen we breathe in enters the blood. This route of circulation goes from the right ventricle of the heart into the right and left pulmonary arteries, which go to the lungs. In the lung, deoxygenated blood that contains carbon dioxide will be expelled from the body and the blood will pick up the oxygen needed by the cells of the body to return it to the left atrium of the heart via the four pulmonary veins.
2) The systemic circulation refers to the functional blood supply which includes all of the oxygenated blood that leaves the left ventricle of the heart through the aortic semilunar valve and goes to the aorta to provide all the organs, tissues and cells of the body with the blood and other vital substances.

The circulatory system of the human body is extremely important in sustaining life. Its proper functioning is responsible for carrying the oxygen and nutrients to the cells for cellular respiration, carries wastes away from cells with lymph and excretory system, carries immune cells to infection sites, distributes heat, maintains levels of body fluids with kidneys, as well as maintenance of optimum pH, and the mobility of the elements, proteins and cells, of the immune system.

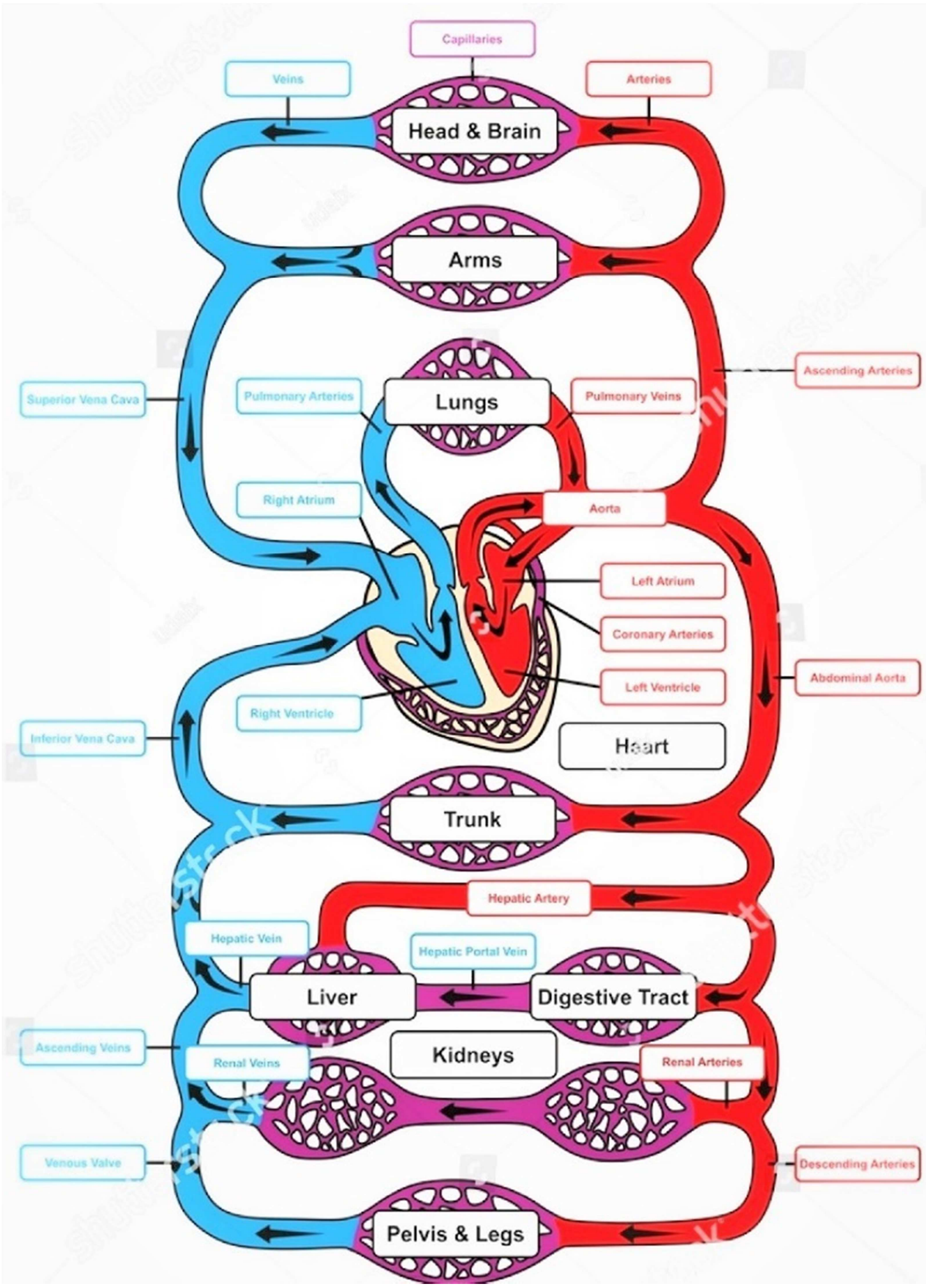

Figure 1. *Blood circulation of human body.*

3.2.2. Representing the Cardiovascular System by Graph:

Now, through the medical application, the cardiovascular system can be classified into set of vertices and set of edges. The vertices represent the organs of human body which receive and send the blood through them. The edges represent the pathway of blood through the cardiovascular system as shown as in Figure 2.

The vertices are v_1 is Superior vena cava, v_2 is Inferior Vena Cava, v_3 is R. Atrium, v_4 is R. Ventricle, v_5 is Pulmonary Arteries, v_6 is Right Lung, v_7 is Left Lung, v_8 is Pulmonary Veins, v_9 is L. Atrium, v_{10} is L. Ventricle, v_{11} is Aorta, v_{12} is Arms, v_{13} is Brain, v_{14} is Liver, v_{15} is Stomach and Intestines, v_{16} is Kidney and finally, v_{17} is Genitals and Legs.

The edges are e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23} that represent channels of blood through the human body.

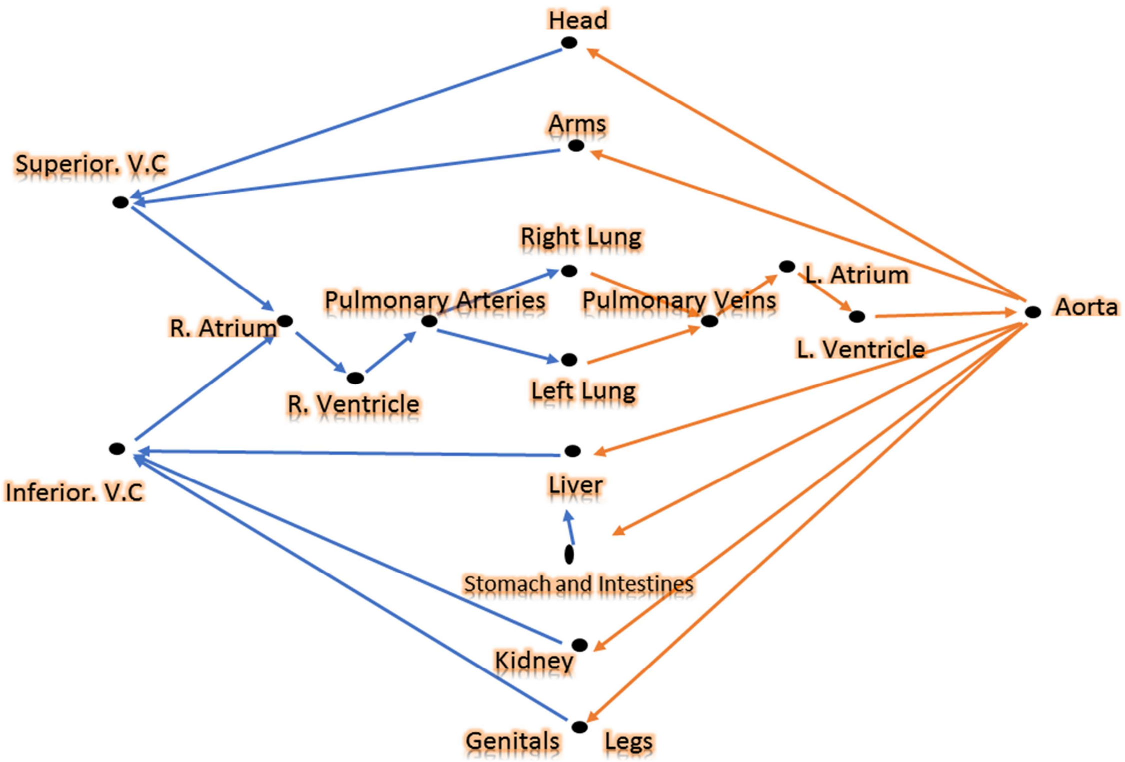

Figure 2. *Representing the cardiovascular system.*

Example 3.1 Consider a crisp simple connected graph $G = (V, E)$ as shown in Figure 2

Firstly, as shown in Figure 2, the circulatory system has been represented as set of vertices and set of edges. So, it's easy to apply the new definitions and properties on it.

Secondly, Let A= $\{v_{12}, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}\} \subseteq V$ be a parameter set and (S, A) be a soft set with its approximation function S: A→ $\wp(V)$ is defined as

$\mathfrak{S}(X) = \{\, y \in V \mid xRy \ \Leftrightarrow xCy \,\}$, xCy where x and y on same closed cycle. For all x ∈ A and ($\mathfrak{Y}$, A) be a soft set over E with its approximation function $\mathfrak{Y}$: A→ $\wp(E)$ is defined as

$\mathfrak{Y}(X) = \{uv \in E \mid u, v \subseteq \mathfrak{S}(X)\}$, For all x ∈ A.

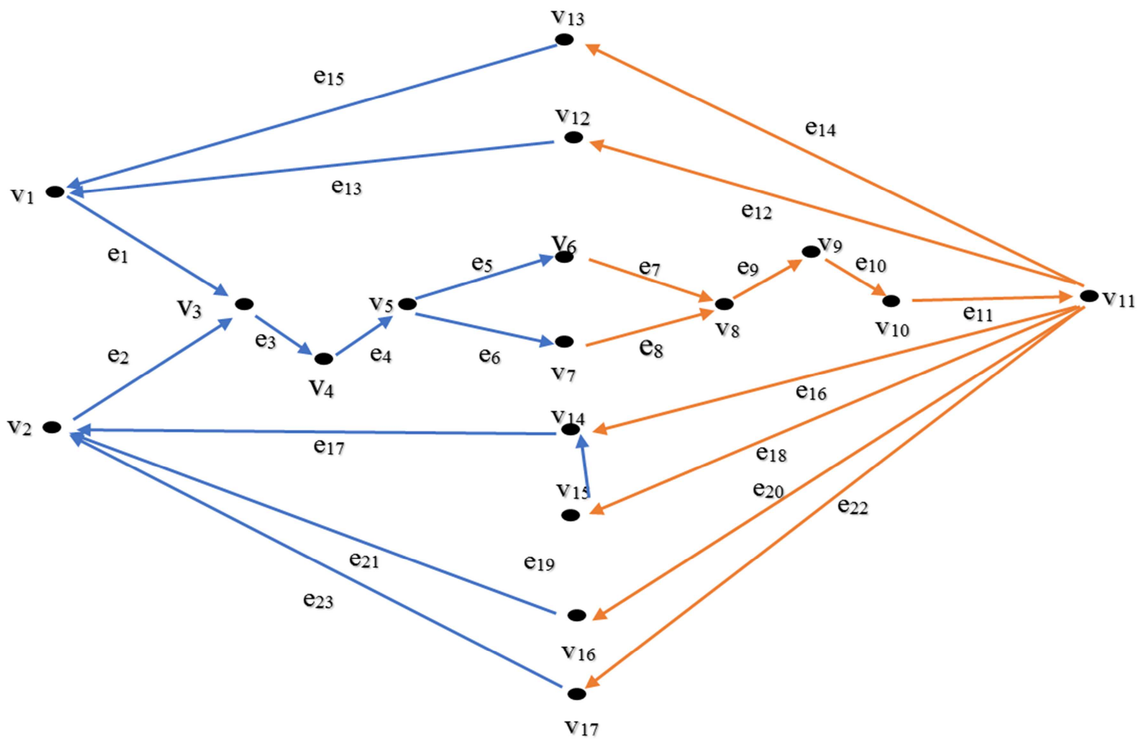

Figure 3. *The Cardiovascular system as a graph.*

Then, $\mathfrak{G}$ (v_{12}) = {$v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12}$}
$\mathfrak{G}$ (v_{13}) = {$v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{13}$},
$\mathfrak{G}$ (v_{14}) = {$v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14}$},
$\mathfrak{G}$ (v_{15}) = {$v_{14},v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{15}$},
$\mathfrak{G}$ ($_{16}$) = {$v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{16}$},
$\mathfrak{G}$ (v_{17}) = {$v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{17}$}
And,

$\mathfrak{Y}$ (v_{12}) = {$e_{13},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12}$},
$\mathfrak{Y}$(v_{13}) ={$e_{15},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{14}$},
$\mathfrak{Y}$(v_{14})={$e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16}$},
$\mathfrak{Y}$(v_{15})={$e_{19},e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{18}$}
$\mathfrak{Y}$(v_{16}) = {$e_{21},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{20}$},
$\mathfrak{Y}$ (V_{17}) = {$e_{23},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{22}$ }

Table 1. *Tabular representation of a soft set.*

A\V	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}	v_{17}
v_{12}	1	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
v_{13}	1	0	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0
v_{14}	0	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0
v_{15}	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0
v_{16}	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	0
v_{17}	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	1

A\E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
v_{12}	1	0	1	1	1	1	1	1	1	1	1	1
v_{13}	1	0	1	1	1	1	1	1	1	1	1	0
v_{14}	0	1	1	1	1	1	1	1	1	1	1	0
v_{15}	0	1	1	1	1	1	1	1	1	1	1	0
v_{16}	0	1	1	1	1	1	1	1	1	1	1	0
v_{17}	0	1	1	1	1	1	1	1	1	1	1	0

A\E	e_{13}	e_{14}	e_{15}	e_{16}	e_{17}	e_{18}	e_{19}	e_{20}	e_{21}	e_{22}	e_{23}
v_{12}	1	0	0	0	0	0	0	0	0	0	0
v_{13}	0	1	1	0	0	0	0	0	0	0	0
v_{14}	0	0	0	1	1	0	0	0	0	0	0
v_{15}	0	0	0	0	1	1	1	0	0	0	0
v_{16}	0	0	0	0	0	0	0	1	1	0	0
v_{17}	0	0	0	0	0	0	0	0	0	1	1

Thus, $\mathbb{H}(v_{12}) = (\mathfrak{G}(v_{12}), \mathfrak{Y}(v_{12}))$, $\mathbb{H}(v_{13}) = (\mathfrak{G}(v_{13}), \mathfrak{Y}(v_{13}))$, $\mathbb{H}$ (v_{14}) = ($\mathfrak{G}(v_{14}),\mathfrak{Y}(v_{14})$), $\mathbb{H}(v_{15})$ = ($\mathfrak{G}$ (v_{15}), $\mathfrak{Y}$ (v_{15})), $\mathbb{H}$ (v_{16}) = ($\mathfrak{G}$ (v_{16}), $\mathfrak{Y}$ (v_{16})) and $\mathbb{H}$ (v_{17}) = ($\mathfrak{G}$ (v_{17}), $\mathfrak{Y}$ (v_{17})) are subgraphs of G as shown in Figure 4. Hence G^* = { $\mathbb{H}$ (v_{12}), $\mathbb{H}$ (v_{13}), $\mathbb{H}$ (v_{14}), $\mathbb{H}$ (v_{15}), $\mathbb{H}$ (v_{16}), $\mathbb{H}$ (v_{17})} is a soft graph of G

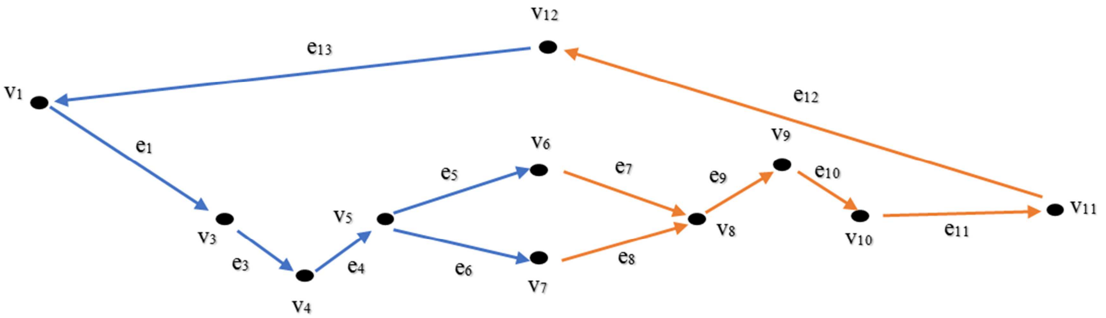

Figure 4. $\mathbb{H}$ *(v_{12}) is corresponding to vertex v_{12}.*

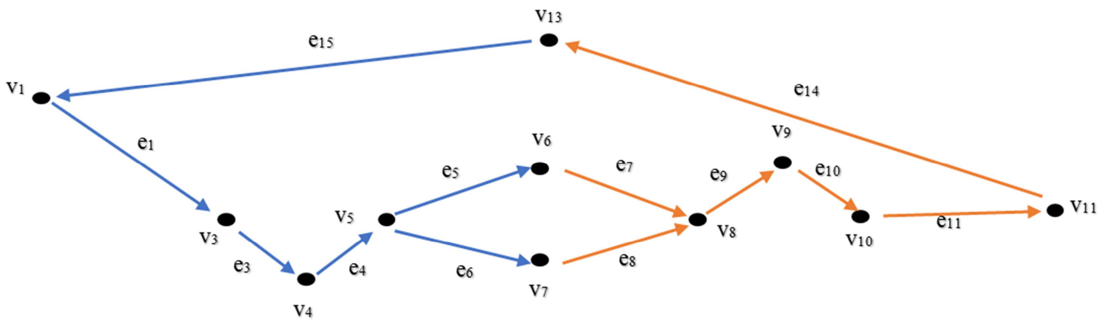

Figure 5. $\mathbb{H}$ *(v_{13}) is corresponding to vertex v_{13}.*

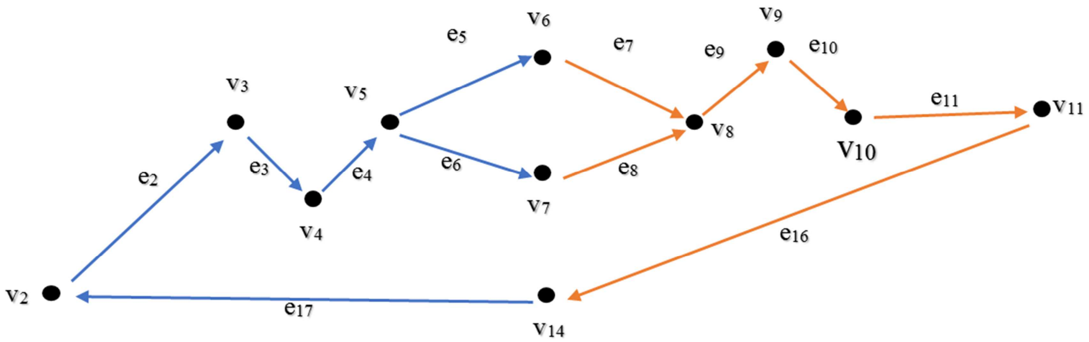

Figure 6. *$\mathbb{H}$ (v_{14}) is corresponding to vertex v_{14}.*

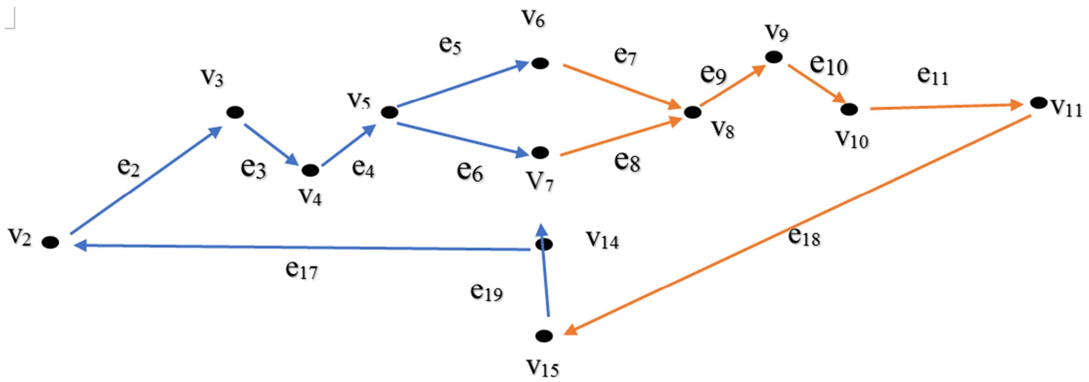

Figure 7. *$\mathbb{H}$ (v_{15}) is corresponding to vertex v_{15}.*

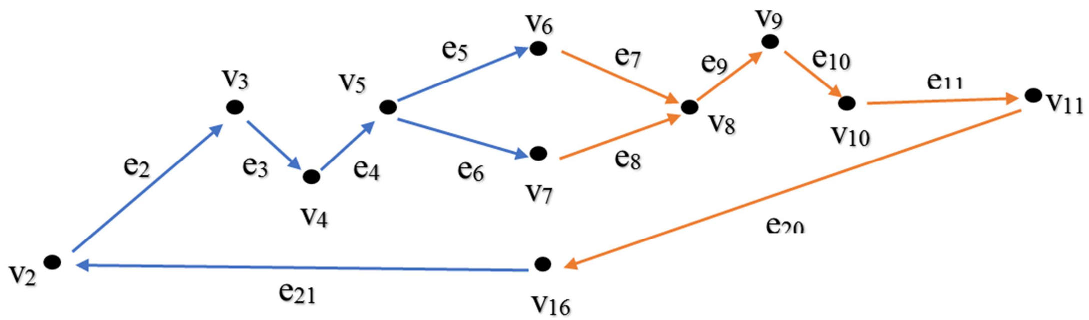

Figure 8. *$\mathbb{H}$ (v_{16}) is corresponding to vertex v_{16}.*

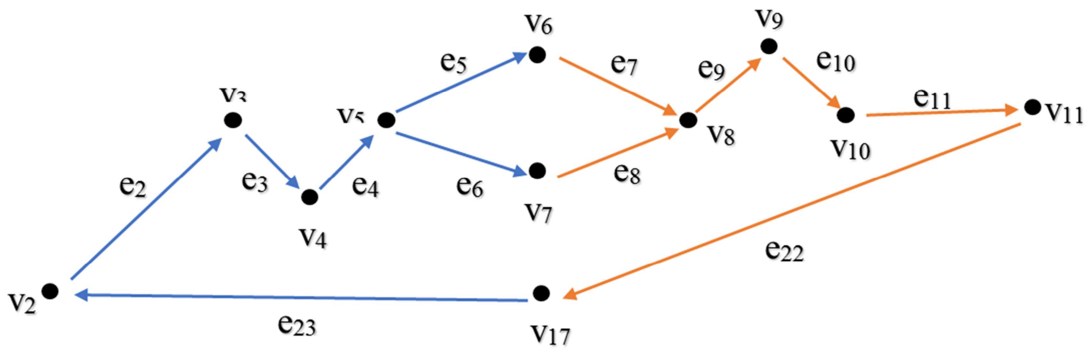

Figure 9. *$\mathbb{H}$ (v_{17}) is corresponding to vertex v_{17}.*

In the previous example. Mathematically, each soft subgraph is deduced from the new definition to generate the soft graph that refers to a closed directed cycle as shown in the previous figures. Medically, each soft graph refers to a complete blood circulation of each human organ mentioned in the example by receiving the oxygenated blood from the heart and returning it back as non-oxidized blood after each organ after each organ has finished performing its function, for examples:

In Figure 5, The resultant cycled soft subgraph is called $\mathbb{H}$ (v_{13}) = ($\mathfrak{G}$ (v_{13}), $\mathfrak{Y}$ (v_{13})) indicates Cerebral circulation, it is responsible for the movement of blood that contain oxygen and glucose through a network of cerebral arteries and veins supplying the brain which needs about 15 percent of your heart's cardiac output to get the oxygen and glucose it needs to stay healthy. Then the deoxygenated blood is carried by cerebral veins to the heart to remove carbon dioxide, lactic acid, and other metabolic products.

In Figure 7, The resultant cycled soft subgraph is called $\mathbb{H}$ (v_{15}) = ($\mathfrak{G}$ (v_{15}), $\mathfrak{Y}$ (v_{15})) indicates the hepatic portal circulation, it is a series of veins that deliver de-oxygenated blood from the capillaries of Gastrointestinal tract which consists of the stomach, intestine, spleen, and pancreas to capillaries in the liver to be detoxified further before it returns to the heart to complete its systemic circulation and pulmonary circulation.

In Figure 8, The resultant cycled soft subgraph is called $\mathbb{H}$ (v_{16}) = ($\mathfrak{G}$ (v_{16}), $\mathfrak{Y}$ (v_{16})) indicates the renal circulation on the human body, it is responsible for suppling the blood flow which is rich with oxygen to the kidneys to perform its function such as regulation of total blood volume and vascular tone, produce hormone that stimulates red blood cell production and other hormones produced to help regulate blood pressure of the human body and remove waste products and excess fluid from the body through the urine. Then backing the deoxygenated blood and other substances again to the heart to complete its cardiovascular circulation.

After illustrating some examples from the point of view of mathematical and medical phenomena as explained above, the induced soft subgraph is a subgraph that defined group of vertices to create set of parameters provided that they are located in the closed directed cycle.

4. New Result on Rough Soft Graph

The concept of soft rough graphs introduced in [1] which based on equivalence classes, that contain parameterized subsets of vertices and edges that serve the aim of finding the lower and upper approximations.

In our present section, we will present how to set the lower and upper approximations of the vertices and edges of a part of a graph using a general relationship, namely that the vertices lie on a closed path and through the set of parameters which is a property that links a set of vertices to each other. Sets can be formed from vertices or edges for use in finding lower approximations and upper approximations, as well as studying the properties of containments, intersections, and unions on the resulting approximations.

Definition 4.1 Let G = (V, E) be crisp simple connected graph, ($\mathfrak{G}$, A) be a set over V and ($\mathfrak{Y}$, A) be a set over E defined as $\mathfrak{Y}$ (x) = { $e_{ij} \in$ E, $e_{ij} \subseteq \mathfrak{G}(x)$}.

The Lower and upper rough soft approximation of vertex are defined as:

$$\mathfrak{G}_L\ (\mathrm{V}_1) = \cap\,\{\mathfrak{G}(v_i) \colon v_i \in \mathrm{V}_1, \mathfrak{G}(v_i) \subseteq \mathrm{V}_1, \mathrm{G}_1\ \subseteq \mathrm{G}\}$$

$$\mathfrak{G}^u\ (\mathrm{V}_1) = \cup\,\{\mathfrak{G}(v_i) \colon v_i \in\ \mathrm{V}_1, \mathfrak{G}(v_i) \cap \mathrm{V}_1 \neq \emptyset, , \mathrm{G}_1\ \subseteq \mathrm{G}\}$$

where $\mathrm{G}_1\ = (\mathrm{V}_1, \mathrm{E}_1)$.

The Lower and upper rough soft approximation of edge are defined as:

$$\mathfrak{Y}_L\ (\mathrm{E}_1) = \cap\,\{\ e_{ij} \colon e_{ij} \in \mathrm{E}_1, \mathfrak{Y}(e_{ij}) \subseteq \mathrm{E}_1, \mathrm{G}_1\ \subseteq \mathrm{G}\}$$

$$\mathfrak{Y}^u(\mathrm{E}_1) = \cup\,\{\ e_{ij} \colon e_{ij} \in\ \mathrm{E}_1, \mathfrak{Y}(e_{ij}) \cap \mathrm{E}_1 \neq \emptyset, \mathrm{G}_1\ \subseteq \mathrm{G}\}$$

Definition 4.2 A rough soft graph $\mathrm{G}^* = \langle \mathfrak{G}_L, \mathfrak{G}^u, \mathfrak{Y}_L, \mathfrak{Y}^u, \mathrm{A} \rangle$ is said to be

Lower vertex induced by subgraph if

$\mathbb{H}_L(\mathrm{V}_1) = (\mathfrak{G}_L(\mathrm{V}_1), \mathfrak{Y}_L\ (\mathrm{E}_1)) = \langle\ \mathfrak{G}_L(\mathrm{V}_1)\ \rangle$ for all $\mathrm{V}_1 \in A$.

Upper vertex induced by subgraph if

$\mathbb{H}^u(\mathrm{V}_1) = (\mathfrak{G}^u(\mathrm{V}_1), \mathfrak{Y}^u(\mathrm{E}_1)) = \langle \mathfrak{G}^u(\mathrm{V}_1) \rangle$ for all $\mathrm{V}_1 \in A$.

Lower edge induced by subgraph if

$\mathbb{H}_L(\mathrm{E}_1) = (\mathfrak{G}_L(\mathrm{V}_1), \mathfrak{Y}_L\ (\mathrm{E}_1)) = \langle \mathfrak{Y}_L\ (\mathrm{E}_1)\ \rangle$ for all $\mathrm{E}_1 \in A$.

Upper edge induced by subgraph if

$\mathbb{H}^u(\mathrm{E}_1) = (\mathfrak{G}^u(\mathrm{V}_1), \mathfrak{Y}^u(\mathrm{E}_1)) = \langle\ \mathfrak{Y}^u(\mathrm{E}_1) \rangle$ for all $\mathrm{E}_1 \in A$.

Definition 4.3 A Graph $\mathrm{G}^* = (\mathrm{G}, \mathfrak{G}_L, \mathfrak{G}^u, \mathfrak{Y}_L, \mathfrak{Y}^u, \mathrm{A})$ is said to be rough soft graph if it satisfies:

1) G = (V, E) is a simple graph.
2) A be a non-empty set of parameters.
3) $(\mathfrak{G}_L, \mathfrak{G}^u, \mathrm{A})$ be a rough soft set over V.
4) $(\mathfrak{Y}_L, \mathfrak{Y}^u, \mathrm{A})$ be a rough soft set over E.
5) $\mathbb{H}_L = (\mathfrak{G}_L, \mathfrak{Y}_L)$ and $\mathbb{H}^u = (\mathfrak{G}^u, \mathfrak{Y}^u)$ are subgraphs of G for all $\mathrm{G}_1 \in$ A.

The set of all rough soft graphs of G is denoted by $\mathfrak{R}\mathfrak{S}\mathrm{G}^*(\mathrm{G})$ that can represent by

$$\mathrm{G}^* = (\mathfrak{G}_\mathrm{L}, \mathfrak{G}^\mathrm{u}, \mathfrak{Y}_\mathrm{L}, \mathfrak{Y}^\mathrm{u}, \mathrm{A}) = \{\mathbb{H}_\mathrm{L}(\mathrm{x}), \mathbb{H}^\mathrm{u}(\mathrm{x})\}$$

In the following example, the blood circulation is transformed into set of vertices and set of edges where the vertices represent the organs of human body which receive and send the blood flow through them and the edges represent the pathway of blood through the cardiovascular system as shown in Figure 3 and by specifying a part to focus on for medical purposes to find the lower and upper approximation for it.

Example 4.1: Consider crisp simple connected graph $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ as shown in Figure 3. Let G_1 be a subgraph with $\mathrm{V}(\mathrm{G}_1)$= {v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13}} and E (G_1)= {e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}}.

Let A = {v_{12},v_{13},v_{14},v_{15},v_{16},v_{17}} be a set of parameters. We define an approximate function: A $\rightarrow \wp$ (V) such that $\mathfrak{G}(X) = \{\ y \in V \mid\ xRy\ \Leftrightarrow xCy\ \}$ and $\mathfrak{Y}(X) = \{uv \in E \mid\ u, v\ \subseteq\ \mathfrak{G}(X)\ \}$

Then,$\mathfrak{G}$(v_{12})={v_{12},v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9, v_{10},v_{11}},

$\mathfrak{G}$ (v_{13}) = {v_{13}, v_1,v_3, v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}},

$\mathfrak{G}$ (v_{14}) = {v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14} },

$\mathfrak{G}$ (v_{15}) ={v_2,v_3,v_4,v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{14},,v_{15}},

$\mathfrak{G}$($_{16}$) = {v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{16}} and

$\mathfrak{G}$ (v_{17}) = {v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}, v_{17}}.

Also,$\mathfrak{Y}$(v_{12})={e_{13},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12}},

$\mathfrak{Y}$(v_{13})={e_{15},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{14}},

$\mathfrak{Y}(v_{14})$ = {e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16}},
$\mathfrak{Y}$ (v_{15}) ={e_{19},e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11}, e_{18} },
$\mathfrak{Y}$ (v_{16}) = {e_{21}, e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11}, e_{20}} and
$\mathfrak{Y}$ (v_{17}) ={e_{23},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{22}}.

The lower and upper rough soft vertex approximation are $\mathfrak{G}_L(V_1)$= {v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}} and $\mathfrak{G}^u(V_1)$= {v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13},v_{14},v_{15},v_{16},v_{17}}. The lower and upper rough soft edge approximation are $\mathfrak{Y}_L(E_1)$= {e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11}} and $\mathfrak{Y}^u(E_1)$= E(G). Thus, $\mathbb{H}_L(V_1) = (\mathfrak{G}_L(V_1), \mathfrak{Y}_L(E_1)), \mathbb{H}_L(E_1) = (\mathfrak{G}_L(V_1), \mathfrak{Y}_L(E_1))$, $\mathbb{H}^u(V_1) = (\mathfrak{G}^u(V_1), \mathfrak{Y}^u(E_1))$ and $\mathbb{H}^u(E_1) = (\mathfrak{G}^u(V_1), \mathfrak{Y}^u(E_1))$ are subgraphs of G^*.

In the previous example, we notice that from the new definition of the lower rough soft approximation of vertices and edges that the resultant lower rough approximation of vertices and edges is the intersection of all the set of parameters of vertices and edges that fall entirely within the set of vertices and edges of sub-graph G_1 which are $\mathbb{H}_L(V_1) = (\mathfrak{G}_L(V_1), \mathfrak{Y}_L(E_1))$.

By comparing these vertices and the edges represented on the blood circulation, as previously, they refer to the most important part of the blood circulation, which is the entry of unoxidized blood from all organs of the body after each of them has finished its function to the heart through the inferior vena cava to the right ventricle into the right and left pulmonary arteries, which go to the lungs. In the lung, deoxygenated blood that contains carbon dioxide will be expelled from the body and the blood will pick up the oxygen needed by the cells of the body to return it to the left atrium of the heart via the four pulmonary veins to the aortic semilunar valve which carry the oxygenated blood to the aorta to provide all the organs, tissues and cells of the body with the blood and other vital substances.

Also, we notice that, from the new definition of the upper rough soft approximation of vertices and edges which is the union of the set of vertices and edges parameters that its intersected with the set of vertices and edges from the sub-graph G_1 is equal to $\emptyset$ this indicates medically to complete the blood circulation of the human body entirely.

Proposition 4.1

Let $G = (V, E)$ be a crisp simple connected graph where $G^* = \langle \mathfrak{G}_L, \mathfrak{G}^u, \mathfrak{Y}_L, \mathfrak{Y}^u, A \rangle$ is rough soft graph of G then the following satisfy:

a) $\mathfrak{G}_L(V_1) \subseteq V_1 \subseteq \mathfrak{G}^u(V_1)$
b) $\mathfrak{Y}_L(E_1) \subseteq E_1 \subseteq \mathfrak{Y}^u(E_1)$

Proof:

1) Let $v_i \in \mathfrak{G}_L(V_1) \Longrightarrow \exists \cap \mathfrak{G}(v_j); \ni v_i \in \cap \mathfrak{G}(v_j)$; from the definition of $\mathfrak{G}_L$:
$\mathfrak{G}_L(V_1) = \cap \{\mathfrak{G}(v_i): v_i \in V_1, \mathfrak{G}(v_i) \subseteq V_1\}$, we get that $v_i \subseteq V_1$.
2) Let $v_i \in V_1 \Longrightarrow \exists \mathfrak{G}(v_j); \ni v_i \in \mathfrak{G}(v_j)$, from the definition of $\mathfrak{G}^u$:
$\mathfrak{G}^u(V_1) = \cup \{\mathfrak{G}(v_i): v_i \in V_1, \mathfrak{G}(v_i) \cap V_1 \neq \emptyset\}$. we conclude that $v_i \in \mathfrak{G}^u(V_1)$.
1) Let $e_{ij} \in \mathfrak{Y}_L(V_1) \Longrightarrow \exists \cap \mathfrak{Y}(e_{ij}); \ni e_i \in \cap \mathfrak{Y}(e_{ij})$; from the definition of $\mathfrak{Y}_L$:
$\mathfrak{Y}_L(E_1) = \cap \{e_{ij}: e_{ij} \in E_1, \mathfrak{Y}(e_{ij}) \subseteq E_1\} \Longrightarrow e_{ij} \subseteq E_1$.
2) Let $e_{ij} \in E_1 \Longrightarrow \exists \mathfrak{G}(e_i); \ni e_{ij} \in \mathfrak{Y}(e_{ij})$, from the definition of $\mathfrak{Y}^u$:
$\mathfrak{Y}^u(E_1) = \cup \{e_{ij}: e_{ij} \in E_1, \mathfrak{Y}(e_{ij}) \cap E_1 \neq \emptyset\}$, we will get that $e_{ij} \in \mathfrak{Y}^u(E_1)$.

The following new results are obtained from which we obtain containment relationships between approximate soft subgraph through the containment and equivalence relationships.

Definition 4.4 Let $G_1^* = \langle \mathfrak{G}_{1L}, \mathfrak{G}_1{}^u, \mathfrak{Y}_{1L}, \mathfrak{Y}_1{}^u, A \rangle$ and $G_2^* = \langle \mathfrak{G}_{2L}, \mathfrak{G}_2{}^u, \mathfrak{Y}_{2L}, \mathfrak{Y}_2{}^u, B \rangle$ are two rough soft graphs of G. Then G_1^* is a rough soft subgraph of G_2^* if

a) $A \subseteq B$
b) $\mathbb{H}_{1L}(V_1) = (\mathfrak{G}_{1L}(V_1), \mathfrak{Y}_{1L}(E_1))$ is a subgraph of $\mathbb{H}_{2L}(V_1) = (\mathfrak{G}_{2L}(V_1), \mathfrak{Y}_{2L}(E_1))$ for all $G_1 \in A$.
c) $\mathbb{H}_1{}^u(V_1) = (\mathfrak{G}_1{}^u(V_1), \mathfrak{Y}_1{}^u(E_1))$ is a subgraph of $\mathbb{H}_2{}^u(V_1) = (\mathfrak{G}_2{}^u(V_1), \mathfrak{Y}_2{}^u(E_1))$ for all $G_1 \in A$.

Example 4.2: Consider crisp simple connected graph $G = (V, E)$ as shown in the following Figure 10. Let G_1 be a subgraph with V (G_1) = {v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{13},v_{14},v_{15}} and E (G_1)= {e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{14},e_{15},e_{17},e_{18}}.

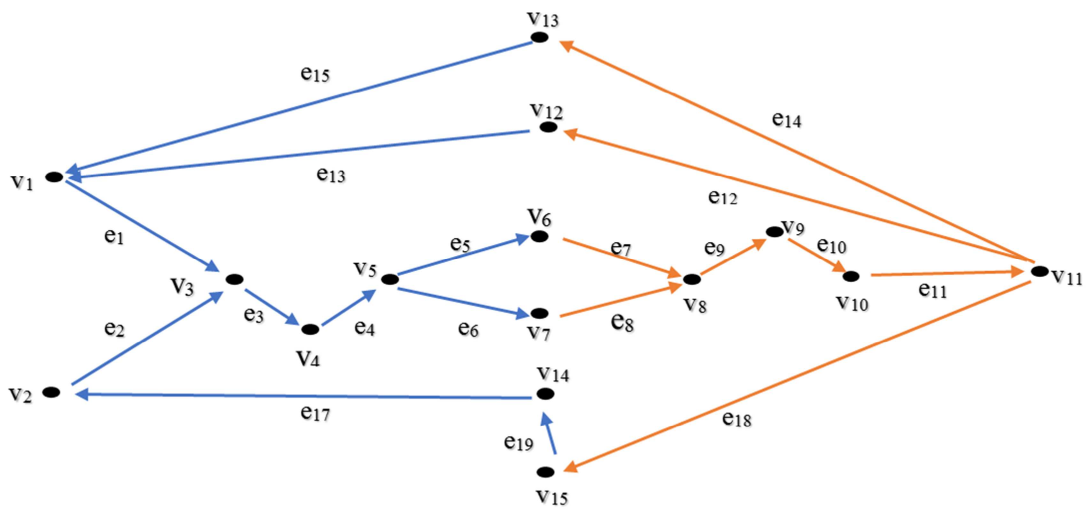

Figure 10. *Example 4.2.*

Let A = {v_{12},v_{13},v_{15}} be a set of parameter an approximate function $\mathfrak{G}_1$: A → ℘ (V) is defined as $\mathfrak{G}_1(X) = \{ y \in V \mid xRy \Leftrightarrow xCy\}$. Then,
$\mathfrak{G}_1(v_{12})$ ={v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12}},
$\mathfrak{G}_1(v_{13})$= {v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{13}}and $\mathfrak{G}_1$ (v_{15}) ={v_{14},v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{15}}.

Also, $\mathfrak{Y}_1(v_{12})$={e_{13},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12}},
$\mathfrak{Y}_1(v_{13})$={e_{15},e_1,e_3,e_4,e_5,e_6,e_7,e_8 e_9,e_{10},e_{11},e_{14}} and
$\mathfrak{Y}_1$ (v_{15}) = {e_{19},e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{18}}. The lower and upper soft vertex are approximation are $\mathfrak{G}_L(V_1)$= {v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}} and $\mathfrak{G}^u(V_1)$= V(G). The lower and upper soft edges are approximation are $\mathfrak{Y}^u(E_1)$ =E(G) and $\mathfrak{Y}_L(E_1)$={e_3,e_4, e_5,e_6,e_7,e_8,e_9,e_{10},e_{11}}.
we will notice that $\mathbb{H}_1{}^u(V_1) = (\mathfrak{G}_1{}^u(V_1), \mathfrak{Y}_1{}^u(E_1))$ and $\mathbb{H}_{1L}(V_1) = (\mathfrak{G}_{1L}(V_1), \mathfrak{Y}_{1L}(E_1))$ are subgraphs of G for all $V(G_1)$ ∈ A. So, $G_1{}^*$ is rough soft graph of G.

Let B={v_{13},v_{15} }be a set of parameter an approximate function $\mathfrak{G}_2$: B → ℘ (V) is defined as $\mathfrak{G}_2(X) = \{y \in V \mid xRy \Leftrightarrow xCy\}$. Then,
$\mathfrak{G}_1(v_{13})$= {v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{13}} and
$\mathfrak{G}_2(v_{15})$ ={v_{14},v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{15}}.
Also,$\mathfrak{Y}_2(v_{13})$ = {e_{15},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10}, e_{11},e_{14}} and
$\mathfrak{Y}_2$ (v_{15}) ={e_{19},e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{18} }. The lower and upper soft vertex are approximation are $\mathfrak{G}_2{}^u(V_1)$= V(G) and $\mathfrak{G}_{2L}(V_1)$= {v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}}. The lower and upper soft edges are approximation are $\mathfrak{Y}_2{}^u(V_1)$=E(G) and $\mathfrak{Y}_{2L}(V_1)$ ={e_3, e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11}}.
we will notice that $\mathbb{H}_{2L}(V_1) = \left(\mathfrak{G}_{2L}(V_1), \mathfrak{Y}_{2L}(E_1)\right)$ and $\mathbb{H}_2{}^u(V_1) = (\mathfrak{G}_2{}^u(V_1), \mathfrak{Y}_2{}^u(E_1))$ are subgraphs of G for all $V(G_1)$ ∈ B. So, $G_2{}^*$ is rough soft graph of G. Since A ⊆ B, $\mathbb{H}_{1L}(V_1)$ is a subgraph of $\mathbb{H}_{2L}(V_1)$ and $\mathbb{H}_1{}^u(V_1)$ is a subgraph of $\mathbb{H}_2{}^u(V_1)$. Hence $G_1{}^*$ is a rough soft subgraph of $G_2{}^*$

Proposition 4.2 Let $G = (V, E)$ be a crisp simple connected graph $G_1{}^* = \langle \mathfrak{G}_{1L}(V_1), \mathfrak{G}_1{}^u(V_1), \mathfrak{Y}_{1L}(E_1), \mathfrak{Y}_1{}^u(E_1), A\rangle$ and $G_2{}^* = \langle \mathfrak{G}_{2L}(V_1), \mathfrak{G}_2{}^u(V_1), \mathfrak{Y}_{2L}(V_1), \mathfrak{Y}_2{}^u(V_1), B\rangle$ are two rough soft graphs of G. Then $G_1{}^*$ is a rough soft subgraph of $G_2{}^*$ if and only if

a) $\mathfrak{G}_{1L}(V_1) \subseteq \mathfrak{G}_{2L}(V_1)$ and $\mathfrak{Y}_{1L}(E_1) \subseteq \mathfrak{Y}_{2L}(E_1)$
b) $\mathfrak{G}_1{}^u(V_1) \subseteq \mathfrak{G}_2{}^u(V_1)$ and $\mathfrak{Y}_1{}^u(E_1) \subseteq \mathfrak{Y}_2{}^u(E_1)$

Proof

a) Let $G_1{}^*$ is a rough soft graph of $G_2{}^*$. Then by the definition 4.4, we have that:
   1) A ⊆ B
   2) $\mathbb{H}_{1L}(V_1) = (\mathfrak{G}_{1L}(V_1), \mathfrak{Y}_{1L}(E_1))$ is a subgraph of $\mathbb{H}_{2L}(V_1) = (\mathfrak{G}_{2L}(V_1), \mathfrak{Y}_{2L}(E_1))$ for all V_1, E_1 ∈ A.
   3) $\mathbb{H}_1{}^u(V_1) = (\mathfrak{G}_1{}^u(V_1), \mathfrak{Y}_1{}^u(E_1))$ is a subgraph of $\mathbb{H}_2{}^u(V_1) = (\mathfrak{G}_2{}^u(V_1), \mathfrak{Y}_2{}^u(E_1))$ for all V_1, E_1 ∈ A.

Since $\mathbb{H}_{1L}(V_1)$ is a subgraph of $\mathbb{H}_{2L}(V_1)$and $\mathbb{H}_1{}^u(V_1)$ is a subgraph of $\mathbb{H}_2{}^u(V_1)$ for all V_1∈ A. Thus $\mathfrak{G}_{1L}(V_1) \subseteq \mathfrak{G}_{2L}(V_1), \mathfrak{Y}_{1L}(E_1) \subseteq \mathfrak{Y}_{2L}(E_1), \mathfrak{G}_1{}^u(V_1) \subseteq \mathfrak{G}_2{}^u(V_1)$ and $\mathfrak{Y}_1{}^u(E_1) \subseteq \mathfrak{Y}_2{}^u(E_1)$.

b) Conversely, assume that $\mathfrak{G}_{1L}(V_1) \subseteq \mathfrak{G}_{2L}(V_1), \mathfrak{Y}_{1L}(E_1) \subseteq \mathfrak{Y}_{2L}(E_1), \mathfrak{G}_1{}^u(V_1) \subseteq \mathfrak{G}_2{}^u(V_1)$ and $\mathfrak{Y}_1{}^u(E_1) \subseteq \mathfrak{Y}_2{}^u(E_1)$ for all V_1 ∈ A. Since $G_1{}^*$ is a rough soft graph of G, $\mathbb{H}_{1L}(V_1)$ and $\mathbb{H}_1{}^u(V_1)$ are subgraphs of G for all V_1 ∈ A. Since $G_2{}^*$ is a rough soft graph of G, $\mathbb{H}_{2L}(V_1)$ and $\mathbb{H}_2{}^u(V_1)$are subgraphs of G for all V_1 ∈ B.

So, $\mathbb{H}_{1L}(V_1) = (\mathfrak{G}_{1L}(V_1), \mathfrak{Y}_{1L}(E_1))$ is a subgraph of $\mathbb{H}_{2L}(V_1)$= $(\mathfrak{G}_{2L}(V_1), \mathfrak{Y}_{2L}(E_1))$ for all V_1 ∈ A and $\mathbb{H}_1{}^u(V_1)$ = $(\mathfrak{G}_1{}^u(V_1), \mathfrak{Y}_1{}^u(E_1))$ is a subgraph of $\mathbb{H}_2{}^u(V_1) = (\mathfrak{G}_2{}^u(V_1)$, $\mathfrak{Y}_2{}^u(E_1))$ for all A. Therefore, $G_1{}^*$ is a rough soft subgraph of $G_2{}^*$.

Proposition 4.3 Let $G = (V, E)$ be a crisp simple connected graph where
$G_1{}^* = \langle \mathfrak{G}_{1L}(V_1), \mathfrak{G}_1{}^u(V_1), \mathfrak{Y}_{1L}(E_1), \mathfrak{Y}_1{}^u(E_1), A\rangle$ and $G_2{}^* = \langle \mathfrak{G}_{2L}(V_1), \mathfrak{G}_2{}^u(V_1), \mathfrak{Y}_{2L}(E_1), \mathfrak{Y}_2{}^u(E_1), B\rangle$ are two rough soft graphs of G, then the following properties satisfy:

a) $\mathfrak{G}_L(G_1{}^* \cup G_2{}^*) \subseteq \mathfrak{G}_L(G_1{}^*) \cup \mathfrak{G}_L(G_2{}^*)$
b) $\mathfrak{G}^u(G_1{}^* \cup G_2{}^*) \supseteq \mathfrak{G}^u(G_1{}^*) \cup \mathfrak{G}^u(G_2{}^*)$

Proof

a) Let $v_i \in \mathfrak{G}_L(G_1{}^* \cup G_2{}^*)$, $v_i \in G_1{}^*$ or $v_i \in G_2{}^*$
$v_i \in \mathfrak{G}_1(x_i)$, $\mathfrak{G}_1(x_i)$ is a mapping of sets of parameters,
From the definition of $\mathfrak{G}_L$, $\mathfrak{G}_1(x_i) \subset V(G_1)$. So, $v_i \in \mathfrak{G}_L(G_1)$
Or
$v_i \in \mathfrak{G}_2(x_i)$, $\mathfrak{G}_2(x_i)$ is a mapping of sets of parameters,
From the definition of $\mathfrak{G}_L$, $\mathfrak{G}_1(x_i) \subset V(G_1)$. So, $v_i \in \mathfrak{G}_L(G_1)$

b) Let $v_i \in V_1 \Rightarrow \exists\, \mathfrak{G}(v_j)$; ∋ $v_i \in \mathfrak{G}(v_j)$, from the definition of $\mathfrak{G}^u$:
$\mathfrak{G}^u(V_1) = \cup\{\mathfrak{G}(v_i): v_i \in V_1, \mathfrak{G}(v_i) \cap V_1 \neq \emptyset\}$. we conclude that $v_i \in \mathfrak{G}^u(V_1)$.

Example 4.3: Consider crisp simple connected graph $G = (V, E)$ as shown in Figure 3.

Let $G_1 = (V_1, E_1)$ be a subgraph with V_1 = {v_1,v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{14}}and
E_1= {e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12},e_{13},e_{16},e_{17}}.

a) Let A = {v_{12},v_{13}} be a set of parameter an approximate function $\mathfrak{G}_1$: A → ℘ (V) is defined as $\mathfrak{G}_1(X) = \{y \in V \mid xRy \Leftrightarrow xCy\}$ that are
$\mathfrak{G}_1(v_{12})$={v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12}},
$\mathfrak{G}_1$ (v_{13}) ={v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{13}},
$\mathfrak{Y}_1(v_{12})$= {e_{13},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12}}
And $\mathfrak{Y}_1(v_{13})$={e_{15},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{14}}.

The lower and upper soft vertex are approximation are $\mathfrak{G}_L(V_1)$ = {v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12}} and $\mathfrak{G}^u(V_1)$ = {v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13}}. The lower and upper soft edges are approximation are $\mathfrak{Y}_L(E_1)$ = {e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12},e_{13}} and $\mathfrak{Y}^u(E_1)$= {e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12},e_{13},e_{14},e_{15}}.

So, we will notice that $\mathbb{H}_1{}^u = (\mathfrak{G}_1{}^u(V_1), \mathfrak{Y}_1{}^u(V_1))$ and $\mathbb{H}_{1L}(V_1) = (\mathfrak{G}_{1L}(V_1), \mathfrak{Y}_{1L}(E_1)$ are subgraphs of G for all $V(G_1)$ ∈ A. So,$G_1{}^*$ is rough soft graph of G.

Let B={v_{14},v_{15},v_{16},v_{17} } } be a set of parameter an approximate function $\mathfrak{G}$ 2: B → ℘ (V) is defined as $\mathfrak{G}_2(X) = \{y \in V \mid xRy \Leftrightarrow xCy\}$ that are
$\mathfrak{G}_2$ (v_{14}) = {v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14}},
$\mathfrak{G}_2$ (v_{15})={v_{14},v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{15}},

$\mathfrak{G}_2(v_{16})=\{v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{16}\}$,
$\mathfrak{G}_2(v_{17})=\{v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{17}\}$,
$\mathfrak{Y}_2(v_{14})=\{e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16}\}$,
$\mathfrak{Y}_2(v_{15})=\{e_{19},e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{18}\}$,
$\mathfrak{Y}_2(v_{16})=\{e_{21},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{20}\}$
and
$\mathfrak{Y}_2(v_{17})=\{e_{23},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{22}\}$. The lower and upper soft vertex are approximation are $\mathfrak{G}_{2L}(V_1)=\{v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14}\}$ and

$$\mathfrak{G}_2{}^u(V_1)=\{v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14},v_{15},v_{16},v_{17}\}.$$

The lower and upper soft edges are approximation are $\mathfrak{Y}_{2L}(E_1)=\{e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16}\}$ and

$$(E_1)=\{e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16},e_{17},e_{18},e_{19},e_{20},e_{21},e_{22},e_{23}\}.$$

So, we will notice $\mathbb{H}_{2L}(V_1)=(\mathfrak{G}_{2L}(V_1),\mathfrak{Y}_{2L}(E_1))$ and $\mathbb{H}_2{}^u(V_1)=(\mathfrak{G}_2{}^u(V_1),\mathfrak{Y}_2{}^u(E_1))$ are subgraphs of G for all $V(G_1)\in A$. So,
$G_2{}^*$ is rough soft graph of G.

b) The lower and upper soft vertex approximation are $\mathfrak{G}_L(V_1)=\{v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}\}$ and $\mathfrak{G}^u(V_1)=V(G)$. The lower and upper soft edges are approximation are

$\mathfrak{Y}_L(E_1)=\{e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11}\}$ and $\mathfrak{Y}^u(V_1)=E(G)$.

Finally, we can deduct from a, b, c that
$\mathfrak{G}_L(V_1)=\{v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}\}\subseteq(\mathfrak{G}_L(V_1)=\{v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12}\}\cup$
$\mathfrak{G}_{2L}(V_1)=\{v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14}\}$
$\mathfrak{Y}_L(E_1)=\{e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11}\}\subseteq(\mathfrak{Y}_L(E_1)=\{e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12},e_{13}\}\cup(\mathfrak{Y}_{2L}(E_1)=\{e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16}\})$
Then $\mathfrak{G}_L(G_1{}^*\cup G_2{}^*)\subseteq\mathfrak{G}_L(G_1{}^*)\cup\mathfrak{G}_L(G_2{}^*)$
and

$$\mathfrak{G}^u(V_1)=V(G)\supseteq(\mathfrak{G}^u(V_1)=(\{v_1,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{12},v_{13}\}\cup\mathfrak{G}_2{}^u(V_1)=\{v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14},v_{15},v_{16},v_{17}\})$$

$$\mathfrak{Y}^u((E_1)=E(G)\supseteq(\mathfrak{Y}^u(E_1)=\{e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12},e_{13},e_{14},e_{15}\}\cup\mathfrak{Y}_2{}^u(E_1)=\{e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16},e_{17},e_{18},e_{19},e_{20},e_{21},e_{22},e_{23}\}$$

Then $\mathfrak{G}^u(G_1{}^*\cup G_2{}^*)\supseteq\mathfrak{G}^u(G_1{}^*)\cup\mathfrak{G}^u(G_2{}^*)$.

Proposition 4.4 Let $G=(V,E)$ be a crisp simple connected graph where
$G_1{}^*=\langle\mathfrak{G}_{1L}(V_1),\mathfrak{G}_1{}^u(V_1),\mathfrak{Y}_{1L}(E_1),\mathfrak{Y}_1{}^u(E_1),A\rangle$ and $G_2{}^*=\langle\mathfrak{G}_{2L}(V_1),\mathfrak{G}_2{}^u(V_1),\mathfrak{Y}_{2L}(E_1),\mathfrak{Y}_2{}^u(E_1),B\rangle$ are two rough soft graphs of G, then the following properties satisfy:
a) $\mathfrak{G}_L(G_1{}^*\cap G_2{}^*)\supseteq\mathfrak{G}_L(G_1{}^*)\cap\mathfrak{G}_L(G_2{}^*)$
b) $\mathfrak{G}^u(G_1{}^*\cap G_2{}^*)\subseteq\mathfrak{G}^u(G_1{}^*)\cap\mathfrak{G}^u(G_2{}^*)$
Proof: obliviously
Proposition 4.5
Let $G=(V,E)$ be a simple connected graph where $G^*=\langle\mathfrak{G}_L,\mathfrak{G}^u,\mathfrak{Y}_L,\mathfrak{Y}^u,A\rangle$ is rough soft graph of G and $G_1=(V_1,E_1)$ is a subgraph, then the following satisfy:
a) $\mathfrak{G}^u(V(G-V_1)=(V(G))-\mathfrak{G}_L(V_1)$
b) $\mathfrak{G}_L(V(G-V_1)=(V(G))-\mathfrak{G}^u(V_1)$
Proof: obliviously
Proposition 4.6
Let $G=(V,E)$ be a crisp simple connected graph where $G^*=\langle\mathfrak{G}_L,\mathfrak{G}^u,\mathfrak{Y}_L,\mathfrak{Y}^u,A\rangle$ is rough soft graph of G and $G_1=(V_1,E_1)$ is a subgraph, then the following satisfy:

$$\frac{1}{m^2}\binom{||\mathfrak{G}^u(V_2)||}{2}\le|\mathfrak{Y}^u(E_1)|\le\binom{||\mathfrak{G}^u(V_1)||}{2}$$

Where $V_2=V_1-v_i$ and m is the number of edges
Proof: obliviously

5. Unions and Intersections Operations of Rough Soft Graph

The aim of this section is introducing some operations on rough soft graph by presenting examples to demonstrate the new concept of approximation soft graph.

Definition 5.1 Let
$G_1{}^*=\langle\mathfrak{G}_{1L}(V_1),\mathfrak{G}_1{}^u(V_1),\mathfrak{Y}_{1L}(E_1),\mathfrak{Y}_1{}^u(E_1),A\rangle$ and
$G_2{}^*=\langle\mathfrak{G}_{2L}(V_1),\mathfrak{G}_2{}^u(V_1),\mathfrak{Y}_{2L}(E_1),\mathfrak{Y}_2{}^u(E_1),B\rangle$ are two rough soft graphs of G. The extended union of $G_1{}^*$ and $G_2{}^*$, denoted by $G_1{}^*\cup_E G_2{}^*$, is defined as
$G_1{}^*\cup_E G_2{}^*=\langle\mathfrak{G}_L(V_1),\mathfrak{G}^u(V_1),\mathfrak{Y}_L(E_1),\mathfrak{Y}^u(E_1),C\rangle$ where C = A∪B and for all $\mathcal{C}$ is belong to C.

$$\mathfrak{G}_L\ X(\mathcal{C})=\begin{cases}\mathfrak{G}_{1L}(\mathcal{C}),\text{if }\mathcal{C}\in A/B\\ \mathfrak{G}_{2L}(\mathcal{C}),\text{if }\mathcal{C}\in B/A\\ \mathfrak{G}_{1L}(\mathcal{C})\cup\mathfrak{G}_{2L}(\mathcal{C}),\text{if }\mathcal{C}\in A\cap B\end{cases}$$

$$\mathfrak{G}^u\ X(\mathcal{C})=\begin{cases}\mathfrak{G}_1{}^u(\mathcal{C}),\text{if }\mathcal{C}\in A/B\\ \mathfrak{G}_2{}^u(\mathcal{C}),\text{if }\mathcal{C}\in B/A\\ \mathfrak{G}_1{}^u(\mathcal{C})\cup\mathfrak{G}_2{}^u(\mathcal{C}),\text{if }\mathcal{C}\in A\cap B\end{cases}$$

$$\mathfrak{Y}_L\ X(\mathcal{C})=\begin{cases}\mathfrak{Y}_{1L}(\mathcal{C}),\text{if }\mathcal{C}\in A/B\\ \mathfrak{Y}_{2L}(\mathcal{C}),\text{if }\mathcal{C}\in B/A\\ \mathfrak{Y}_{1L}(\mathcal{C})\cup\mathfrak{Y}_{2L}(\mathcal{C}),\text{if }\mathcal{C}\in A\cap B\end{cases}$$

$$\mathfrak{Y}^u\ X(\mathcal{C})=\begin{cases}\mathfrak{Y}_1{}^u,(\mathcal{C}),\text{if }\mathcal{C}\in A/B\\ \mathfrak{Y}_2{}^u(\mathcal{C}),\text{if }\mathcal{C}\in B/A\\ \mathfrak{Y}_1{}^u,(\mathcal{C})\cup\mathfrak{Y}_2{}^u(\mathcal{C}),\text{if }\mathcal{C}\in A\cap B\end{cases}$$

So, $\mathbb{H}^u(\mathcal{C})=(\mathfrak{G}^u(\mathcal{C}),\mathfrak{Y}^u(\mathcal{C}))$ and $\mathbb{H}_L(\mathcal{C})=(\mathfrak{G}_L(\mathcal{C}),\mathfrak{Y}_L(\mathcal{C}))$. That is, $G_1{}^*\cup_E G_2{}^*=\{\mathbb{H}^u(\mathcal{C}),\sqcap\mathbb{H}_L(\mathcal{C}):\mathcal{C}\in C\}$

Definition 5.2 Let
$G_1{}^*=\langle\mathfrak{G}_{1L}(V_1),\mathfrak{G}_1{}^u(V_1),\mathfrak{Y}_{1L}(E_1),\mathfrak{Y}_1{}^u(E_1),A\rangle$ and
$G_2{}^*=\langle\mathfrak{G}_{2L}(V_1),\mathfrak{G}_2{}^u(V_1),\mathfrak{Y}_{2L}(E_1),\mathfrak{Y}_2{}^u(E_1),B\rangle$ are two rough soft graphs of G such that $A\cap B\neq\emptyset$. The restricted union of $G_1{}^*$ and $G_2{}^*$, denoted by $G_1{}^*\cup_R G_2{}^*$, is defined as
$G_1{}^*\cup_R G_2{}^*=G^*=\langle\mathfrak{G}_L(V_1),\mathfrak{G}^u(V_1),\mathfrak{Y}_L(E_1),\mathfrak{Y}^u(E_1),C\rangle$ where C = A ∩ B and for all $\mathcal{C}$ is belong to C, when
i. $\mathfrak{G}_L\ X(\mathcal{C})=\mathfrak{G}_{1L}(\mathcal{C})\cup\mathfrak{G}_{2L}S_2(\mathcal{C})$

ii. $\mathfrak{Y}_L$ X($\mathcal{C}$)= $\mathfrak{Y}_{1L}(\mathcal{C}) \cup \mathfrak{Y}_{2L}(\mathcal{C})$
iii. $\mathfrak{G}^u$ X($\mathcal{C}$)= ${\mathfrak{G}_1}^u(\mathcal{C}) \cup {\mathfrak{G}_2}^u(\mathcal{C})$
iv. $\mathfrak{Y}^u$ X($\mathcal{C}$)= ${\mathfrak{Y}_1}^u(\mathcal{C}) \cup {\mathfrak{Y}_2}^u(\mathcal{C})$

Definition 5.3 Let ${G_1}^* = \langle \mathfrak{G}_{1L}(\mathrm{V}_1), {\mathfrak{G}_1}^u(\mathrm{V}_1), \mathfrak{Y}_{1L}(\mathrm{E}_1), {\mathfrak{Y}_1}^u(\mathrm{E}_1), \mathrm{A}\rangle$ and ${G_2}^* = \langle \mathfrak{G}_{2L}(\mathrm{V}_1), {\mathfrak{G}_2}^u(\mathrm{V}_1), \mathfrak{Y}_{2L}(\mathrm{E}_1), {\mathfrak{Y}_2}^u(\mathrm{E}_1), \mathrm{B}\rangle$ are two rough soft graphs of G. The extended intersection of ${G_1}^*$ and ${G_2}^*$, denoted by ${G_1}^* \cap_E {G_2}^*$, is defined as

${G_1}^* \cap_E {G_2}^* = \langle \mathfrak{G}_L(\mathrm{V}_1), \mathfrak{G}^u(\mathrm{V}_1), \mathfrak{Y}_L(\mathrm{E}_1), \mathfrak{Y}^u(\mathrm{E}_1), \mathrm{C}\rangle$ where C = A∪B and for all $\mathcal{C}$ is belong to C.

$$\mathfrak{G}_L\ \mathrm{X}(\mathcal{C}) = \begin{cases} \mathfrak{G}_{1L}(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A/B} \\ \mathfrak{G}_{2L}(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{B/A} \\ \mathfrak{G}_{1L}(\mathcal{C}) \cap \mathfrak{G}_{2L}(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

$$\mathfrak{G}^u\ \mathrm{X}(\mathcal{C}) = \begin{cases} {\mathfrak{G}_1}^u(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A/B} \\ {\mathfrak{G}_2}^u(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{B/A} \\ {\mathfrak{G}_1}^u(\mathcal{C}) \cap {\mathfrak{G}_2}^u(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

$$\mathfrak{Y}_L\ \mathrm{X}(\mathcal{C}) = \begin{cases} \mathfrak{Y}_{1L}(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A/B} \\ \mathfrak{Y}_{2L}(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{B/A} \\ \mathfrak{Y}_{1L}(\mathcal{C}) \cap \mathfrak{Y}_{2L}(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

$$\mathfrak{Y}^u\ \mathrm{X}(\mathcal{C}) = \begin{cases} {\mathfrak{Y}_1}^u, (\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A/B} \\ {\mathfrak{Y}_2}^u(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{B/A} \\ {\mathfrak{Y}_1}^u, (\mathcal{C}) \cap {\mathfrak{Y}_2}^u(\mathcal{C}), \text{if } \mathcal{C} \in \mathrm{A} \cap \mathrm{B} \end{cases}$$

So, $\mathbb{H}^u(\mathcal{C}) = (\mathfrak{G}^u(\mathcal{C}), \mathfrak{Y}^u(\mathcal{C}))$ and $\mathbb{H}_L(\mathcal{C}) = (\mathfrak{G}_L(\mathcal{C}), \mathfrak{Y}_L(\mathcal{C}))$. That is, ${G_1}^* \cup_E {G_2}^* = \{ \mathbb{H}^u(\mathcal{C}), \sqcap\ \mathbb{H}_L(\mathcal{C}) : \mathcal{C} \in \mathrm{C}\}$

Definition 5.4 Let ${G_1}^* = \langle \mathfrak{G}_{1L}(\mathrm{V}_1), {\mathfrak{G}_1}^u(\mathrm{V}_1), \mathfrak{Y}_{1L}(\mathrm{E}_1), {\mathfrak{Y}_1}^u(\mathrm{E}_1), \mathrm{A}\rangle$ and ${G_2}^* = \langle \mathfrak{G}_{2L}(\mathrm{V}_1), {\mathfrak{G}_2}^u(\mathrm{V}_1), \mathfrak{Y}_{2L}(\mathrm{E}_1), {\mathfrak{Y}_2}^u(\mathrm{E}_1), \mathrm{B}\rangle$ are two rough soft graphs of G such that A ⋂ B ≠ ∅. The restricted intersection of ${G_1}^*$ and ${G_2}^*$, denoted by ${G_1}^* \cap_R {G_2}^*$, is defined as

${G_1}^* \cap_R {G_2}^* = G^* = \langle \mathfrak{G}_L(\mathrm{V}_1), \mathfrak{G}^u(\mathrm{V}_1), \mathfrak{Y}_L(\mathrm{E}_1), \mathfrak{Y}^u(\mathrm{E}_1), \mathrm{C}\rangle$ where C = A ∩ B and for all $\mathcal{C}$ is belong to C, when

i. $\mathfrak{G}_L$ X($\mathcal{C}$) = $\mathfrak{G}_{1L}(\mathcal{C}) \cap \mathfrak{G}_{2L}\mathrm{S}_2(\mathcal{C})$
ii. $\mathfrak{Y}_L$ X($\mathcal{C}$) = $\mathfrak{Y}_{1L}(\mathcal{C}) \cap \mathfrak{Y}_{2L}(\mathcal{C})$
iii. $\mathfrak{G}^u$ X($\mathcal{C}$) = ${\mathfrak{G}_1}^u(\mathcal{C}) \cap {\mathfrak{G}_2}^u(\mathcal{C})$
iv. $\mathfrak{Y}^u$ X($\mathcal{C}$) = ${\mathfrak{Y}_1}^u(\mathcal{C}) \cap {\mathfrak{Y}_2}^u(\mathcal{C})$

Proposition 5.1

Let $G = (\mathrm{V}, \mathrm{E})$ be a simple connected graph where ${G_1}^* = \langle \mathfrak{G}_{1L}(\mathrm{V}_1), {\mathfrak{G}_1}^u(\mathrm{V}_1), \mathfrak{Y}_{1L}(\mathrm{E}_1), {\mathfrak{Y}_1}^u(\mathrm{E}_1), \mathrm{A}\rangle$ and ${G_2}^* = \langle \mathfrak{G}_{2L}(\mathrm{V}_1), {\mathfrak{G}_2}^u(\mathrm{V}_1), \mathfrak{Y}_{2L}(\mathrm{E}_1), {\mathfrak{Y}_2}^u(\mathrm{E}_1), \mathrm{B}\rangle$ are two rough soft graphs G such that A ⋂ B ≠ ∅. then ${G_1}^* \cup_E {G_2}^*$ is rough soft set of G.

Proof

Since ${G_1}^*$ is a rough soft graph of G, ($\mathfrak{G}_{1L}(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{E}_1)$) and (${\mathfrak{G}_1}^u(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{E}_1)$) are subgraphs of G for all x ∈ A\B. Since ${G_2}^*$ is a rough soft graph of G, ($\mathfrak{G}_{2L}(\mathrm{V}_1)$, ${\mathfrak{Y}_2}^u(\mathrm{E}_1)$) and (${\mathfrak{G}_2}^u(\mathrm{V}_1)$, ${\mathfrak{Y}_2}^u(\mathrm{E}_1)$) are subgraphs of G for all x ∈ B\A.

Let x ∈ A ∩ B, ($\mathfrak{G}_L(\mathrm{V}_1)$, $\mathfrak{Y}_L(\mathrm{E}_1)$) = ($\mathfrak{G}_{1L}(\mathrm{V}_1) \cup \mathfrak{G}_{2L}(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{E}_1) \cup \mathfrak{Y}_{2L}(\mathrm{V}_1)$) and ($\mathfrak{G}^u(\mathrm{V}_1)$, $\mathfrak{Y}^u(\mathrm{E}_1) = {\mathfrak{Y}_1}^u(\mathrm{E}_1)$) = (${\mathfrak{G}_1}^u(\mathrm{V}_1) \cup {\mathfrak{G}_2}^u(\mathrm{V}_1)$, ${\mathfrak{Y}_1}^u(\mathrm{E}_1) \cap {\mathfrak{Y}_2}^u(\mathrm{E}_1)$). Since ($\mathfrak{G}_{1L}(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{E}_1)$), (${\mathfrak{G}_1}^u(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{E}_1)$),($\mathfrak{G}_{2L}(\mathrm{V}_1)$, ${\mathfrak{Y}_2}^u(\mathrm{E}_1)$) and (${\mathfrak{G}_2}^u(\mathrm{V}_1)$, ${\mathfrak{Y}_2}^u(\mathrm{E}_1)$) are subgraphs of G. Thus, ($\mathfrak{G}_L(\mathrm{V}_1)$, $\mathfrak{Y}_L(\mathrm{E}_1)$) and ($\mathfrak{G}^u(\mathrm{V}_1)$, $\mathfrak{Y}^u(\mathrm{E}_1)$) are subgraphs of G for all x ∈ A ∩ B. Hence ${G_1}^* \cup_E {G_2}^*$ = G∗ = $\langle \mathfrak{G}_L(\mathrm{V}_1), \mathfrak{Y}_L(\mathrm{E}_1), \mathfrak{G}^u(\mathrm{V}_1), \mathfrak{Y}^u(\mathrm{E}_1), \mathrm{C}\rangle$ is a rough soft graph of G.

Proposition 5.2

Let $G = (\mathrm{V}, \mathrm{E})$ be a simple connected graph where ${G_1}^* = \langle \mathfrak{G}_{1L}(\mathrm{V}_1), {\mathfrak{G}_1}^u(\mathrm{V}_1), \mathfrak{Y}_{1L}(\mathrm{E}_1), {\mathfrak{Y}_1}^u(\mathrm{E}_1), \mathrm{A}\rangle$ and ${G_2}^* = \langle \mathfrak{G}_{2L}(\mathrm{V}_1), {\mathfrak{G}_2}^u(\mathrm{V}_1), \mathfrak{Y}_{2L}(\mathrm{E}_1), {\mathfrak{Y}_2}^u(\mathrm{E}_1), \mathrm{B}\rangle$ are two rough soft graphs G such that A ⋂ B ≠ ∅. $\mathfrak{G}_{1L}(\mathcal{C}) \cap \mathfrak{G}_{2L}(\mathcal{C}) \neq \emptyset$ and ${\mathfrak{G}_1}^u(\mathcal{C}) \cap {\mathfrak{G}_2}^u(\mathcal{C}) \neq \emptyset$ for all $\mathcal{C} \in$ A ⋂ B. then ${G_1}^* \cap_E {G_2}^*$ for all is rough soft set of G.

Proof

Since ${G_1}^*$ is a rough soft graph of G, ($\mathfrak{G}_{1L}(\mathrm{V}_1), \mathfrak{Y}_{1L}(\mathrm{E}_1)$) and (${\mathfrak{G}_1}^u(\mathrm{V}_1), {\mathfrak{Y}_1}^u(\mathrm{E}_1)$) are subgraphs of G for all x ∈ A\B. Since ${G_2}^*$ is a rough soft graph of G, ($\mathfrak{G}_{2L}(\mathrm{V}_1), \mathfrak{Y}_{2L}(\mathrm{E}_1)$) and (${\mathfrak{G}_2}^u(\mathrm{V}_1), {\mathfrak{Y}_2}^u(\mathrm{E}_1)$) are subgraphs of G for all x ∈ B\A.

Let x ∈ A ∩ B, ($\mathfrak{G}_L(\mathrm{V}_1)$, $\mathfrak{Y}_L(\mathrm{E}_1)$) = ($\mathfrak{G}_{1L}(\mathrm{V}_1) \cup \mathfrak{G}_{2L}(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{V}_1) \cup \mathfrak{Y}_{2L}(\mathrm{V}_1)$) and ($\mathfrak{G}^u(\mathrm{V}_1)$, $\mathfrak{Y}^u(\mathrm{E}_1)$) = (${\mathfrak{G}_1}^u(\mathrm{V}_1) \cup {\mathfrak{G}_2}^u(\mathrm{V}_1)$, ${\mathfrak{Y}_1}^u(\mathrm{E}_1) \cap {\mathfrak{Y}_2}^u(\mathrm{E}_1)$). Since ($\mathfrak{G}_{1L}(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{E}_1)$), (${\mathfrak{G}_1}^u(\mathrm{V}_1)$, $\mathfrak{Y}_{1L}(\mathrm{E}_1)$),($\mathfrak{G}_{2L}(\mathrm{V}_1)$, ${\mathfrak{Y}_2}^u(\mathrm{E}_1)$) and (${\mathfrak{G}_2}^u(\mathrm{V}_1)$, ${\mathfrak{Y}_2}^u(\mathrm{E}_1)$) are subgraphs of G and by assumption $\mathfrak{G}_{1L}(\mathrm{V}_1) \cup \mathfrak{G}_{2L}(\mathrm{V}_1) = \emptyset$ and ${\mathfrak{G}_1}^u(\mathrm{V}_1) \cup {\mathfrak{G}_2}^u(\mathrm{V}_1) = \emptyset$ for all x ∈ A ∩ B. Thus, ($\mathfrak{G}_L(\mathrm{V}_1)$, $\mathfrak{Y}_L(\mathrm{E}_1)$)and ($\mathfrak{G}^u(\mathrm{V}_1)$, $\mathfrak{Y}^u(\mathrm{E}_1)$) are subgraphs of G for all x ∈ A ∩ B. Hence

${G_1}^* \cap_E {G_2}^* = G^* = \langle \mathfrak{G}_L(\mathrm{V}_1), \mathfrak{G}^u(\mathrm{V}_1), \mathfrak{Y}_L(\mathrm{E}_1), \mathfrak{Y}^u(\mathrm{E}_1), \mathrm{C}\rangle$ is a rough soft graph of G.

Example 5.1: Consider crisp simple connected graph $\mathrm{G} = (\mathrm{V}, \mathrm{E})$ as shown in Figure 3. Let $G_1 = (V_1, E_1)$ be a subgraph with $V_1 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{14}\}$ and $E_1 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{16}, e_{17}\}$.

a) Let A = $\{v_{12}, v_{13}, v_{14}, v_{15}\}$ be a set of parameters an approximate function $\mathfrak{G}_1$: A → ℘ (V) is defined as $\mathfrak{G}_1(X) = \{ y \in V \mid xRy \Leftrightarrow xCy \}$ that are

$\mathfrak{G}_1(v_{12}) = \{v_1, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$, $\mathfrak{G}_1(v_{13}) = \{v_1, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{13}\}$, $\mathfrak{G}_1(v_{14}) = \{v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{14}\}$, $\mathfrak{G}_1(v_{15}) = \{v_{14}, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{15}\}$, $\mathfrak{Y}_1(v_{12}) = \{e_{13}, e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}\}$, $\mathfrak{Y}_1(v_{13}) = \{e_{15}, e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{14}\}$, $\mathfrak{Y}_1(v_{14}) = \{e_{17}, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{16}\}$ and $\mathfrak{Y}_1(v_{15}) = \{e_{19}, e_{17}, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{18}$.

The lower and upper soft vertex are approximation are $\mathfrak{G}_L(\mathrm{V}_1) = \{v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}\}$ and $\mathfrak{G}^u(\mathrm{V}_1) = \{v_1, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$. The lower and upper soft edges are approximation are $\mathfrak{Y}_L(\mathrm{E}_1) = \{e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}\}$ and $\mathfrak{Y}^u(\mathrm{E}_1) = \{e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}\}$.

So, we will notice that ${\mathbb{H}_1}^u = ({\mathfrak{G}_1}^u(\mathrm{V}_1), {\mathfrak{Y}_1}^u(\mathrm{V}_1))$ and $\mathbb{H}_{1L}(\mathrm{V}_1) = (\mathfrak{G}_{1L}(\mathrm{V}_1), \mathfrak{Y}_{1L}(\mathrm{E}_1))$ are subgraphs of G for all $\mathrm{V}(G_1) \in$ A. So, ${G_1}^*$ is rough soft graph of G

b) Let B={v_{14},v_{15},v_{16},v_{17} } } be a set of parameter an approximate function $\mathfrak{G}_2$: B⟶ ℘ (V) is defined as $\mathfrak{G}_2(X) = \{y \in V \mid xRy \Leftrightarrow xCy\}$ that are

$\mathfrak{G}_2$ (v_{14}) = {v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14}}, $\mathfrak{G}_2$ (v_{15})={v_{14},v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{15}}, $\mathfrak{G}_2$ (v_{16})= {v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{16}},

$\mathfrak{G}_2$ (v_{17})={v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{17}},

$\mathfrak{Y}_2$(v_{14}) ={e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{16} },

$\mathfrak{Y}_2$(v_{15}) ={e_{19},e_{17},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{18}},

$\mathfrak{Y}_2$(v_{16}) ={e_{21},e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{20}} and

$\mathfrak{Y}_2$ (v_{17}) ={e_{23},e_1,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{22} }. The lower and upper soft vertex are approximation are $\mathfrak{G}_{2L}(\mathrm{V}_1) =$ { v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14}} and ${\mathfrak{G}_2}^u(\mathrm{V}_1) =${$v_2$,$v_3$,$v_4$,$v_5$,$v_6$,$v_7$,$v_8$,$v_9$,$v_{10}$,$v_{11}$,$v_{14}$,$v_{15}$,$v_{16}$,$v_{17}$ }. The lower and upper soft edges are approximation are $\mathfrak{Y}_{2L}$ (E_1)= { e_{17}, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10},e_{11},e_{16}} and ${\mathfrak{Y}_2}^u(\mathrm{E}_1)$= { e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23} }.

By noticing that, $\mathbb{H}_{2L}(\mathrm{V}_1) = \left(\mathfrak{G}_{2L}(\mathrm{V}_1), \mathfrak{Y}_{2L}(\mathrm{E}_1)\right)$ and ${\mathbb{H}_2}^u(\mathrm{V}_1) = \left({\mathfrak{G}_2}^u(\mathrm{V}_1), {\mathfrak{Y}_2}^u(\mathrm{E}_1)\right)$ are subgraphs of G for all $\mathrm{V}(G_1)$ ∈ A. So, ${G_2}^*$ is rough soft graph of G.

Then, we can get that A\B={v_{12},v_{13}}, B\ A= {v_{16},v_{17}},A∩B = {v_{14},v_{15}},and

A∪ B = {v_{12},v_{13},v_{14},v_{15},v_{16},v_{17}}.

The extended union of ${G_1}^* \cup_E {G_2}^*$:

a) $\mathfrak{G}_L(\mathrm{V}_1) = \mathfrak{G}_{1L}(\mathrm{V}_1) \cup \mathfrak{G}_{2L}(\mathrm{V}_1) =$ {v_2,v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11},v_{14}}

b) $\mathfrak{Y}_L(\mathrm{V}_1) = \mathfrak{Y}_{1L}(\mathrm{E}_1) \cup \mathfrak{Y}_{2L}(\mathrm{E}_1) =${$e_{17}$, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11},e_{16}}

c) $\mathfrak{G}^u(\mathrm{V}_1) = {\mathfrak{G}_1}^u(\mathrm{V}_1) \cup {\mathfrak{G}_2}^u(\mathrm{V}_1)$ = V(G)

d) $\mathfrak{Y}^u(\mathrm{V}_1) = {\mathfrak{Y}_1}^u(\mathrm{E}_1) \cup {\mathfrak{Y}_2}^u(\mathrm{E}_1)$ = E(G)

The extended intersection of ${G_1}^* \cap_E {G_2}^*$:

a) $\mathfrak{G}_L(\mathrm{V}_1) = \mathfrak{G}_{1L}(\mathrm{V}_1) \cap \mathfrak{G}_{2L}(\mathrm{V}_1) =$ {v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}}

b) $\mathfrak{Y}_L(\mathrm{V}_1) = \mathfrak{Y}_{1L}(\mathrm{E}_1) \cap \mathfrak{Y}_{2L}(\mathrm{E}_1) =${ e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11} }

c) $\mathfrak{G}^u(\mathrm{V}_1) = {\mathfrak{G}_1}^u(\mathrm{V}_1) \cap {\mathfrak{G}_2}^u(\mathrm{V}_1))$ = {v_3,v_4,v_5,v_6,v_7,v_8,v_9,v_{10},v_{11}}

d) $\mathfrak{Y}^u(\mathrm{V}_1) = {\mathfrak{Y}_1}^u(\mathrm{E}_1) \cap {\mathfrak{Y}_2}^u(\mathrm{E}_{1_1}) =${ e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11} }

6. Conclusion

In this paper, the circulatory system on the human body is represented as a graph. We used the concepts of soft graph on this graph to know the parts affecting the blood flow through the veins, arteries and the important organs in the human body. The future suggestion of our work is to link the concepts of soft set, rough set and graph theory to find a mathematical model for treating some diseases in the circulatory system or for modelling the transmission of nerve waves within neurons. Some important operation can be presented such as cartesian product, join and composition on rough soft graph. Introducing some new results from the previous concepts through different approximation set such as intuitionistic set, fuzzy set and picture fuzzy set. Finally, it is possible to study some of the previous operators in a different way that depends on the interpretation of the type of approximation used.

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