Pure and Applied Mathematics Journal
2015; 4(4): 147-154
Published online June 17, 2015 (http://www.sciencepublishinggroup.com/j/pamj)
doi: 10.11648/j.pamj.20150404.12
ISSN: 2326-9790 (Print); ISSN: 2326-9812 (Online)

Mathematical Methodologies in Physics and Their Applications in Derivation of Velocity and Acceleration Theories

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To cite this article:
Edward T. H. Wu. Mathematical Methodologies in Physics and Their Applications in Derivation of Velocity and Acceleration Theories. *Pure and Applied Mathematics Journal*. Vol. 4, No. 4, 2015, pp. 147-154. doi: 10.11648/j.pamj.20150404.12

Abstract: The principles to use variables and mathematical methodologies in physics are addressed. A set of refined definitions with designated variables are used to derive the Velocity and Acceleration Theories in Distance Field and Vector Space. Mathematical methodologies such as Linear Algebra and Vector Calculus are used systematically in a step by step derivation process. The proof of the theories can be easily achieved by substitution of the designated variables with a set of parameters that matches the same assumptions and conditions in every step of the derivation process.

Keywords: Variables, Parameters, Velocity, Acceleration, Linear Algebra, Vector Calculus, Mathematical Methodology

1. Variable and Parameter

A designated variable can be used to represent any individual number (element) of a specific domain in mathematics, or any individual parameter of a specific property of a substance in physics, for the purpose of stating a function, operation and correlation between the domains in mathematics or the properties of a substance in physics. In another word, every definition, principle and theory in mathematics and physics can be represented and correlated by a group of variables as long as each one of them is designated exclusively to a specific domain in mathematics or a specific property of a substance in physics.

2. Mathematical Methodology

The correlations and theories between a group of physical properties with parameters containing numerical quantities and directions, at a specific state and under certain assumptions and conditions, of the same or different substances can be derived by designated variables through the operation and calculation processes of mathematical methodologies such as Linear Algebra (Ref. 1) (Ref. 2) (Ref. 3) (Ref. 4), Vector Calculus (Ref. 5) (Ref. 6) (Ref. 7), etc.

3. Derivation of Theories

A standard procedure to derive a theory in physics involves the following processes:

1. *VARIABLES* – Each designated variable is assigned to a property at a specific state of a substance.
2. *IF* – Assumptions and conditions are adapted.
3. *FACT*–Definitions, principles and theories in mathematics and physics are applied.
4. *DERIVATION*–Experience, experiments, logics and mathematical methodologies are used in the derivation processes.
5. *THEORY* – Correlations between the properties of the same or different substances are obtained.

Example 1. For a motion in Distance Field, at any initial state (x_i, t_i) & final state (x_f, t_f) where $x_i < x_f$,

$$x_f - x_i = \int_{t_i}^{t_f} v dt$$

[DERIVATION]

1. *VARIABLES*

Assign "t" to time, "x" to distance and "v" to velocity. Also assign t_i to the initial time, x_i to the initial distance and v_i to the initial velocity; t_f to the final time, x_f to the final distance and v_f to the final velocity, etc.

2. *IF*

The motion in Distance Field is a continuous function F(t) = x, where "x" is distance and "t" is time. At any time "t", there is a corresponding distance "x", together they form a function set (x, t). Similarly at its adjacent time t', where t' > t & (t' – t) = $\Delta t \rightarrow 0$, there is a corresponding distance x', also together they form a function set (x', t').

3. *FACT*

Appling the following definitions:

dt = t' – t

dx = x' – x

v = dx/dt

for any two time t_i and t_f, where $t_i < t_f$, we can find (x_i, t_i) and (x_f, t_f) and a group of sequential numbers t_i, t_1, t_2, …, t_m, t_f in the time domain between t_i & t_f, where $(t_1 - t_i) = \Delta t_i \rightarrow 0$, $(t_2 - t_1) = \Delta t_1 \rightarrow 0$, $(t_2 - t_1) = \Delta t_2 \rightarrow 0$, …, $(t_f - t_m) = \Delta t_m \rightarrow 0$, and also a group of corresponding sequential numbers x_i, x_1, x_2, …, x_m, x_f in the distance domain between x_i & x_f, to form a group of function sets (x_i, t_i), (x_1, t_1), (x_2, t_2), …, (x_m, t_m), (x_f, t_f), such that:

$$dt_i = t_1 - t_i, \quad dt_1 = t_2 - t_1, \ldots, \quad dt_m = t_f - t_m$$

$$dx_i = x_1 - x_i, \quad dx_1 = x_2 - x_1, \ldots, \quad dx_m = x_f - x_m$$

$$dx_i = v_i dt_i, \quad dx_1 = v_1 dt_1, \ldots, \quad dx_m = v_m dt_m$$

4. *DERIVATION*

$$\int_{x_i}^{x_f} dx = dx_i + dx_1 + \ldots + dx_m = (x_1 - x_i) + (x_2 - x_1) + \ldots + (x_f - x_m) = x_f - x_i$$

$$\int_{t_i}^{t_f} vdt = v_i dt_i + v_1 dt_1 + \ldots + v_m dt_m$$

Therefore,

$$x_f - x_i = \int_{t_i}^{t_f} vdt$$

5. *THEORY*

The derivation process is completed and the Theory is derived:

For a motion in Distance Field, at any initial state (x_i, t_i) & final state (x_f, t_f) where $x_i < x_f$,

$$x_f - x_i = \int_{t_i}^{t_f} vdt$$

4. Proof of Theory and Solution

To prove a theory or to verify the solution of a problem, "Substitution Principle" is used with the following procedures:

1. Apply *SUBSTITUTION* to the designated variables in the derivation or solution process by a set of parameters of the properties at a specific state of the substance that meets the *IF* conditions and assumptions.
2. Adapt the same *FACT* and *DERIVATION* process as that in deriving the theory or solving the problem in every steps of the proving process.
3. *PROOF* of the theory and solution can be achieved by obtaining the same answer and result at the end of the proving process.

The proof of the Theory and Solution is quite straight forward because the DERIVATION process leads to the same answers and results for every set of the parameters of the properties, at a specific state of the substance, that can satisfy the IF assumptions and conditions.

Example 2. For a motion in Distance Field, If $x = 5t + 2t^2$, what is the acceleration?

[SOLVE PROBLEM]

1. Assign *VARIABLE*:

"t" is the time from the starting point

$$\Delta t > 0 \ \& \ \Delta t \rightarrow 0$$

$$t' = t + \Delta t$$

$$t'' = t' + \Delta t$$

2. Adapt *IF*:

$$x = 5t + 2t^2$$

$$x' = 5t' + 2(t')^2$$

$$x'' = 5t'' + 2(t'')^2$$

3. Apply *FACT* in *DERIVATION* process:

$$dt = t' - t = \Delta t$$

$$dx = x' - x = 5(t' - t) + 2((t')^2 - t^2)$$

$$= 5\Delta t + 2(t^2 + 2t\Delta t + \Delta t^2 - t^2) = 5\Delta t + 4t\,\Delta t$$

$$v = dx/dt = dx/\Delta t = 5 + 4t$$

$$dt' = t'' - t' = \Delta t$$

$$dx' = x'' - x' = 5(t'' - t') + 2((t'')^2 - (t')^2)$$

$$= 5\Delta t + 2(t'^2 + 2t'\Delta t + \Delta t^2 - t'^2) = 5\Delta t + 4t'\,\Delta t$$

$$v' = dx'/dt' = dx'/\Delta t = 5 + 4t'$$

$$dv = v' - v = 4(t' - t) = 4\,dt$$

$$a = dv/dt = 4$$

4. Obtain SOLUTION:

The acceleration a = 4.

[PROVE RESULT]

1. Apply *SUBSTITUTION*:

Taking any time in the time domain, for example t = 3 sec from the starting point. With a finite $\Delta t = 0.01$ sec, all of the variables at time "t" can thus be substituted as follows:

$$t = 3$$

$$\Delta t = 0.01$$

$$\Delta t > 0 \ \& \ \Delta t \rightarrow 0$$

$t' = 3 + 0.01$

$t'' = t' + 0.01 = (3 + 0.01) + 0.01$

2. Adapt *IF*:

$x = 5\,(3) + 2\,(3)^2$

$x' = 5\,(3 + 0.01) + 2(3 + 0.01)^2$

$x'' = 5\,((3 + 0.01) + 0.01) + 2\,[(3 + 0.01) + 0.01]^2$

3. Apply *FACT* in *DERIVATION* process:

$dt = t' - t = (3 + 0.01) - 3 = 0.01$

$dx = x' - x = 5\,[(3 + 0.01) - 3] + 2\,[(3 + 0.01)^2 - 3^2]$

$= 5(0.01) + 2\,[(3)^2 + 2(3)\,(0.01) + (0.01)^2 - 3^2]$

$= 5(0.01) + 4(3)\,(0.01)$

$v = dx/dt = [5(0.01) + 4(3)\,(0.01)]/\,0.01 = 5 + 4(3)$

$dt' = t'' - t' = [(3 + 0.01) + 0.01] - (3 + 0.01) = 0.01$

$dx' = x'' - x' = 5\,\{[(3 + 0.01) + 0.01] - (3 + 0.01)\} + 2\,\{[(3 + 0.01) + 0.01]^2 - (3 + 0.01)^2\}$

$= 5\,(0.01) + 2\,[(3 + 0.01)^2 + 2(3 + 0.01)\,(0.01) + (0.01)^2 - (3 + 0.01)^2]$

$= 5(0.01) + 4(3 + 0.01)\,(0.01)$

$v' = dx'/dt' = [5(0.01) + 4(3 + 0.01)\,(0.01)]/\,0.01$

$= 5 + 4(3 + 0.01)$

$dv = v' - v = 4[(3 + 0.01) - 3] = 4(0.01)$

$a = dv/dt = 4(0.01)/0.01 = 4$

4. Proof of *SOLUTION*:

Because every step in the proving process is true and also follows exactly the same steps as that in the derivation process, therefore the same results as that of the derivation process can be readily achieved.

All variables can also be replaced by numerical numbers through substitutions and calculations (Table 1). However, it makes no difference to the final results.

Table 1. *Substitution of variables with numerical numbers.*

t	x	t'	x'	dt=t'-t	dx=x'-x	v=dx/dt	t"	x"	dt'=t"-t'	dx'=x"-x'	v'=dx'/dt'	dv=v'-v	a=dv/dt
t1	x1	t1'	x1'	dt1=t1'-t1	dx1=x1'-x1	v1=dx1/dt1	t1"	x1"	dt1'=t1"-t1'	dx1'=x1"-x1'	v1'=dx1'/dt1'	dv1=v1'-v1	a1=dv1/dt1
3.000	33.000	3.010	33.170	0.010	0.170	17.020	3.020	33.341	0.010	0.171	17.060	0.040	4.000

5. Motion in Distance Field

The motion of an object in Distance Field (Fig. 1) can be expressed by a function x = F(t). At any time "t", there is a pair of real numbers (x, t) where "t" is the time from the origin, and "x" is the distance along the motion from the origin. Since the starting distance at origin "x_0" is 0 and the starting time at origin "t_0" is 0, therefore (x_0, t_0) at origin can be designated as (0, 0).

5.1. Definitions

For a motion in Distance Field x = F(t), at any time "t", there are a corresponding distance "x", and two randomly chosen subsequent times t' & t", where t < t' < t" and t' = t + Δt (Δt→0) and t" = t' + Δt' (Δt'→0), and their corresponding distances x' & x". In another word, at any time "t", we can find a position function set (x, t) and two adjacent position function sets (x', t') and (x", t"), where t < t' < t" and t' = t + Δt (Δt→0) & t" = t' + Δt' (Δt'→0).

To be more specifically, there is a motion in Distance Field, where x = F(t), such that:

"t" is any time in the time domain of x = F(t)

Δt > 0 and Δt →0 & Δt' > 0 and Δt' →0

t → x: find x from t and F(x, t)

t → t': find t' from t and Δt by t' = t + Δt

t'→ x': find x' from t' and F(x, t)

t'→ t": find t" from t' and Δt' by t" = t' + Δt'

t" → x": find x" from t" and F(x, t)

Giving the above variables t, x, t', x', t", and x", we can apply the following definitions:

Differential of Time:

$$dt = t' - t \qquad dt' = t'' - t'$$

Differential of Distance:

$$dx = x' - x \qquad dx' = x'' - x'$$

Velocity:

$$v = dx/dt \qquad v' = dx'/dt'$$

$$dx = vdt \qquad dx' = v'dt$$

Differential of Velocity:

$$dv = v' - v$$

Acceleration:

$$a = dv/dt$$

$$dv = adt$$

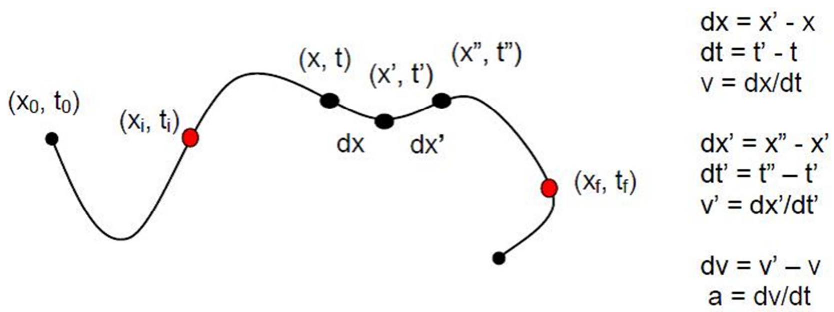

Fig. 1. *Motion in Distance Field.*

5.2. Velocity

5.2.1. Theory 1

For a motion in the Distance Field, at any two time t_i and t_f ($t_i < t_f$) in the time domain, there are two corresponding distances x_i and x_f such that:

$$x_f - x_i = \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} vdt \tag{1}$$

Also, for a motion in the Distance Field with origin (0, 0), at any time t in the time domain, there is a corresponding distance x such that:

$$x = \int_0^t vdt \tag{2}$$

[PROOF]

For a motion in the Distance Field x = F(t), at any time "t", there are (x, t) and (x', t'), where t' > t & (t' – t) = Δt → 0.

Also by applying the following definitions:

$$dt = t' - t$$

$$dx = x' - x$$

$$dx = vdt$$

for a motion in Distance Field, at any two times t_i and t_f that $t_i < t_f$, we can find (x_i, t_i) and (x_f, t_f) and a group of continuous sequential numbers t_i, t_1, t_2, …, t_m, t_f in the time domain between t_i & t_f, where $(t_1 - t_i) = \Delta t_i \to 0$, $(t_2 - t_1) = \Delta t_1 \to 0$, $(t_2 - t_1) = \Delta t_2 \to 0$, …, $(t_f - t_m) = \Delta t_m \to 0$, and also a group of corresponding continuous sequential numbers x_i, x_1, x_2, …, x_m, x_f in the distance domain between x_i & x_f, to form position function sets (x_i, t_i), (x_1, t_1), (x_2, t_2), …, (x_m, t_m), (x_f, t_f), such that:

$$dt_i = t_1 - t_i, \quad dt_1 = t_2 - t_1, \ \ldots, \quad dt_m = t_f - t_m$$

$$dx_i = x_1 - x_i, \quad dx_1 = x_2 - x_1, \ \ldots, \quad dx_m = x_f - x_m$$

$$dx_i = v_i dt_i, \quad dx_1 = v_1 dt_1, \ \ldots, \quad dx_m = v_m dt_m$$

$$\int_{x_i}^{x_f} dx = dx_i + dx_1 + \ldots + dx_m = (x_1 - x_i) + (x_2 - x_1) + \ldots + (x_f - x_m) = x_f - x_i$$

$$\int_{t_i}^{t_f} vdt = v_i dt_i + v_1 dt_1 + \ldots + v_m dt_m$$

Therefore,

For any (x_i, t_i) & (x_f, t_f)

$$x_f - x_i = \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} vdt$$

For any (x, t) with origin (0, 0), where $x_i = 0$ & $t_i = 0$ and $x_f = x$ & $t_f = t$

$$x = \int_0^t vdt$$

5.2.2. Theory 2

For a motion in Distance Field, at any two time t_i and t_f, ($t_i < t_f$) in the time domain, there are two corresponding distances x_i and x_f such that:

For a constant velocity v,

$$x_f - x_i = v(t_f - t_i) \tag{3}$$

Also, for a motion in Distance Field with origin (0, 0), for any time t in the time domain, there is a corresponding distance x such that:

For a constant velocity v,

$$x = vt \tag{4}$$

[PROOF]

Because v is a constant, $v = v_i = v_1 = v_2 = \ldots = v_m$

$$\int_{t_i}^{t_f} vdt = v_i dt_i + v_1 dt_1 + \ldots + v_m dt_m = v dt_i + v dt_1 + v dt_2 + \ldots + v dt_m = v\,(dt_i + dt_1 + dt_2 + \ldots + dt_m) = v\,(t_f - t_i)$$

$$\int_{x_i}^{x_f} dx = dx_i + dx_1 + \ldots + dx_m = x_f - x_i$$

$$\int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} vdt$$

Therefore, for any (x_i, t_i) & (x_f, t_f), and constant v,

$$x_f - x_i = v(t_f - t_i)$$

For any time t with origin (0, 0), and constant v, where $x_i = 0$ & $t_i = 0$ and $x_f = x$ & $t_f = t$

$$x = vt$$

5.3. Acceleration

5.3.1. Theory 1

For a motion in Distance Field, at any two times t_i and t_f, ($t_i < t_f$) in the time domain, there are two corresponding velocities v_i and v_f such that:

$$\int_{t_i}^{t_f} adt = v_f - v_i \tag{5}$$

Also, for a motion in Distance Field with origin (0, 0), at any time t in the time domain, there is a corresponding velocity v such that:

$$\int_0^t adt = v - v_0 \tag{6}$$

[PROOF]

For a motion in Distance Field, at any time t, there are (x, t), (x', t') and (x'', t'') where t' > t & (t' – t) = $\Delta t \to 0$ and $t'' > t'$ & $(t'' - t') = \Delta t' \to 0$.

Also by applying the following definitions:

$$dt = t' - t,\ dt' = t'' - t'$$

$$dx = x' - x\ ,\ dx' = x'' - x'$$

$$v = dx/dt\ ,\ v' = dx'/dt'$$

$$dv = v' - v$$

$$dv = adt$$

for a motion in Distance Field, at any two times t_i and t_f, that $t_i < t_f$, we can find (x_i, t_i) and (x_f, t_f) and a group of continuous sequential numbers t_i, t_1, t_2, …, t_m, t_f in the time domain between t_i & t_f, where $(t_1 - t_i) = \Delta t_i \to 0$, $(t_2 - t_1) = \Delta t_1 \to 0$, $(t_2 - t_1) = \Delta t_2 \to 0$, …, $(t_f - t_m) = \Delta t_m \to 0$, and also a group of corresponding continuous sequential numbers x_i, x_1, x_2, …,x_m, x_f in the distance domain between x_i & x_f, to form position function sets (x_i, t_i), (x_1, t_1), (x_2, t_2), …, (x_m, t_m), (x_f, t_f), such that:

$$dt_i = t_1 - t_i,\ dt_1 = t_2 - t_1,\ \ldots,\ dt_m = t_f - t_m$$

$$dx_i = x_1 - x_i,\ dx_1 = x_2 - x_1,\ \ldots,\ dx_m = x_f - x_m$$

$$v_i = dx_i/dt_i,\ v_1 = dx_1/dt_1,\ \ldots,\ v_m = dx_m/dt_m$$

$$dv_i = v_1 - v_i,\ dv_1 = v_2 - v_1,\ \ldots,\ dv_m = v_f - v_m$$

$$a_i = dv_i/dt_i,\ a_1 = dv_1/dt_1,\ \ldots,\ a_m = dv_m/dt_m$$

$$dv_i = a_i dt_i,\ dv_1 = a_1 dt_1,\ \ldots,\ dv_m = a_m dt_m$$

$$\int_{v_i}^{v_f} dv = dv_i + dv_1 + \ldots + dv_m = (v_1 - v_i) + (v_2 - v_1) + \ldots$$

$$+ (v_f - v_m) = v_f - v_i$$

$$\int_{t_i}^{t_f} adt = a_i dt_i + a_1 dt_1 + \ldots + a_m dt_m$$

Therefore,

For any (v_i, t_i) & (v_f, t_f)

$$\int_{t_i}^{t_f} adt = v_f - v_i$$

For any time t with origin (0, 0), where $x_i = 0$, $t_i = 0$, $v_i = v_0$ and $x_f = x$, $t_f = t$, $v_f = v$

$$\int_0^t adt = v - v_0$$

5.3.2. Theory 2

For a motion in Distance Field, at any two times t_i and t_f ($t_i < t_f$) in the time domain, there are two corresponding velocities v_i and v_f such that:

For a constant acceleration a,

$$v_f - v_i = a(t_f - t_i) \tag{7}$$

Also, for a motion in the Distance Field with origin (0, 0), at any time t in the time domain, there is a corresponding velocity v such that:

For a constant acceleration a,

$$v = v_0 + at \tag{8}$$

[PROOF]

Because,

$$v_f - v_i = \int_{t_i}^{t_f} adt$$

$$a = a_i = a_1 = a_2 = \ldots = a_m$$

$$v_f - v_i = \int_{t_i}^{t_f} adt = a_i dt_i + a_1 dt_1 + \ldots + a_m dt_m$$

$$= adt_i + adt_1 + \ldots + adt_m = a\,(t_f - t_i)$$

Therefore, for any (v_i, t_i) & (v_f, t_f), and a constant acceleration a,

$$v_f - v_i = a(t_f - t_i)$$

Also, for any time t with origin (0, 0), and a constant acceleration a, where $x_i = 0$, $t_i = 0$, $v_i = v_0$ and $x_f = x$, $t_f = t$, $v_f = v$, then

$$v = v_0 + at$$

5.3.3. Theory 3

For a motion in Distance Field with origin (0, 0), at any time t in the time domain, there is a corresponding distance x such that:

For a constant acceleration a,

$$x = v_0 t + ½ at^2 \tag{9}$$

[PROOF]

Because for a motion in Distance Field with origin (0, 0), at any time t in the time domain, for a constant acceleration a,

$$v = v_0 + at$$

$$x = \int_0^t vdt$$

$$x = \int_0^t vdt = \int_0^t (v_0 + at)dt = \int_0^t v_0 dt + \int_0^t atdt = v_0 t + \int_0^t atdt$$

[Theory]
$d(x^n) = nx^{n-1}dx$
$d(x^n) = (x')^n - x^n = (x + dx)^n - x^n$
$= (x^n + nx^{n-1}dx + ?dx^2 + \ldots) - x^n = nx^{n-1}dx$
Therefore,

$$\int_0^t atdt = \int_0^t \frac{1}{2} a(2tdt) = \frac{1}{2} a \int_0^t dt^2 = \frac{1}{2} at^2$$

$$x = v_0 t + ½ at^2$$

5.3.4. Theory 4

For a motion in Distance Field with origin (0, 0), at any time t in the time domain, there is corresponding distance x and velocity v such that:

For a constant acceleration a,

$$v^2 = v_0^2 + 2\ ax \tag{10}$$

[PROOF]
Because,

$$v = v_0 + at$$

$$t = (v - v_0)/a$$

$$x = v_0 t + ½\ at^2$$

$$x = v_0[(v - v_0)/a] + ½\ a\ (v - v_0)^2/a^2 = vv_0/a - v_0^2/a + ½\ (v^2 - 2v\ v_0 + v_0^2)/a = ½\ (v^2 - v_0^2)/a$$

Therefore,

$$v^2 = v_0^2 + 2ax$$

5.3.5. Theory 5

For a motion in Distance Field with origin (0, 0), at any time t in the time domain, there is a corresponding distance x and velocity v such that:

For a constant acceleration a,

$$x = ½\ (v + v_0)\ t \tag{11}$$

[PROOF]
Because,

$$v = v_0 + at$$

$$x = v_0 t + ½\ at^2$$

$$x = ½\ v_0 t + (½\ v_0 t + ½\ at^2) = ½\ v_0 t + ½\ vt$$

Therefore,

$$x = ½\ (v + v_0)\ t$$

6. Motion in Vector Space

The motion of an object in Vector Space (Fig. 2) (Ref. 8) (Ref. 9) (Ref. 10) can be expressed by a continuous function X = F(t). At any time "t", there is a pair of position vector and real number (X, t) where "t" is the time from the origin and "X" is the position vector from the origin. Since the starting position vector at origin "X_0" is "0" and the starting time at origin "t_0" is 0, therefore (X_0, t_0) at origin can be designated as (0, 0).

6.1. Definitions

For a motion in Vector Space, at any time "t" , there are a corresponding position vector "X", and two randomly chosen subsequent times t' & t" where t < t' < t" and t' = t + Δt (Δt→0) and t" = t' + Δt' (Δt'→0), and their corresponding position vectors X' & X". In another word, at any time "t", we can find a position function set (X, t) and two adjacent position function sets (X', t') and (X", t"), where t < t' < t" and Δt & Δt' →0.

To be more specifically, there is a motion in Vector Space, where X = F(t), such that:

"t" is any time in the time domain of X = F(t)
Δt > 0 and Δt →0 & Δt' > 0 and Δt' →0
t → X: find X from t and F(X, t)
t → t': find t' from t and Δt by t' = t + Δt
t'→ X': find X' from t' and F(X, t)
t'→ t": find t" from t' and Δt' by t" = t' + Δt'
t" → X": find X" from t" and F(X, t)

With the above variables t, X, t', X', t", and X", we can apply the following definitions:

Differential of Time:

$$dt = t' - t \qquad dt' = t'' - t'$$

Differential of Position Vector:

$$dX = X' - X \qquad dX' = X'' - X'$$

Velocity Vector:

$$V = dX/dt \qquad V' = dX'/dt'$$

$$dX = Vdt \qquad dX' = V'dt'$$

Differential of Velocity Vector:

$$dV = V' - V$$

Acceleration Vector:

$$A = dV/dt$$

$$dV = Adt$$

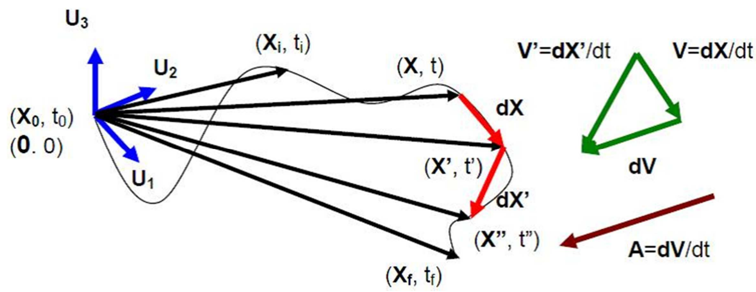

Fig. 2. *Motion in Vector Space.*

Replace the real umber variables x, dx, v, dv, a in the motion of Distance Field by corresponding vector variables X, dX, V, dV, A in the motion of Vector Space as follows:

x → X
dx → dX
v → V
dv → dV
a → A

Then, all the derivation processes and theories in the motion of Vector Space will be exactly identical as that in the motion of Distance Field.

6.2. Velocity

6.2.1. Theory 1

For a motion in Vector Space, at any two time t_i and t_f ($t_i < t_f$), there are two corresponding position vectors X_i and X_f such that:

$$X_f - X_i = \int_{t_i}^{t_f} V dt \tag{12}$$

Also, for a motion in Vector Space with origin (**0**, 0), at any time t there is a corresponding position vector **X** such that:

$$X = \int_0^t V dt \tag{13}$$

6.2.2. Theory 2

For a motion in Vector Space, for any two time t_i and t_f ($t_i < t_f$) in the time domain, there are two corresponding position vectors X_i and X_f such that:

For a constant velocity vector V,

$$X_f - X_i = V (t_f - t_i) \tag{14}$$

Also, for a motion in Vector Space with origin (0, 0), at any time t in the time domain there is a corresponding position vector X such that:

For a constant velocity vector V,

$$X = V t \tag{15}$$

6.3. Acceleration

6.3.1. Theory 1

For a motion in Vector Space, at any two time t_i and t_f ($t_i < t_f$) in the time domain, there are two corresponding velocity vectors V_i and V_f such that:

$$\int_{t_i}^{t_f} A dt = V_f - V_i \tag{16}$$

For a motion in Vector Space with origin (0, 0), at any time t in the time domain, there is a corresponding velocity vector V such that:

$$\int_0^t A dt = V - V_0 \tag{17}$$

6.3.2. Theory 2

For a motion in Vector Space, at any two time t_i and t_f ($t_i < t_f$) in the time domain, there are two corresponding velocity vectors V_i and V_f such that:

For a constant acceleration vector A,

$$V_f - V_i = A (t_f - t_i) \tag{18}$$

For a motion in Vector Space with origin (0, 0), at any time t in the time domain, there is a corresponding velocity vector V such that:

For a constant acceleration vector A,

$$V = V_0 + A t \tag{19}$$

6.3.3. Theory 3

For a motion in Vector Space with origin (0, 0), at any time t in the time domain, there is a corresponding position vector X such that:

For a constant acceleration vector A,

$$X = V_0 t + ½ A t^2 \tag{20}$$

6.3.4. Theory 4

For a motion in Vector Space with origin (0, 0), at any time t in the time domain, there is a position vector X and velocity vector V such that:

For a constant acceleration vector A,

$$V^2 = V_0^2 + 2 AX \tag{21}$$

6.3.5. Theory 5

For a motion in Vector Space with origin (0, 0), at any time t in the time domain, there is a position vector X and velocity vector V such that:

For a constant acceleration vector A,

$$X = ½ (V + V_0) t \tag{22}$$

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