Pure and Applied Mathematics Journal
2015; 4(4): 178-188
Published online August 11, 2015 (http://www.sciencepublishinggroup.com/j/pamj)
doi: 10.11648/j.pamj.20150404.17
ISSN: 2326-9790 (Print); ISSN: 2326-9812 (Online)

Fourier Coefficients of a Class of Eta Quotients of Weight 18 with Level 12

Baris Kendirli

Dept. of Math,. Aydın University, Istanbul, Turkey

Email address:
baris.kendirli@gmail.com

To cite this article:
Barış Kendirli. Fourier Coefficients of a Class of Eta Quotients of Weight 18 with Level 12. *Pure and Applied Mathematics Journal*. Vol. 4, No. 4, 2015, pp. 178-188. doi: 10.11648/j.pamj.20150404.17

Abstract: Williams [16] and later Yao, Xia and Jin[15] discovered explicit formulas for the coefficients of the Fourier series expansions of a class of eta quotients. Williams expressed all coefficients of 126 eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$ and Yao, Xia and Jin, following the method of proof of Williams, expressed only even coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$. Here, we will express the even Fourier coefficients of 324 eta quotients in terms of $\sigma_{17}(n), \sigma_{17}\left(\frac{n}{2}\right), \sigma_{17}\left(\frac{n}{3}\right), \sigma_{17}\left(\frac{n}{4}\right), \sigma_{17}\left(\frac{n}{6}\right)$ and $\sigma_{17}\left(\frac{n}{12}\right)$.

Keywords: Dedekind Eta Function, Eta Quotients, Fourier Series

1. Introduction

The divisor function $\sigma_i(n)$ is defined for a positive integer i by

$$\sigma_i(n) := \sum_{d \text{ positive integer}, d|n} d^i, \text{ if } n \text{ is a positive integer, and} \quad (1)$$

$$\sigma_i(n) := 0 \text{ if n is not a positive integer.}$$

The Dedekind eta function is defined by

$$\eta(z) := q^{1/24} \prod_{n=1}^{\infty} (1 - q^n), \quad (2)$$

Where

$$q := e^{2\pi i z}, z \in H = \{x + iy : y > 0\} \quad (3)$$

and an eta quotient of level n is defined by

$$f(z) := \prod_{m|n} \eta(mz)^{a_m}, n, m \in \mathbb{N}, a_m \in Z. \quad (4)$$

It is interesting and important to determine explicit formulas of the Fourier coefficients of eta quotients, because they are the building blocks of modular forms of level n and weight k. The book of Köhler [13] (Chapter 3, pg.39) describes such expansions by means of Hecke Theta series and develops algorithms for the determination of suitable eta quotients. One can find more information in [3], [6], [14]. I have determined the Fourier coefficients of the theta series associated to some quadratic forms, see [7], [8], [9][10], [11] and [12].

It is known that Williams, see [16] discovered explicit formulas for the coefficients of Fourier series expansions of a class of 126 eta quotients in terms of $\sigma(n), \sigma(\frac{n}{2}), \sigma(\frac{n}{3})$ and $\sigma(\frac{n}{6})$. One example is as follows:

$$\frac{\eta^2(2z)\eta^4(4z)\eta^6(6z)}{\eta^2(z)\eta^2(3z)\eta^4(12z)}$$

gives the expansion found by Williams.

Then Yao, Xia and Jin [15] expressed the even Fourier coefficients of 104 eta quotients in terms of $\sigma_3(n), \sigma_3(\frac{n}{2}), \sigma_3(\frac{n}{3})$ and $\sigma_3(\frac{n}{6})$.One example is as follows:

$$\frac{\eta^{25}(2z)\eta^4(3z)}{\eta^{12}(z)\eta^5(4z)\eta^3(6z)\eta(12z)},$$

where the even coefficients are obtained. After that we find that we can express the even Fourier coefficients of 324 eta quotients in terms of $\sigma_{17}(n), \sigma_{17}\left(\frac{n}{2}\right), \sigma_{17}\left(\frac{n}{3}\right), \sigma_{17}\left(\frac{n}{4}\right), \sigma_{17}\left(\frac{n}{6}\right)$ and $\sigma_{17}\left(\frac{n}{12}\right)$, see Table 3. One example is as follows:

$$\eta^8(2z)\eta^{20}(4z)\eta^4(6z)\eta^4(12z).$$

We also see that the odd Fourier coefficients of 650 eta quotients are zero and even coefficients can be expressed by simple formula.

2. Main Body

Now let

$$f_1 := \sum_{n=0}^{\infty} f_1(n) = \frac{\eta^{17}(4z)\eta^{19}(6z)\eta^{13}(12z)}{\eta^{13}(2z)},$$

$$f_2 = \sum_{n=0}^{\infty} f_2(n) = \frac{\eta^{4}(4z)\eta^{20}(6z)\eta^{20}(12z)}{\eta^{8}(2z)},$$

$$f_3 = \sum_{n=0}^{\infty} f_3(n) = \frac{\eta^{16}(4z)\eta^{20}(6z)\eta^{8}(12z)}{\eta^{8}(2z)},$$

$$f_4 = \sum_{n=0}^{\infty} f_4(n) = \frac{\eta^{18}(4z)\eta^{6}(6z)\eta^{18}(12z)}{\eta^{6}(2z)},$$

$$f_5 = \sum_{n=0}^{\infty} f_5(n) = \frac{\eta^{13}(4z)\eta^{11}(6z)\eta^{17}(12z)}{\eta^{5}(2z)},$$

$$f_6 = \sum_{n=0}^{\infty} f_6(n) = \frac{\eta^{8}(4z)\eta^{16}(6z)\eta^{16}(12z)}{\eta^{4}(2z)},$$

$$f_7 = \sum_{n=0}^{\infty} f_7(n) = \frac{\eta^{20}(4z)\eta^{16}(6z)\eta^{4}(12z)}{\eta^{4}(2z)},$$

$$f_8 = \sum_{n=0}^{\infty} f_8(n) = \frac{\eta^{6}(2z)\eta^{18}(4z)\eta^{18}(6z)}{\eta^{6}(12z)},$$

$$f_9 = \sum_{n=0}^{\infty} f_9(n) = \frac{\eta^{7}(2z)\eta^{13}(4z)\eta^{17}(12z)}{\eta(6z)},$$

$$f_{10} = \sum_{n=0}^{\infty} f_{10}(n) = \eta^{8}(2z)\eta^{20}(4z)\eta^{4}(6z)\eta^{4}(12z),$$

$$f_{11} = \sum_{n=0}^{\infty} f_{11}(n) = \frac{\eta^{18}(4z)\eta^{18}(6z)\eta^{18}(12z)}{\eta^{18}(2z)},$$

$$f_{12} = \sum_{n=0}^{\infty} f_{12}(n) = \frac{\eta^{11}(2z)\eta^{17}(4z)\eta^{13}(12z)}{\eta^{5}(6z)},$$

$$f_{13} = \sum_{n=0}^{\infty} f_{12}(n) = \frac{\eta^{11}(2z)\eta^{17}(4z)\eta^{19}(6z)}{\eta^{11}(12z)},$$

$$f_{14} = \sum_{n=0}^{\infty} f_{14}(n) = \frac{\eta^{20}(2z)\eta^{20}(4z)\eta^{4}(12z)}{\eta^{8}(6z)},$$

$$f_{15} = \sum_{n=0}^{\infty} f_{15}(n) = \frac{\eta^{11}(2z)\eta^{19}(6z)\eta^{13}(12z)}{\eta^{7}(4z)},$$

$$f_{16} = \sum_{n=0}^{\infty} f_{16}(n) = \frac{\eta^{20}(2z)\eta^{16}(6z)\eta^{16}(12z)}{\eta^{16}(4z)},$$

$$f_{17} = \sum_{n=0}^{\infty} f_{17}(n) = \frac{\eta^{17}(2z)\eta^{13}(6z)\eta^{13}(12z)}{\eta^{7}(4z)},$$

$$f_{18} = \sum_{n=0}^{\infty} f_{18}(n) = \eta^{17}(2z)\eta^{5}(4z)\eta^{13}(6z)\eta(12z),$$

$$f_{19} = \sum_{n=0}^{\infty} f_{19}(n) = \frac{\eta^{18}(4z)\eta^{12}(6z)\eta^{18}(12z)}{\eta^{12}(2z)},$$

$$f_{20} = \sum_{n=0}^{\infty} f_{20}(n) = \frac{\eta^{13}(4z)\eta^{17}(6z)\eta^{17}(12z)}{\eta^{11}(2z)},$$

$$f_{21} = \sum_{n=0}^{\infty} f_{21}(n) = \frac{\eta^{17}(4z)\eta^{13}(6z)\eta^{13}(12z)}{\eta^{7}(2z)},$$

$$f_{22} = \sum_{n=0}^{\infty} f_{22}(n) = \frac{\eta^{4}(4z)\eta^{14}(6z)\eta^{20}(12z)}{\eta^{2}(2z)},$$

$$f_{23} = \sum_{n=0}^{\infty} f_{23}(n) = \frac{\eta^{16}(4z)\eta^{14}(6z)\eta^{8}(12z)}{\eta^{2}(2z)},$$

$$f_{24} = \sum_{n=0}^{\infty} f_{24}(n) = \frac{\eta^{18}(2z)\eta^{18}(6z)\eta^{12}(12z)}{\eta^{12}(4z)},$$

$$f_{25} = \sum_{n=0}^{\infty} f_{25}(n) = \eta^{18}(4z)\eta^{18}(12z),$$

$$f_{26} = \sum_{n=0}^{\infty} f_{26}(n) = \eta(2z)\eta^{13}(4z)\eta^{5}(6z)\eta^{17}(12z),$$

$$f_{27} = \sum_{n=0}^{\infty} f_{27}(n) \frac{\eta^{7}(2z)\eta^{19}(4z)\eta^{11}(6z)}{\eta(12z)},$$

$$f_{28} = \sum_{n=0}^{\infty} f_{27}(n) \frac{\eta^{12}(2z)\eta^{18}(4z)\eta^{18}(12z)}{\eta^{12}(6z)},$$

$$f_{29} = \sum_{n=0}^{\infty} f_{27}(n) \frac{\eta^{12}(2z)\eta^{18}(4z)\eta^{12}(6z)}{\eta^{6}(12z)}$$

$$f_{30} = \sum_{n=0}^{\infty} f_{30}(n) = \frac{\eta^{13}(2z)\eta^{13}(4z)\eta^{17}(12z)}{\eta^{7}(6z)},$$

$$f_{31} = \sum_{n=0}^{\infty} f_{31}(n) \frac{\eta^{18}(2z)\eta^{12}(4z)\eta^{18}(6z)}{\eta^{12}(12z)}.$$

Now we can state our main Theorem:

Theorem 1. Let $b_1, b_2, \cdots, b_5$ be non-negative integers satisfying

$$b_1 + b_2 + \cdots + b_5 \leq 36. \tag{5}$$

Define the integers $a_1, a_2, a_3, a_4, a_6, a_{12}$ by

$$a_1 := -b_1 + 2b_2 - 2b_3 - 4b_4 - b_5 + 36, \tag{6}$$

$$a_2 := 3b_1 + b_2 + 3b_3 + 10b_4 + b_5 - 90, \tag{7}$$

$$a_3 := 3b_1 + 2b_2 + 6b_3 + 4b_4 + 3b_5 - 108, \tag{8}$$

$$a_4 := -2b_1 - b_2 - b_3 - 4b_4 + 2b_5 + 36, \tag{9}$$

$$a_6 := -9b_1 - 7b_2 - 9b_3 - 10b_4 - 7b_5 + 270, \tag{10}$$

$$a_{12} := 6b_1 + 3b_2 + 3b_3 + 4b_4 + 2b_5 - 108. \tag{11}$$

They are functions of q by (3). Now define integers

$$k_0, k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}, k_{11}, k_{12}, k_{13}, k_{14}, k_{15}, k_{16}, k_{17}, k_{18},$$

$$k_{19}, k_{20}, k_{21}, k_{22}, k_{23}, k_{24}, k_{25}, k_{26}, k_{27}, k_{28}, k_{29}, k_{30}, k_{31}, k_{32}, k_{33}, k_{34}, k_{35}, \text{and } k_{36}$$

by

$$\frac{1}{2^{b_1+b_5}} x^{b_1}(1-x)^{b_2}(1+x)^{b_3}(1+2x)^{b_4}(2+x)^{b_5} = k_0 + k_1 x + k_2 x^2 + k_3 x + k_4 x^4 + k_5 x^5 + k_6 x^6 + k_7 x^7 + k_8 x^8 \tag{12}$$

$$+k_9 x^9 + k_{10} x^{10} + k_{11} x^{11} + k_{12} x^{12} + k_{13} x^{13} + k_{14} x^{14} + k_{15} x^{15} \tag{13}$$

$$+k_{16} x^{16} + k_{17} x^{17} + k_{18} x^{18} + k_{19} x^{19} + k_{20} x^{20} \tag{14}$$

$$+k_{21} x^{21} + k_{22} x^{22} + k_{23} x^{23} + k_{24} x^{24} + k_{25} x^{25} + k_{26} x^{26} + k_{27} x^{27} \tag{15}$$

$$+k_{28} x^{28} + k_{29} x^{29} + k_{30} x^{30} + k_{31} x^{31} + k_{32} x^{32} + k_{33} x^{33} + k_{34} x^{34} + k_{35} x^{35} + k_{36} x^{36}. \tag{16}$$

Define the rational numbers

$$c_1, c_2, c_3, c_4, c_6, c_{12}, r_1, r_2, \ldots, r_{30}$$

and r_{31} as in www.fatih.edu.tr/~bkendirli/weight18/Table 1. Here $\{f_1, \ldots f_{31}\} \backslash \{f_{11}\} \in S_{18}(\Gamma_0(12))$, $f_{11} \in M_{18}(\Gamma_0(12)) \backslash S_{18}(\Gamma_0(12))$ and

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z) = \delta(b_1) + \sum_{n=1}^{\infty} c(n) q^n,$$

where for n∈ ℕ,

$$c(n) = -c_1\sigma_{17}(n) - c_2\sigma_{17}\left(\frac{n}{2}\right) - c_3\sigma_{17}\left(\frac{n}{3}\right) - c_4\sigma_{17}\left(\frac{n}{4}\right) - c_6\sigma_{17}\left(\frac{n}{6}\right) - c_{12}\sigma_{17}\left(\frac{n}{12}\right) + r_1 f_1(n) + \cdots + r_{31} f_{31}(n).$$

In particular,

$$c(2n) = -c_1\sigma_{17}(2n) - c_2\sigma_{17}(n) - c_4\sigma_{17}\left(\frac{n}{2}\right) - (262\,145 c_3 + c_6)\sigma_{15}\left(\frac{n}{3}\right) - (c_{12} - 262\,144 c_3)\sigma_{15}\left(\frac{n}{6}\right) + r_1 f_1(2n) + \cdots + r_{16} f_{16}(2n),$$

$$c(2n-1) = -c_1\sigma_{15}(2n-1) - c_3\sigma_{15}\left(\frac{2n-1}{3}\right) + r_{17} f_{17}(2n-1) + \cdots + r_{31} f_{31}(2n-1),$$

for n∈ ℕ.

Proof. It follows from (6-11) that

$$a\_1 + 2a\_2 + 3a\_3 + 4a\_4 + 6a\_6 + 12a\_12 = 24b\_1, \tag{17}$$

$$a_1 + a_2 + a_3 + a_4 + a_6 + a_{12} = 36, \tag{18}$$

$$-\frac{a_1}{6} - \frac{a_2}{3} - \frac{a_3}{6} - \frac{2a_4}{3} - \frac{a_6}{3} - \frac{2a_{12}}{3} = -b_1 - b_5.$$

Now we will use p-k parametrization of Alaca, Alaca and Williams, see [1]:

$$p(q) := \frac{\varphi^2(q) - \varphi^2(q^3)}{2\varphi^2(q^3)}, k(q) := \frac{\varphi^3(q^3)}{\varphi(q)}, \tag{19}$$

where the theta function $\varphi(q)$ is defined by

$$\varphi(q)=\sum_{-\infty}^{\infty} q^{n^2}.$$

etting $x=p$ in (12), and multiplying both sides by k^{18},we obtain

$$\frac{k^{18}}{2^{b_1+b_5}}p^{b_1}(1-p)^{b_2}(1+p)^{b_3}(1+2p)^{b_4}(2+p)^{b_5}$$

$$\begin{aligned}=(&k_0+k_1p+k_2p^2+k_3p^3+k_4p^4+k_5p^5+k_6p^6+k_7p^7+k_8p^8+k_9p^9+k_{10}p^{10}+k_{11}p^{11}++k_{12}p^{12}+k_{13}p^{13}\\&+k_{14}p^{14}+k_{15}p^{15}+k_{16}p^{16}+k_{17}p^{17}+k_{18}p^{18}+k_{19}p^{19}+k_{20}p^{20}+k_{21}p^{21}+k_{22}p^{22}+k_{23}p^{23}+k_{24}p^{24}\\&+k_{25}p^{25}+k_{26}p^{26}+k_{27}p^{27}+k_{28}p^{28}+k_{29}p^{29}+k_{30}p^{30}+k_{31}p^{31}+k_{32}p^{32}+k_{33}p^{33}+k_{34}p^{34}\\&+k_{35}p^{35}+k_{36}p^{36})k^{18}.\end{aligned}$$

Alaca, Alaca and Williams [2] have established the following representations in terms of p and k:

$$\eta(q)=2^{-1/6}p^{1/24}(1-p)^{1/2}(1+p)^{1/6}(1+2p)^{1/8}(2+p)^{1/8}k^{1/2}, \tag{20}$$

$$\eta(q^2)=2^{-1/3}p^{1/12}(1-p)^{1/4}(1+p)^{1/12}(1+2p)^{1/4}(2+p)^{1/4}k^{1/2}, \tag{21}$$

$$\eta(q^3)=2^{-1/6}p^{1/8}(1-p)^{1/6}(1+p)^{1/2}(1+2p)^{1/24}(2+p)^{1/24}k^{1/2}, \tag{22}$$

$$\eta(q^4)=2^{-2/3}p^{1/6}(1-p)^{1/8}(1+p)^{1/24}(1+2p)^{1/8}(2+p)^{1/2}k^{1/2}, \tag{23}$$

$$\eta(q^6)=2^{-1/3}p^{1/4}(1-p)^{1/12}(1+p)^{1/4}(1+2p)^{1/12}(2+p)^{1/12}k^{1/2}, \tag{24}$$

$$\eta(q^{12})=2^{-2/3}p^{1/2}(1-p)^{1/24}(1+p)^{1/8}(1+2p)^{1/24}(2+p)^{1/6}k^{1/2}, \tag{25}$$

$$E_6(q):=1-504\sum_{n=1}^{\infty}\sigma_5(n)q^n$$

$$\begin{aligned}&=(1-246p-5532p^2-38614p^3-135369p^4-276084p^5\\&-348024p^6-276084p^7-135369p^8-38614p^9-5532p^{10}\\&-246p^{11}+p^{12})k^6,\end{aligned}$$

$$E_4(q):=1+240\sum_{n=1}^{\infty}\sigma_3(n)q^n$$

$$\begin{aligned}&=(1+124p+964p^2+2788p^3+3910p^4+2788p^5\\&+964p^6+124p^7+p^8)k^4.\end{aligned}$$

Therefore, since

$$E_{18}(q)=\frac{5500}{43867}E_6^3(q)+\frac{38367}{43867}E_6(q)E_4^3(q),$$

we have

$$\begin{aligned}E_{18}(q)=(&p^{36}+\frac{775\,242}{43\,867}p^{35}-\frac{935\,300\,808}{43\,867}p^{34}-\frac{479\,718\,421\,878}{43\,867}p^{33}\\&-\frac{38\,618\,625\,180\,573}{43\,867}p^{32}-\frac{1234\,640\,783\,535\,120}{43\,867}p^{31}\\&-\frac{21\,916\,011\,691\,473\,792}{43\,867}p^{30}-\frac{252\,643\,315\,041\,469\,584}{43\,867}p^{29}\\&-\frac{2065\,064\,598\,427\,463\,748}{43\,867}p^{28}-\frac{12\,645\,150\,195\,540\,418\,600}{43\,867}p^{27}\\&-\frac{60\,191\,705\,453\,414\,612\,832}{43\,867}p^{26}-\frac{228\,609\,396\,856\,786\,345\,128}{43\,867}p^{25}\\&-\frac{706\,047\,862\,569\,538\,312\,980}{43\,867}p^{24}-\frac{1798\,300\,198\,147\,477\,624\,560}{43\,867}p^{23}\\&-\frac{3817\,086\,982\,353\,184\,063\,104}{43\,867}p^{22}-\frac{6804\,790\,928\,276\,409\,773\,424}{43\,867}p^{21}\end{aligned}$$

$$
\begin{aligned}
&-\frac{10\,245\,612\,527\,911\,949\,392\,998}{43\,867}p^{20}-\frac{13\,077\,984\,442\,388\,649\,094\,404}{43\,867}p^{19}\\
&-\frac{14\,183\,115\,793\,553\,780\,830\,128}{43\,867}p^{18}-\frac{13\,077\,984\,442\,388\,649\,094\,404}{43\,867}p^{17}\\
&-\frac{10\,245\,612\,527\,911\,949\,392\,998}{43\,867}p^{16}-\frac{6804\,790\,928\,276\,409\,773\,424}{43\,867}p^{15}\\
&-\frac{3817\,086\,982\,353\,184\,063\,104}{43\,867}p^{14}-\frac{1798\,300\,198\,147\,477\,624\,560}{43\,867}p^{13}\\
&-\frac{706\,047\,862\,569\,538\,312\,980}{43\,867}p^{12}-\frac{228\,609\,396\,856\,786\,345\,128}{43\,867}p^{11}\\
&-\frac{60\,191\,705\,453\,414\,612\,832}{43\,867}p^{10}-\frac{12\,645\,150\,195\,540\,418\,600}{43\,867}p^{9}\\
&-\frac{2065\,064\,598\,427\,463\,748}{43\,867}p^{8}-\frac{252\,643\,315\,041\,469\,584}{43\,867}p^{7}\\
&-\frac{21\,916\,011\,691\,473\,792}{43\,867}p^{6}-\frac{1234\,640\,783\,535\,120}{43\,867}p^{5}\\
&-\frac{38\,618\,625\,180\,573}{43\,867}p^{4}-\frac{479\,718\,421\,878}{43\,867}p^{3}-\frac{935\,300\,808}{43\,867}p^{2}\\
&+\frac{775\,242}{43\,867}p+1)k^{18},
\end{aligned}
$$

$$
\begin{aligned}
E_{18}(q^2)={}&(p^{36}+18p^{35}+\frac{6309\,666}{43\,867}p^{34}+\frac{28\,961\,727}{43\,867}p^{33}\\
&-\frac{308\,674\,899}{87\,734}p^{32}-\frac{3639\,905\,496}{43\,867}p^{31}-\frac{84\,415\,605\,936}{43\,867}p^{30}\\
&-\frac{975\,153\,257\,352}{43\,867}p^{29}-\frac{7871\,740\,659\,954}{43\,867}p^{28}\\
&-\frac{48\,180\,925\,598\,836}{43\,867}p^{27}-\frac{229\,616\,693\,343\,792}{43\,867}p^{26}\\
&-\frac{872\,230\,167\,761\,940}{43\,867}p^{25}-\frac{2693\,410\,301\,866\,986}{43\,867}p^{24}\\
&-\frac{6859\,725\,720\,266\,232}{43\,867}p^{23}-\frac{14\,560\,859\,613\,263\,040}{43\,867}p^{22}\\
&-\frac{25\,958\,442\,940\,644\,024}{43\,867}p^{21}-\frac{39\,084\,193\,930\,940\,595}{43\,867}p^{20}\\
&-\frac{49\,888\,462\,700\,809\,602}{43\,867}p^{19}-\frac{54\,103\,968\,749\,611\,992}{43\,867}p^{18}\\
&-\frac{49\,888\,462\,700\,809\,602}{43\,867}p^{17}-\frac{39\,084\,193\,930\,940\,595}{43\,867}p^{16}\\
&-\frac{25\,958\,442\,940\,644\,024}{43\,867}p^{15}-\frac{14\,560\,859\,613\,263\,040}{43\,867}p^{14}\\
&-\frac{6859\,725\,720\,266\,232}{43\,867}p^{13}-\frac{2693\,410\,301\,866\,986}{43\,867}p^{12}\\
&-\frac{872\,230\,167\,761\,940}{43\,867}p^{11}-\frac{229\,616\,693\,343\,792}{43\,867}p^{10}\\
&-\frac{48\,180\,925\,598\,836}{43\,867}p^{9}-\frac{7871\,740\,659\,954}{43\,867}p^{8}\\
&-\frac{975\,153\,257\,352}{43\,867}p^{7}-\frac{84\,415\,605\,936}{43\,867}p^{6}-\frac{3639\,905\,496}{43\,867}p^{5}\\
&-\frac{308\,674\,899}{87\,734}p^{4}+\frac{28\,961\,727}{43\,867}p^{3}+\frac{6309\,666}{43\,867}p^{2}+18p+1)k^{18},
\end{aligned}
$$

$$
\begin{aligned}
E_{18}(q^3)={}&(p^{36}+18p^{35}+144p^{34}+\frac{29\,080\,230}{43\,867}p^{33}+\frac{81\,862\,371}{43\,867}p^{32}\\
&+\frac{129\,064\,464}{43\,867}p^{31}-\frac{22\,337\,088}{43\,867}p^{30}-\frac{1193\,934\,960}{43\,867}p^{29}\\
&-\frac{6473\,096\,388}{43\,867}p^{28}-\frac{28\,970\,785\,048}{43\,867}p^{27}\\
&-\frac{142\,462\,891\,584}{43\,867}p^{26}-\frac{613\,411\,925\,400}{43\,867}p^{25}
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1922\,036\,606\,100}{43\,867}p^{24}-\frac{4579\,376\,969\,232}{43\,867}p^{23}\\
&-\frac{9461\,957\,464\,512}{43\,867}p^{22}-\frac{17\,636\,769\,114\,768}{43\,867}p^{21}\\
&-\frac{27\,308\,190\,632\,166}{43\,867}p^{20}-\frac{33\,729\,332\,247\,900}{43\,867}p^{19}\\
&-\frac{35\,484\,183\,005\,664}{43\,867}p^{18}-\frac{33\,729\,332\,247\,900}{43\,867}p^{17}\\
&-\frac{27\,308\,190\,632\,166}{43\,867}p^{16}-\frac{17\,636\,769\,114\,768}{43\,867}p^{15}\\
&-\frac{9461\,957\,464\,512}{43\,867}p^{14}-\frac{4579\,376\,969\,232}{43\,867}p^{13}\\
&-\frac{1922\,036\,606\,100}{43\,867}p^{12}-\frac{613\,411\,925\,400}{43\,867}p^{11}\\
&-\frac{142\,462\,891\,584}{43\,867}p^{10}-\frac{28\,970\,785\,048}{43\,867}p^{9}\\
&-\frac{6473\,096\,388}{43\,867}p^{8}-\frac{1193\,934\,960}{43\,867}p^{7}-\frac{22\,337\,088}{43\,867}p^{6}\\
&+\frac{129\,064\,464}{43\,867}p^{5}+\frac{81\,862\,371}{43\,867}p^{4}+\frac{29\,080\,230}{43\,867}p^{3}\\
&+144p^{2}+18p+1)k^{18},
\end{aligned}
$$

$$
\begin{aligned}
E_{18}(q^4)=(&\frac{1}{262\,144}p^{36}+\frac{401\,985}{5749\,735\,424}p^{35}-\frac{233\,699\,517}{2874\,867\,712}p^{34}\\
&+\frac{111\,942\,533\,949}{2874\,867\,712}p^{33}-\frac{11\,657\,555\,492\,571}{5749\,735\,424}p^{32}\\
&+\frac{7703\,600\,322\,195}{359\,358\,464}p^{31}-\frac{6974\,484\,891\,375}{359\,358\,464}p^{30}\\
&-\frac{22\,722\,809\,742\,693}{89\,839\,616}p^{29}+\frac{53\,240\,261\,186\,547}{359\,358\,464}p^{28}\\
&+\frac{232\,858\,848\,626\,717}{179\,679\,232}p^{27}-\frac{4607\,875\,351\,917}{44\,919\,808}p^{26}\\
&-\frac{324\,266\,361\,567\,831}{89\,839\,616}p^{25}-\frac{262\,813\,037\,358\,195}{179\,679\,232}p^{24}\\
&+\frac{215\,260\,077\,592\,395}{44\,919\,808}p^{23}+\frac{112\,489\,707\,780\,261}{44\,919\,808}p^{22}\\
&-\frac{37\,710\,901\,527\,003}{5614\,976}p^{21}-\frac{381\,947\,582\,978\,163}{44\,919\,808}p^{20}\\
&-\frac{65\,142\,446\,901\,093}{22\,459\,904}p^{19}-\frac{11\,327\,286\,134\,973}{11\,229\,952}p^{18}\\
&-\frac{42\,136\,567\,354\,953}{11\,229\,952}p^{17}-\frac{107\,022\,904\,612\,137}{22\,459\,904}p^{16}\\
&-\frac{8257\,547\,688\,291}{2807\,488}p^{15}-\frac{3154\,099\,233\,537}{2807\,488}p^{14}\\
&-\frac{278\,753\,497\,173}{701\,872}p^{13}-\frac{501\,559\,886\,151}{2807\,488}p^{12}\\
&-\frac{91\,138\,556\,457}{1403\,744}p^{11}-\frac{811\,095\,417}{43\,867}p^{10}\\
&-\frac{11\,577\,929\,941}{701\,872}p^{9}-\frac{22\,511\,837\,745}{1403\,744}p^{8}\\
&-\frac{2462\,497\,353}{350\,936}p^{7}+\frac{304\,619\,577}{350\,936}p^{6}+\frac{129\,466\,656}{43\,867}p^{5}\\
&+\frac{81\,919\,827}{43\,867}p^{4}+663p^{3}+144p^{2}+18p+1)k^{18},
\end{aligned}
$$

$$
\begin{aligned}
E_{18}(q^6)=(&p^{36}+18p^{35}+144p^{34}+663p^{33}+\frac{3735}{2}p^{32}+2952p^{31}\\
&+\frac{38\,278\,992}{43\,867}p^{30}-\frac{307\,015\,416}{43\,867}p^{29}-\frac{686\,939\,634}{43\,867}p^{28}\\
&-\frac{515\,629\,228}{43\,867}p^{27}+\frac{438\,109\,344}{43\,867}p^{26}+\frac{1351\,258\,740}{43\,867}p^{25}
\end{aligned}
$$

$$+\frac{1068\,365\,142}{43\,867}p^{24}-\frac{308\,234\,376}{43\,867}p^{23}-\frac{1351\,213\,920}{43\,867}p^{22}$$
$$-\frac{1117\,405\,512}{43\,867}p^{21}-\frac{168\,597\,747}{43\,867}p^{20}+\frac{521\,787\,474}{43\,867}p^{19}$$
$$+\frac{715\,735\,824}{43\,867}p^{18}+\frac{521\,787\,474}{43\,867}p^{17}-\frac{168\,597\,747}{43\,867}p^{16}$$
$$-\frac{1117\,405\,512}{43\,867}p^{15}-\frac{1351\,213\,920}{43\,867}p^{14}-\frac{308\,234\,376}{43\,867}p^{13}$$
$$+\frac{1068\,365\,142}{43\,867}p^{12}+\frac{1351\,258\,740}{43\,867}p^{11}+\frac{438\,109\,344}{43\,867}p^{10}$$
$$-\frac{515\,629\,228}{43\,867}p^{9}-\frac{686\,939\,634}{43\,867}p^{8}-\frac{307\,015\,416}{43\,867}p^{7}$$
$$+\frac{38\,278\,992}{43\,867}p^{6}+2952p^{5}+\frac{3735}{2}p^{4}+663p^{3}+144p^{2}+18p+1)k^{18},$$

$$E_{18}(q^6)=(\frac{1}{262\,144}p^{36}+\frac{9}{131\,072}p^{35}+\frac{9}{16\,384}p^{34}$$
$$+\frac{7271\,853}{2874\,867\,712}p^{33}+\frac{40\,990\,437}{5749\,735\,424}p^{32}$$
$$+\frac{4060\,197}{359\,358\,464}p^{31}-\frac{586\,713}{359\,358\,464}p^{30}$$
$$-\frac{9258\,939}{89\,839\,616}p^{29}-\frac{201\,932\,793}{359\,358\,464}p^{28}$$
$$-\frac{226\,257\,313}{179\,679\,232}p^{27}+\frac{415\,196\,415}{89\,839\,616}p^{26}$$
$$+\frac{4123\,232\,613}{89\,839\,616}p^{25}+\frac{27\,284\,567\,805}{179\,679\,232}p^{24}$$
$$+\frac{7840\,865\,637}{44\,919\,808}p^{23}-\frac{19\,383\,267\,141}{44\,919\,808}p^{22}$$
$$-\frac{11\,414\,735\,505}{5614\,976}p^{21}-\frac{132\,668\,452\,503}{44\,919\,808}p^{20}$$
$$+\frac{17\,700\,651\,225}{22\,459\,904}p^{19}+\frac{54\,470\,572\,833}{5614\,976}p^{18}$$
$$+\frac{157\,667\,087\,871}{11\,229\,952}p^{17}+\frac{33\,347\,831\,031}{22\,459\,904}p^{16}$$
$$-\frac{61\,501\,278\,789}{2807\,488}p^{15}-\frac{82\,956\,848\,247}{2807\,488}p^{14}$$
$$-\frac{4774\,798\,179}{701\,872}p^{13}+\frac{68\,418\,816\,597}{2807\,488}p^{12}$$
$$+\frac{985\,959}{32}p^{11}+\frac{159\,921}{16}p^{10}-\frac{188\,011}{16}p^{9}$$
$$-\frac{501\,075}{32}p^{8}-\frac{55\,989}{8}p^{7}+\frac{6981}{8}p^{6}+2952p^{5}$$
$$+\frac{3735}{2}p^{4}+663p^{3}+144p^{2}+18p+1)k^{18}.$$

We can also similarly determine $f_1,\cdots,f_{30}$ and f_{31} in terms of p and k as in www.fatih.edu.tr/~bkendirli/weight18/Table 3. Obviously, $f_1,\ldots f_{31}$ are functions of , see (3),(19). We see that $\{f_1,\ldots f_{31}\}\backslash\{f_{11}\}\in S_{18}(\Gamma_0(12))$, $f_{11}\in M_{18}(\Gamma_0(12))\backslash S_{18}(\Gamma_0(12))$ by [4]. Now

$$\eta^{a_1}(z)\eta^{a_2}(2z)\eta^{a_3}(3z)\eta^{a_4}(4z)\eta^{a_6}(6z)\eta^{a_{12}}(12z)$$
$$=q^{b_1}\prod_{n=1}^{\infty}(1-q^n)^{a_1}(1-q^{2n})^{a_2}(1-q^{3n})^{a_3}(1-q^{4n})^{a_4}(1-q^{6n})^{a_6}(1-q^{12n})^{a_{12}}$$
$$=2^{-\frac{a_1}{6}-\frac{a_2}{3}-\frac{a_3}{6}-\frac{2a_4}{3}-\frac{a_6}{3}-\frac{2a_{12}}{3}}p^{\frac{a_1}{24}+\frac{a_2}{12}+\frac{a_3}{8}+\frac{a_4}{6}+\frac{a_6}{4}+\frac{a_{12}}{2}}(1-p)^{\frac{a_1}{2}+\frac{a_2}{4}+\frac{a_3}{6}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}}$$
$$(1+p)^{\frac{a_1}{6}+\frac{a_2}{12}+\frac{a_3}{2}+\frac{a_4}{24}+\frac{a_6}{4}+\frac{a_{12}}{8}}(1+2p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{8}+\frac{a_6}{12}+\frac{a_{12}}{24}}(2+p)^{\frac{a_1}{8}+\frac{a_2}{4}+\frac{a_3}{24}+\frac{a_4}{2}+\frac{a_6}{12}+\frac{a_{12}}{6}}$$
$$k^{\frac{a_1+a_2+a_3+a_4+a_6+a_{12}}{2}}=\frac{k^{18}}{2^{b_1+b_5}}p^{b_1}(1-p)^{b_2}(1+p)^{b_3}(1+2p)^{b_4}(2+p)^{b_5}$$

$$= k^{18}(k_0 + k_1p + k_2p^2 + k_3p^3 + k_4p^4 + k_5p^5 + k_6p^6$$
$$+k_7p^7 + k_8p^8 + k_9p^9 + k_{10}p^{10} + k_{11}p^{11}$$
$$+k_{12}p^{12} + k_{13}p^{13} + k_{14}p^{14} + k_{15}p^{15} + k_{16}p^{16}$$
$$k_{17}p^{17} + k_{18}p^{18} + k_{19}p^{19} + k_{20}p^{20} + k_{21}p^{21} + k_{22}p^{22}$$
$$+k_{23}p^{23} + k_{24}p^{24} + k_{25}p^{25} + k_{26}p^{26} + k_{27}p^{27} + k_{28}p^{28}$$
$$+k_{29}p^{29} + k_{30}p^{30} + k_{31}p^{31} + k_{32}p^{32} + k_{33}p^{33} + k_{34}p^{34} + k_{35}p^{35} + k_{36}p^{36})$$

$$= \frac{43867c_1}{28728}\left(1 - \frac{28728}{43867}\sum_{n=1}^{\infty}\sigma_{17}(n)q^n\right) + \frac{43867c_2}{28728}\left(1 - \frac{28728}{43867}\sum_{n=1}^{\infty}\sigma_{17}(n)q^{2n}\right)$$
$$+\frac{43867c_3}{28728}\left(1 - \frac{28728}{43867}\sum_{n=1}^{\infty}\sigma_{17}(n)q^{3n}\right) + \frac{43867c_4}{28728}\left(1 - \frac{28728}{43867}\sum_{n=1}^{\infty}\sigma_{17}(n)q^{4n}\right)$$
$$+\frac{43867c_6}{28728}\left(1 - \frac{28728}{43867}\sum_{n=1}^{\infty}\sigma_{17}(n)q^{6n}\right) + \frac{43867c_{12}}{28728}\left(1 - \frac{28728}{43867}\sum_{n=1}^{\infty}\sigma_{17}(n)q^{12n}\right)$$
$$+r_1q^{13}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{17}(1-q^{6n})^{19}(1-q^{12n})^{13}}{(1-q^{2n})^{13}}$$
$$r_2q^{15}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{4}(1-q^{6n})^{20}(1-q^{12})^{20}}{(1-q^{2n})^{8}}$$
$$+r_3q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{16}(1-q^{6n})^{20}(1-q^{12n})^{8}}{(1-q^{2n})^{8}}$$
$$+r_4q^{13}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{18}(1-q^{6n})^{6}(1-q^{12n})^{18}}{(1-q^{2n})^{6}}$$
$$+r_5.q^{13}\prod_{n=1}^{\infty}\frac{((1-q^{4n})^{13}(1-q^{6n})^{11}(1-q^{12n})^{17}}{(1-q^{2n})^{5}}$$
$$+r_6q^{13}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{8}(1-q^{6n})^{16}(1-q^{12n})^{16}}{(1-q^{2n})^{4}}$$
$$+r_7q^{9}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{20}(1-q^{6n})^{16}(1-q^{12n})^{4}}{(1-q^{2n})^{4}}$$
$$+r_8q^{5}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{6}(1-q^{4n})^{18}(1-q^{6n})^{18}}{(1-q^{12n})^{6}}$$

$$+r_9q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{7}(1-q^{4n})^{13}(1-q^{12n})^{17}}{(1-q^{6n})}$$
$$+r_{10}q^{7}\prod_{n=1}^{\infty}(1-q^{2n})^{8}(1-q^{4n})^{20}(1-q^{6n})^{4}(1-q^{12n})^{4}$$
$$+r_{11}q^{15}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{18}(1-q^{6n})^{18}(1-q^{12n})^{18}}{(1-q^{2n})^{18}}$$
$$+r_{12}q^{9}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{11}(1-q^{4n})^{17}(1-q^{12n})^{13}}{(1-q^{6n})^{5}}$$
$$+r_{13}q^{3}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{11}(1-q^{4n})^{17}(1-q^{6n})^{19}}{(1-q^{12n})^{11}}$$
$$+r_{14}q^{5}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{20}(1-q^{4n})^{20}(1-q^{12n})^{4}}{(1-q^{6n})^{8}}$$
$$+r_{15}q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{11}(1-q^{6n})^{19}(1-q^{12n})^{13}}{(1-q^{4n})^{7}}$$

$$+r_{16}q^{11}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{20}(1-q^{6n})^{16}(1-q^{12n})^{16}}{(1-q^{4n})^{16}}$$
$$+r_{17}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{17}(1-q^{6n})^{13}(1-q^{12n})^{13}}{(1-q^{4n})^{7}}$$
$$+r_{18}q^{6}\prod_{n=1}^{\infty}(1-q^{2n})^{17}(1-q^{4n})^{5}(1-q^{6n})^{13}(1-q^{12n})$$
$$+r_{19}q^{14}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{18}(1-q^{6n})^{12}(1-q^{12n})^{18}}{(1-q^{2n})^{12}}$$
$$+r_{20}q^{14}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{13}(1-q^{6n})^{17}(1-q^{12n})^{17}}{(1-q^{2n})^{11}}$$
$$+r_{21}q^{12}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{17}(1-q^{6n})^{13}(1-q^{12n})^{13}}{(1-q^{2n})^{7}}$$
$$+r_{22}q^{14}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{4}(1-q^{6n})^{14}(1-q^{12n})^{20}}{(1-q^{2n})^{2}}$$
$$+r_{23}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{4n})^{16}(1-q^{6n})^{14}(1-q^{12n})^{8}}{(1-q^{2n})^{2}}$$
$$+r_{24}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{18}(1-q^{6n})^{18}(1-q^{12n})^{12}}{(1-q^{4n})^{12}}$$
$$+r_{25}q^{12}\prod_{n=1}^{\infty}(1-q^{4n})^{18}(1-q^{12n})^{18}$$
$$+r_{26}q^{12}\prod_{n=1}^{\infty}(1-q^{2n})(1-q^{4n})^{13}(1-q^{6n})^{5}(1-q^{12n})^{17}$$
$$+r_{27}q^{6}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{7}(1-q^{4n})^{19}(1-q^{6n})^{11}}{(1-q^{12n})}$$

$$+r_{28}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{12}(1-q^{4n})^{18}(1-q^{12n})^{18}}{(1-q^{6n})^{12}}$$
$$+r_{29}q^{4}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{12}(1-q^{4n})^{18}(1-q^{6n})^{12}}{(1-q^{12n})^{6}}$$
$$+r_{30}q^{10}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{13}(1-q^{4n})^{13}(1-q^{12n})^{17}}{(1-q^{6n})^{7}}$$
$$+r_{31}q^{2}\prod_{n=1}^{\infty}\frac{(1-q^{2n})^{18}(1-q^{4n})^{12}(1-q^{6n})^{18}}{(1-q^{12n})^{12}}$$
$$=\delta(b_1)-\sum_{n=1}^{\infty}(c_1\sigma_{17}(n)+c_2\sigma_{17}\left(\frac{n}{2}\right)+c_3\sigma_{17}\left(\frac{n}{3}\right)+c_4\sigma_{17}\left(\frac{n}{4}\right)$$
$$+c_6\sigma_{17}\left(\frac{n}{6}\right)+c_{12}\sigma_{17}\left(\frac{n}{12}\right))+r_1f_1(n)+\dots.+r_{31}f_{31}(n),$$

where

$$\delta(b_1)=\begin{cases}0 \text{ if } b_1\neq 0\\ 1 \text{ if } b_1=0\end{cases}.$$

So

$$c(n) = -(c_1\sigma_{17}(n) + c_2\sigma_{17}\left(\frac{n}{2}\right) + c_3\sigma_{17}\left(\frac{n}{3}\right) + c_4\sigma_{17}\left(\frac{n}{4}\right)$$
$$+c_6\sigma_{17}\left(\frac{n}{6}\right) + c_{12}\sigma_{17}\left(\frac{n}{12}\right)) + r_1 f_1(n) + \ldots + r_{31} f_{31}(n).$$

Therefore, for n=1,2,…,

$$c(2n) = -c_1\sigma_{17}(2n) - c_2\sigma_{17}(n) - c_4\sigma_{17}\left(\frac{n}{2}\right) - (262\,145c_3 + c_6)\sigma_{17}\left(\frac{n}{3}\right)$$
$$-(c_{12} - 262\,144c_3)\sigma_{17}\left(\frac{n}{6}\right) + r_{17}f_{17}(2n) + \cdots + r_{31}f_{31}(2n),$$

$$c(2n-1) = -c_1\sigma_{15}(2n-1) - c_3\sigma_{15}\left(\frac{2n-1}{3}\right)$$
$$+r_1 f_1(2n-1) + \cdots + r_{14}f_{14}(2n-1),$$

since it is easy to see that

$$\sigma_k\left(\frac{2n}{3}\right) = (2^k+1)\sigma_k\left(\frac{n}{3}\right) - 2^k\sigma_k\left(\frac{n}{6}\right)$$

hence,

$$\sigma_{17}\left(\frac{2n}{3}\right) = 262\,145\sigma_{17}\left(\frac{n}{3}\right) - 262\,144\sigma_{17}\left(\frac{n}{6}\right),$$

and, for n=1,2,…,

$$f_1(2n) = \cdots = f_{14}(2n) = 0,$$
$$f_{15}(2n-1) = \cdots = f_{27}(2n-1) = 0.$$

3. Conclusion

1. We have found 324 eta quotients, see www.fatih.edu.tr/~bkendirli/weight18/Table 4, such that, for n=1,2,⋯,

$$c(2n) = -c_1\sigma_{17}(2n) - c_2\sigma_{17}(n) - c_4\sigma_{17}\left(\frac{n}{2}\right) - (262\,145c_3 + c_6)\sigma_{17}\left(\frac{n}{3}\right)$$
$$-(c_{12} - 262\,144c_3)\sigma_{17}\left(\frac{n}{6}\right)$$

$$c(2n-1) = -c_1\sigma_{17}(2n-1) - c_3\sigma_{17}\left(\frac{2n-1}{3}\right) + r_1 f_1(2n-1) + \cdots + r_{14}f_{14}(2n-1).$$

and 650 eta quotients, such that for n=1,2,⋯,

$$c(2n) = -c_1\sigma_{17}(2n) - c_2\sigma_{17}(n) - c_4\sigma_{17}\left(\frac{n}{2}\right) - c_6\sigma_{17}\left(\frac{n}{3}\right)$$
$$-c_{12}\sigma_{15}\left(\frac{n}{6}\right) + r_{17}f_{17}(2n) + \cdots + r_{31}f_{31}(2n),$$
$$c(2n-1) = 0.$$

2. $S_{18}(\Gamma_0(12))$ is 31 dimensional, $M_{18}(\Gamma_0(12))$ is 37 dimensional,see [5] (Chapter 3, pg.87 and Chapter 5, pg.197), and generated by

$\Delta_{1,18}, \Delta_{1,18}(2z), \Delta_{1,18}(3z), \Delta_{1,18}(4z), \Delta_{1,18}(6z), \Delta_{1,18}(12z),$
$\Delta_{2,18}, \Delta_{2,18}(2z), \Delta_{2,18}(3z), \Delta_{2,18}(6z),$
$\Delta_{3,18,1}, \Delta_{3,18,1}(2z), \Delta_{3,18,1}(4z),$
$\Delta_{3,18,2}(z), \Delta_{3,18,2}(2z), \Delta_{3,18,2}(4z),$
$\Delta_{3,18,3}(z), \Delta_{3,18,3}(2z), \Delta_{3,18,3}(4z)$(conjugate of $\Delta_{3,18,2}$ by $x^2 - 594x - 42912$),
$\Delta_{4,18,1}, \Delta_{4,18,1}(3z),$
$\Delta_{4,18,2}, \Delta_{4,18,2}(3z)$(conjugate of $\Delta_{4,18,1}$ by $x^2 + 5880x - 336440304$),
$\Delta_{6,18,1}, \Delta_{6,18,1}(2z), \Delta_{6,18,2}, \Delta_{6,18,2}(2z), \Delta_{6,18,3}, \Delta_{6,18,3}(2z),$
$\Delta_{12,18,1}, \Delta_{12,18,2},$

where $\Delta_{1,18}$ is the unique newform in $S_{18}(\Gamma_0(1))$; $\Delta_{2,18}$ is the unique newform in $S_{18}(\Gamma_0(2))$; $\Delta_{3,18,1}, \Delta_{3,18,2}$ and $\Delta_{3,18,3}$ are the unique newforms in $S_{18}(\Gamma_0(3))$, $\Delta_{4,18,1}, \Delta_{4,18,2}$ are the unique newforms in $S_{18}(\Gamma_0(4))$, $\Delta_{6,18,1}, \Delta_{6,18,2}$ and $\Delta_{6,18,3}$ are the unique newforms in $S_{18}(\Gamma_0(6))$, and $\Delta_{12,18,1}, \Delta_{12,18,2}$ are the unique newforms in $S_{18}(\Gamma_0(12))$. By taking t as a root of $x^2 - 594x - 42912$, and s as the root of $x^2 + 5880x - 336440304$, we see $f_1, \cdots, f_{31}$ as linear combinations of this basis in www.fatih.edu.tr/~bkendirli/weight18/Table 3.

References

[1] Ayşe Alaca, Şaban Alaca and Kenneth S. Williams, *On the two−dimensional theta functions of Borweins*, Acta Arith. 124 (2006) 177-195. Carleton University, Ottawa, Ontario, Canada K1S 5B6. E-mail: aalaca@math.carleton.ca, salaca@math.carleton.ca, williams@math.carleton.ca

[2] *Evaluation of the convolution sums* $\sum_{l+12m=n} \sigma(l)\sigma(m)$ *and* $\sum_{3l+4m=n} \sigma(l)\sigma(m)$, Adv. Theor. Appl. Math. 1(2006), 27−48.

[3] Basil Gordon, *Some identities in combinatorial analysis*, Quart. J. Math. Oxford Ser.12 (1961), 285-290. University of California, Los Angeles

[4] Basil Gordon[1] and Sınai Robins[2] , *Lacunarity of Dedekind* η *−products,* Glasgow Math. J. 37 (1995), 1-14. [1]University of California, Los Angeles, [2]University of Northern Colorado, Greeley

[5] Fred Diamond[1], Jerry Shurman[2], *A First Course in Modular Forms,*Springer Graduate Texts in Mathematics 228 [1]Brandeis University Waltham, MA 02454 USA, [2]Reed College Portland, OR 97202 USA E-mail: fdiamond@brandeis.edu, jerry@reed.edu

[6] Victor G. Kac, *Infinite−dimensional algebras, Dedekind's* η *−function, classical Möbius function and the very strange formula*, Adv. Math. 30 (1978) 85-136. MIT, Cambridge, Massachusetts 02139

[7] Barış Kendirli, "*Evaluation of Some Convolution Sums by Quasimodular Forms*", European Journal of Pure and Applied Mathematics ISSN 13075543 Vol.8., No. 1, Jan. 2015, pp. 81-110 Aydın University Istanbul/Turkey E-mail:baris.kendirli@gmail.com

[8] "*Evaluation of Some Convolution Sums and Representation Numbers of Quadratic Forms of Discriminant* 135", British Journal of Mathematics and Computer Science, Vol6/6, Jan. 2015, pp. 494-531.

[9] *Evaluation of Some Convolution Sums and the Representation numbers,* Ars Combinatoria Volume CXVI, July, pp 65-91.

[10] *Cusp Forms in* $S_4(\Gamma_0(79))$ *and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant* -79, Bulletin of the Korean Mathematical Society Vol 49/3 2012

[11] *Cusp Forms in* $S_4(\Gamma_0(47))$ *and the number of representations of positive integers by some direct sum of binary quadratic forms with discriminant* -47,Hindawi , International Journal of Mathematics and Mathematical Sciences Vol 2012, 303492 10 pages

[12] *The Bases of* $M_4(\Gamma_0(71))$,$M_6(\Gamma_0(71))$ *and the Number of Representations of Integers,* Hindawi, Mathematical Problems in Engineering Vol 2013, 695265, 34 pages

[13] Günter Köhler, Eta Products and Theta Series Identities (Springer−Verlag, Berlin, 2011). University of Würzburg Am Hubland 97074 Würzburg Germany E-mail:koehler@mathematik.uni-wuerzburg.de

[14] Ian Grant Macdonald, *Affine root systems and Dedekind's* η *−function*, Invent. Math. 15 (1972), 91-143.

[15] Olivia X. M. Yao[1], Ernest X. W. Xia[2] and J. Jin[3], Explicit Formulas for the Fourier coefficients of a class of eta quotients, International Journal of Number Theory Vol. 9, No. 2 (2013) 487-503. *[1,2]Jiangsu University Zhenjiang, Jiangsu, 212013, P. R. China [3]Nanjing Normal University Taizhou College, 225300, Jiangsu, P. R. China E-mail:yaoxiangmei@163.com, ernestxwxia@163.com, jinjing19841@126.com*

[16] Kenneth S. Williams, Fourier series of a class of eta quotients, Int. J. Number Theory 8 (2012), 993-1004. Carleton University, Ottawa, Ontario, Canada K1S 5B6 E-mail: williams@math.carleton.ca.