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The Maximum Principle of Forward Backward Transformation Stochastic Control System

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Abstract: In the paper, we discuss the maximum principle for the forward backward stochastic system. Assume the system follows a coupled forward backward stochastic differential equation modulated by a Marlcov chain and the control domain is convex. By convex variable method, we give the necessary and sufficient conditions for the existence of optimal control.

Keywords: Maximum Principle, Stochastic Control System, Forward Backward Transformation

1. Introduction

By the convex variation method, we give both the necessary and sufficient condition for the optimal control.

In this chapter, we study the maximum principle system forward backward conversion system. The control system is described by forward backward stochastic differential equations with continuous time Markov chain with finite state.

This chapter is structured as follows: in Section 1, we give the preliminary knowledge and problems; in section second, we obtain the necessary and sufficient conditions.

2. The Introduction of Optimal Control Problem

(Ω, F, P) be a probability space, T>0 is a fixed time. $\{B_{t,} 0 \le t \le T\}$ is a D dimension Brown, $\{\omega_{t,} 0 \le t \le T\}$ is a finite state Markov chain, The state space is $\wp = \{1, 2, ..., k\}$ the transfer density function is $\Xi(i.j)$ for $i \neq j$, where $\Xi(i.j)$ is bounded nonnegative function. $\wp(i,i) = -\sum_{j \in \wp \setminus \{i\}} \wp(i,j)$. $\mathbb{F} = (F_t)_{t \in [0,T]}$ is flow field F generated by $\{B_s, \omega_s; 0 \le s \le T\}$ and all P zero sets. ζ^t is the counting measure of ω_t. Where $\zeta^t(j)$ is the number of times that Markova chain jump to state j. within 0 to t. $\zeta^t(j)$ is a saddle measure. At this point, the markov chain can be expressed as

$$d\omega_t = \sum_{g \in \wp} \Xi(\omega_{t-}, g)(g - \omega_{t-})dt + \sum_{g \in \wp} (j - \omega_{t-}) d\tilde{\zeta}(g)$$

We defined the following space:

$$\|W\|_{S^2} := \left(E\left[\sup_{s \in [0,T]} |W_s|^2 \right] \right)^{\frac{1}{2}} < +\infty;$$

$$\|O\|_{\Theta^2} := \left(E\left[\int_0^T |O_s|^2 ds \right] \right)^{\frac{1}{2}} < +\infty;$$

$$\|G\|_{J^2} := \left(E\left[\int_0^T \sum_{g \in \wp} |G_s(g)|^2 1_{\{\omega_{s-} \neq g\}} \Xi(\omega_{s-}, g) ds \right] \right)^{\frac{1}{2}} < +\infty.$$

Consider the following forward and backward stochastic system conversion system:

$$\begin{cases} dX_t = b(t,\omega_t,X_t,u_t)dt + \varpi(t,\omega_t,X_t,u_t)dB_t \\ -dY_t = \Psi(t,\omega_t,X_t,W_t,O_t,G_t(1)n_t(1),\cdots,G_t(w)n_t(w),u_t)dt - O_t dB_t - \sum_{g\in\wp} G_t(g)d\tilde{\zeta}_t(g) \\ X_0 = x, W_T = \Phi(X_T) \end{cases} \tag{2.1}$$

Where $\psi(t,i,x,y,z,q,u)$ are generators of BSDEs with a Markov chain.

In this chapter, for the convenience of marking, we only consider the situation that the X and Y are one dimension and change the backward equation into:

$$-dW_t = \Psi(t,\omega_t,X_t,W_t,O_t,G_t n_t,u_t)dt - O_t dB_t - \sum_{g\in\wp} G_t(g)d\tilde{\zeta}_t(g),$$

$$G_t n_t = \left(G_t(1)n_t(1),\cdots,G_t(w)n_t(w).b:[0,T]\times\wp\times\mathbb{R}\times D\to\mathbb{R},\varpi:[0,T]\times\wp\times\mathbb{R}\times D\to\mathbb{R}^{z\times d} and \Phi:\mathbb{R}\to\mathbb{R}\right)$$

are measurable functions with certainty. D is a nonempty convex subset of $\mathbb{R}$. b,ϖ satisfy Lipschitz Condition as to x. $\Psi(t,i,x,y,z,q,u)$ satisfy the following Lipschitz Condition: $\forall i\in\wp$

$$\left|g(t,i,x,y,z,q,u) - g(t,i,x,y',z',q',u)\right| \le C\left(|y-y'| + |z-z'| + |q-q'|\right). \tag{2.2}$$

$$\left|\left(G_t(g) - G_t'(g)\right)n_t(g)\right| \le C\left|G_t - G_t'\right|_t. \tag{2.3}$$

In this paper, we make the following assumptions:

$$E\int_0^t |u_t|^2 dt < +\infty.$$

$$\lambda(u.) = E\left\{\int_0^T z(t,\omega_t,X_{t},{}_t W,O_t,G_t n_t,u_t)dt + h(X_T) + r(W_0)\right\}, \tag{2.4}$$

3. Maximum Principle

In this part, we will get the maximum principle of stochastic variational method for optimal control problems in Section 1 of the value by classical convex optimal control. We will give the necessary and sufficient condition for the existence.

3.1. Necessary Condition

Let $u(\cdot)$ be an optimal control of the optimal control system (1.1),whose corresponding trajectory is denoted as $(X(\cdot),W(\cdot),O(\cdot),G(\cdot))$. Let $v(\cdot)$ be another adaptation process (not necessarily valued in U) ,which satisfies $u(\cdot)+v(\cdot)\in D$ .Since the control domain D is convex, $\forall 0\le\tau\le 1, u^\tau(\cdot) := u(\cdot)+\tau v(\cdot)\in D$. For the convenience of marking,

$$\xi = \xi(t,\omega_t,X_t,u_t)$$
$$\xi = b,\varpi,b_x,b_u,\varpi_x,\varpi_u$$
$$\Phi = \Phi(t,\omega_t,X_t,W_t,O_t,G_t n_t,u_t)$$
$$\Phi = \Psi,\Psi_t,\Psi_y,\Psi_z,\Psi_{w(g)},\Psi_u,z,z_x,z_y,z_z,z_{w(g)},z_u,$$

where $\Psi_{w(g)} := \Psi G(g)n(g), z_{w(g)} := z_{G(g)n(g)}$.

Introduction of variational equations as follows:

$$\begin{cases} d\vartheta_t = (b_x\vartheta_t + b_u v_t)dt + (\varpi_x\vartheta_t + \varpi_u v_t)dB_t, \\ -d\mathrm{A}_t = \left[\Psi_t\vartheta_t + \Psi_y\mathrm{A}_t + \Psi_z\delta_t + \sum_{g\in\wp}\Psi_{w(g)}\varsigma_t(g)n_t(g) + \Psi_u v_t\right]dt - \varsigma_t dB_t - \sum_{g\in\wp}\varsigma_t(g)d\tilde{\zeta}_t(g), \\ \vartheta_0 = 0, \mathrm{A}_T = \Phi_x(X_T)\vartheta_T. \end{cases} \tag{3.1}$$

Obviously, the above linear FBSDE has a unique solution $(\vartheta,\mathfrak{A},\delta,\tau)\in S^2\times S^2\times\Theta^2\times J^2$

Note $(X^\tau(\cdot),Y^\tau(\cdot),O^\tau(\cdot),G^\tau(\cdot))$ is the rail line corresponding to $u^\tau(\cdot)$. $\forall 0\le t\le T$,

Note

$$\tilde{X}_t^\tau = \frac{X_t^\tau - X_t}{\tau} - \vartheta_t, \tilde{Y}_t^\tau = \frac{Y_t^\tau - Y_t}{\tau} - \mathrm{A}_t, \tilde{Z}_t^\tau = \frac{Z_t^\tau - Z_t}{\tau} - \delta_t, \tilde{G}_t^\tau = \frac{G_t^\tau - G_t}{\tau} - \delta_t.$$

We have a convergence results are as follows:

Lemma 1.

$$\lim_{\tau\to\infty} \sup_{0\le t\le T} E\left|\tilde{X}_t^\tau\right|^2 \to 0, \lim_{\tau\to\infty} \sup_{0\le t\le T} E\left|\tilde{W}_t^\tau\right|^2 \to 0,$$

$$\lim_{\tau\to\infty} E\int_0^T \left|\tilde{Z}_t^\tau\right|^2 \to 0, \lim_{\tau\to\infty} E\int_0^T \left|\tilde{G}_t^\tau\right|_t^2 \to 0.$$

Proof.

We show that the other three equations. Firstly, we have

$$\begin{cases} -d\tilde{W}_t^\tau = \begin{Bmatrix} \tau^{-1}\left[\Psi\begin{pmatrix} t,\omega_t, X_t + \tau\left(\vartheta_t + \tilde{X}_t^\tau\right), Y_t + \tau\left(n_t + \tilde{W}_t^\tau\right), \\ Z_t + \tau\left(\delta_t + \tilde{Z}_t^\tau\right), \left(W_t + \tau\left(\varsigma_t + G_t^\tau\right)\right)n_t, u_t + \tau v_t \end{pmatrix} - \Psi\right] \\ -\Psi_x\vartheta_t - \Psi_y \mathrm{A}_t - \Psi_z\delta_t - \sum_{g\in\wp}\Psi_{w(g)}\varsigma_t(g)n_t(g) - \Psi_u v_t \end{Bmatrix} dt - \tilde{Z}_t^\tau dB_t - \sum_{g\in\wp}\tilde{G}_t^\tau(g)d\tilde{\zeta}_t(g), \\ \tilde{W}_T^\tau = \tau^{-1}\left[\Phi\left(X_T^\tau\right) - \Phi\left(X_T\right)\right] - \Phi_x\left(X_T\right)\vartheta_T. \end{cases} \tag{3.2}$$

Noted

$$A_t^\tau = \int_0^1 \Psi_x\left(t,\omega_t, X_t + \Xi\tau\left(\vartheta_t + \tilde{X}_t^\tau\right), W_t + \Xi\tau\left(\mathrm{A}_t + \tilde{Y}_t^\tau\right), Z_t + \Xi\tau\left(\delta_t + \tilde{Z}_t^\tau\right), \left(G_t + \Xi\tau\left(\varsigma_t + \tilde{G}_t^\tau\right)\right)n_t, u_t + \Xi\tau v_t\right)d\Xi = \int_0^1 \Psi_x(\Delta\Xi)d\Xi$$

Similarly

$$B_t^\tau = \int_0^1 \Psi_y(\Delta\Xi)d\Xi, C_t^\tau = \int_0^1 \Psi_z(\Delta\Xi)d\Xi, D_t^\tau(g) = \int_0^1 \Psi_{w(g)}(\Delta\Xi)d\Xi,$$

$$G_t^\tau = \left(A_t^\tau - \Psi_x\right)\vartheta_t + \left(B_t^\tau - \Psi_y\right)\mathrm{A}_t + \left(C_t^\tau - \Psi_z\right)\delta_t + \sum_{g\in\wp}\left(D_t^\tau(g) - \Psi_{w(g)}\right)\varsigma_t(g)n_t(g) + \int_0^1\left[\Psi_u(\Delta\Xi) - \Psi_u\right]v_t d\Xi$$

This (2.2) could also be written as follows.

$$\begin{cases} -d\tilde{W}_t^\tau = \left[A_t^\tau\tilde{X}_t^\tau + B_t^\tau\tilde{W}_t^\tau + C_t^\tau\tilde{Z}_t^\tau + \sum_{g\in\wp} D_t^\tau(g)\tilde{G}_t^\tau(g)n_t(g) + G_t^\tau\right]dt - \tilde{Z}_t^\tau dB_t - \sum_{g\in\wp}\tilde{G}_t^\tau(g)d\tilde{\zeta}_t(g), \\ \tilde{Y}_T^\tau = \tau^{-1}\left[\Phi\left(X_T^\tau\right) - \Phi\left(X_T\right)\right] - \Phi_x\left(X_T\right)\vartheta_T. \end{cases}$$

Attention to the measurable variation for $\tilde{\zeta}(g)$

$$d\left\langle\tilde{\zeta}(g), \tilde{\zeta}(g)\right\rangle_t = n_t(g)dt = 1_{\{\omega_{t-}\neq g\}}\Xi(\omega_{t-}, g)dt \tag{3.3}$$

And $\left|\left(G_t(g) - G_t^{'}(g)\right)n_t(g)\right| \le c\left|G_t - G_t^{'}\right|_t$, we apply division integral formula to $\left|\tilde{Y}_t^\tau\right|$

And we can get the following from (2.3) and (1.3)

$$E\left|\tilde{W}_t^\tau\right|^2 + E\int_t^T\left|\tilde{Z}_t^\tau\right|^2 ds + \sum_{g\in\wp} E\int_t^T\left|\tilde{G}_s^\tau(g)\right|^2 n_s(g)ds$$

$$= 2E\int_t^T\left\langle\tilde{W}_s^\tau, A_s^\tau\tilde{X}_s^\tau + B_s^\tau\tilde{Y}_s^\tau + C_s^\tau\tilde{Z}_s^\tau + \sum_{g\in\wp} D_s^\tau(g)\tilde{G}_s^\tau(g)n_s(g) + G_s^\tau\right\rangle ds + E\left[\tau^{-1}\left(\Phi\left(X_T^\tau\right) - \Phi\left(X_T\right)\right) - \Phi_x\left(X_T\right)\vartheta_T\right]$$

$$\le CE\int_t^T\left|\tilde{W}_s^\tau\right|^2 ds + \frac{1}{2}E\int_t^T\left|\tilde{Z}_s^\tau\right|^2 ds + \frac{1}{2}\sum_{g\in\wp}E\int_t^T\left|\tilde{G}_s^\tau(g)\right|^2 n_s(g)ds + \lambda_\tau$$

where

$$\lambda_\tau = E\int_t^T\left(\left|A_s^\tau\tilde{x}_s^\tau\right|^2 + \left|G_s^\tau\right|^2\right)ds + E\left[\int_0^1\Phi_x\left(X_T + \Xi\tau\left(\tilde{X}_T^\tau + \vartheta_T\right)\right)\left(\tilde{X}_T^\tau + \vartheta_T\right)d\Xi - \Phi_x\left(X_T\right)\vartheta_T\right].$$

We know that $\lim_{\tau\to 0}\lambda_\tau = 0$, By Gronwall inequality, we can prove the three convergence results.

Because. $u(.)$ is an optimal control, we have

$$\tau^{-1}\left[\lambda\left(u^{\tau}(.)\right)-\lambda\left(u(.)\right)\right]\leq 0 \qquad (3.4)$$

The following variational inequality was established

we prove the following variational inequalities,

Lemma .2. We suppose that theorem $(A1-A3)$ are tenable ,

$$E\int_0^T\left[z_x\vartheta_t+z_y\mathrm{A}_t+z_z\delta_t+\sum_{g\in\wp}z_{w(g)}\varsigma_t(g)n_t(g)+z_uv_t\right]dt+E\gamma_x(X_T)\vartheta_T+Er_y(W_0)\mathrm{A}_0\leq 0 \qquad (3.5)$$

Proof.

The convergence results of lemma 1

$$\tau^{-1}E\left(\gamma\left(X_T^{\tau}\right)-\gamma\left(X_T\right)\right)=E\int_0^1\gamma_x\left[X_T+\Xi\tau\left(\tilde{X}_T^{\tau}+\vartheta_T\right)\right]\left(\tilde{X}_T^{\tau}+\vartheta_T\right)d\Xi\to E\gamma_x(X_T)\vartheta_T$$

Similarly, we have

$$\tau^{-1}E\left(r\left(W_0^{\tau}\right)-r\left(W_0\right)\right)\to Er_y(W_0)\mathrm{A}_0,$$

$$\tau^{-1}E\int_0^T\left[z\left(t,\omega_t,X_t^{\tau},W_t^{\tau},O_t^{\tau},G_t^{\tau}n_t,u_t^{\tau}\right)-z\left(t,\omega_t,X_t,W_t,O_t,G_tn_t,u_t\right)\right]dt\to E\int_0^T\left[z_x\vartheta_t+z_y\mathrm{A}_t+z_z\delta_t+\sum_{g\in\wp}z_{w(g)}\varsigma_t(g)n_t(g)+z_uv_t\right]dt$$

Thus (3.5) can be obtained by (3.4)

we introduce the dual equations as follows,

$$\begin{cases}-dP_t=\left(b_xP_t-\Psi_xU_t+\varpi_xI_t+z_x\right)dt-I_tdB_t-\sum_{g\in\wp}\Theta_t(g)d\tilde{\zeta}_t(g)\\ dU_t=\left(\Psi_yU_t-z_y\right)dt+\left(\Psi_zU_t-z_z\right)dB_t+\sum_{g\in\wp}\left(\Psi_{w(g)}(t-)k_{t-}-z_{w(g)}(t-)\right)d\tilde{\zeta}_t(g)\\ P_T=-\Phi_x(X_T)U_T+\gamma_x(X_T),U_0=-r_y(W_0)\end{cases} \qquad (3.6)$$

The define of $\psi_{\omega(g)}(t-),z_{\omega(g)}(t-)$ is

$$J(t,i,x,y,z,w,u,q,w,k)=\left(q,b(t,i,x,u)\right)+\left(w,\varpi(t,i,x,u)\right)-\left(k,\Psi(t,i,x,y,z,wn,u)\right)+z\left(t,i,x,y,z,wn_t,u\right) \qquad (3.7)$$

$$\begin{cases}-dP_t=J_xdt-I_tdB_t-\sum_{g\in\wp}\Theta_t(g)d\tilde{\zeta}_t(g)\\ dU_t=-J_ydt-J_zdB_t-\sum_{g\in\wp}J_{w(g)}(t-)d\tilde{\zeta}_t(g)\\ P_T=-\Phi_x(X_T)U_T+\gamma_x(X_T),U_0=-r_y(W_0)\end{cases} \qquad (3.8)$$

correspondingly. $(P(\cdot),I(\cdot),U(\cdot))$ is the only solution of the dual equations(3.6).thus for any $v\in Q$, the following can be obtain,

$$J_u\bullet(v-u_t)\leq 0,a.e.,a.s.. \qquad (3.9)$$

Proof.

We apply division integral formula to $\vartheta_TP_T+\mathrm{A}_TU_T$

Now we can prove stochastic maximum principle

Theorem 1 (maximum principle). Set $u(\bullet)$ as an optimal control, $(X(\cdot),Y(\cdot),O(\cdot),G(\cdot))$ Is the rail line

$$\begin{aligned}E\left(\vartheta_TP_T+\mathrm{A}_TU_T-\mathrm{A}_0U_0\right)&=E\left[\vartheta_TJ_x(X_T)+\mathrm{A}_0r_y(W_0)\right]\\&=E\int_0^T\left[-\left(z_x\vartheta_t+z_y\mathrm{A}_t+z_z\delta_t+\sum_{g\in\wp}z_{w(g)}\varsigma_t(g)n_t(g)+P_tb_uv_t+I_t\varpi_uv_t-U_t\Psi_uv_t\right)\right]dt\end{aligned}$$

According the Lemma .2.

$$E\int_0^TJ_u\cdot v_tdt\leq 0$$

We suppose that v_t is the following form,

$$v_s=\begin{cases}os\notin[t,t+\varepsilon]\\ v_s'-u_ss\in[t,t+\varepsilon]\end{cases}$$

Where $v'(\bullet)\in Q$, Thus we can obtain that

$$\varepsilon^{-1}E\int_t^{t+\varepsilon} J_u \cdot \left(v_s' - u_s\right) ds \le 0$$

We suppose that $\varepsilon \to 0$, Thus we can obtain that

$$E\left[J_u \cdot \left(v_t' - u_t\right)\right] \le 0, a.e..$$

For $v \in Q$ and $A \in F_t$, We suppose that $v_t' = v1_A + u_t 1_A$, Thus we can obtain that

$$E\left[J_u \cdot (v - u_t) 1_A\right] \le 0, a.e..$$

Thus

$$E\left\{J_u \cdot (v - u_t) \middle| F_t\right\} = J_u \cdot (v - u_t) \le 0, a.e., a.s..$$

3.2. Sufficient Conditions

In this section, under certain conditions, we obtain sufficient conditions for the existence of the optimal control.

Theorem 2. We suppose that theorem $(A1 - A3)$ are tenable, h, r, H are concave to (x, y, z, u, w) and $W_T = \Phi(X_T)$ can be written as $W_T = I(\omega_T) X_T$, where I is the measurable function with certainty. $(P(\cdot), I(\cdot), U(\cdot), \Theta(\cdot))$ is the solution of the Dual equations as to the control $u(\cdot)$, which will be an optimal control if and on if theorem (3.9) is satisfied.

Proof

We suppose that $v(\cdot)$ is arbitrary feasible control, and noted $\left(X_t^v, Y_t^v, W_t^v, Q_t^v\right)$ is rail line correspondingly. In order to mark the simple, we note:

$$\lambda^v := \lambda\left(t, \omega_t, X_t^v, v_t\right), \text{when } \lambda = b, \varpi$$
$$\theta^v := \theta\left(t, \omega_t, X_t^v, W_t^v, O_t^v, G_t^v n_t, v_t\right), \text{when } \theta = z, \Psi$$
$$J^v := J\left(t, \omega_t, X_t^v, W_t^v, O_t^v, G_t^v n_t, v_t, P_t, I_t, U_t\right),$$

Thus

$$\lambda\left(v(\cdot)\right) - \lambda\left(u(\cdot)\right) = E\int_0^T \left(z^v - z\right) dt + E\left(\gamma\left(X_T^v\right) - \gamma\left(X_T\right)\right) + E\left(r\left(W_0^v\right) - r\left(W_0\right)\right).$$

We could know from the concavity of the Υ, γ, J.

$$E\left(\gamma\left(X_T^v\right) - \gamma\left(X_T\right)\right) \ge E\left(P_T + I(\omega_T) U_T\right)\left(X_T^v - X_T\right),$$
$$E\left(r\left(W_0^v\right) - r\left(W_0\right)\right) \ge -E\left(U_0\left(W_0^v - W_0\right)\right)$$

$$E\int_0^T \left(z^v - z\right) dt = E\int_0^T \left[\left(J^v - J\right) - P_t\left(b^v - b\right) - I_t\left(\varpi^v - \varpi\right) + U_t\left(\Psi^v - \Psi\right)\right] dt$$
$$\ge E\int_0^T \left[J_x\left(X_t^v - X_t\right) + J_y\left(W_t^v - W_t\right) + J_z\left(O_t^v - O_t\right) + \sum_{g \in \wp} J_{w(g)}\left(G_t^v - G_t\right) + J_u\left(v_t - u_t\right) - P_t\left(b^v - b\right) - I_t\left(\varpi^v - \varpi\right) + U_t\left(\Psi^v - \Psi\right)\right] dt$$

We apply the division integral formula to $P_t \cdot \left(X_t^v - X_t\right)$ and $U_t \cdot \left(Y_t^v - Y_t\right)$

$$EP_T\left(X_T^v - X_T\right) = E\int_0^T \left[P_t\left(b^v - b\right) - J_x\left(X_t^v - X_t\right) + J_t\left(\varpi^v - \varpi\right)\right] dt,$$

$$EU_0\left(W_0^v - W_0\right) = EU_T\left(\Phi\left(X_T^v\right) - \Phi\left(X_T\right)\right) - E\int_0^T \begin{bmatrix} -U_t\left(\Psi^v - \Psi\right) - \left(W_t^v - W_t\right)\left(\Psi_y U_t - z_y\right) - \left(O_t^v - O_t\right)\left(\Psi_z U_t - z_y\right) \\ -\sum_{g \in \wp}\left(G_t^v(g) - G_t(g)\right)\left(\Psi_{w(g)}\Theta_t - z_{w(g)}\right) n_t(g) \end{bmatrix} dt$$

By the condition (3.9) can be obtained

$$\lambda\left(v(\cdot)\right) - \lambda\left(u(\cdot)\right) \le 0$$

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