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MHD Micropolar Fluid Near a Vertical Plate with Newtonian Heating and Thermal Radiation in the Presence of Mass Diffusion

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Abstract: The effects of chemical reaction and thermal radiation on unsteady free convection flow of a micropolar fluid past a semi-infinite vertical plate embedded in a porous medium in the presence of heat absorption with Newtonian heating have been investigated. Both physically important boundary conditions of uniform wall concentration (UWC) and uniform mass flux (UMF) are considered. Rosseland diffusion approximation is used to describe the radiative heat flux in the energy equation. Numerical results of velocity profiles of micropolar fluids are compared with the corresponding flow problems for a Newtonian fluid in UWC and UMF cases. Graphical results for velocity, temperature and concentration profiles of both phases based on the analytical solutions are presented and discussed. Finally the effects of the pertinent parameters on the skin friction, couple stress and the rate of heat transfer coefficient at the plate are discussed.

Keywords: Thermal Radiation, Chemical Reaction, Mass Diffusion, Newtonian Heating

1. Introduction

The theory of micropolar fluids developed by Eringen [1] describes some physical systems which do not satisfy the Navier-Stokes equations. This general theory of micropolar fluids deviates from that of Newtonian fluids by adding two new variables to the velocity. These variables are microrotations that are spin and microinertia tensors describing the distributions of atoms and molecules inside the microscopic fluid particles. This theory may be applied to the explanation for the phenomenon of the flow of colloidal fluids, liquid crystals, polymeric suspensions, animal blood, etc. An excellent review of micropolar fluids and their applications was given by Ariman et al. [2]. Gorla [3] discussed the steady state heat transfer in a micropolar fluid flow over a semi-infinite plate, and the analysis is based on similarity variables. Rees and Pop [4] studied the free convection boundary layer flow of micropolar fluid from a vertical flat plate. Singh [5] has studied the free convection flow of a micropolar fluid past an infinite vertical plate using the finite difference method.

The study of flow and heat transfer for an electrically conducting micropolar fluid past a porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems, such as magneto hydrodynamic (MHD) generators, plasma studies, nuclear reactors, oil exploration, geothermal energy extractions and the boundary layer control in the field of aerodynamics. Also, the porous media heat transfer problems have several practical engineering applications, such as the crude oil extraction, the ground water pollution, and many other practical applications, i.e., in biomechanical problems (e.g., blood, flow in the pulmonary alveolar sheet) and in the filtration transpiration cooling. Hiremath and Patil [6] studied the effect of free convection currents on the oscillatory flow of the polar fluid through a porous medium, which is bounded by the vertical plane surface with a constant temperature. Kim [7] investigated the unsteady free convection flow of a micropolar fluid past a vertical plate embedded in a porous medium. Bakr and Raizah [8] studied the unsteady MHD mixed convection flow of a viscous dissipating micropolar fluid in a boundary layer slip flow regime with Joule heating.

At the macroscopic level, it is well accepted that the boundary condition for a viscous fluid at a solid wall is one of no-slip, i.e., the fluid velocity matches the velocity of the solid boundary. While the no-slip condition has been processed experimentally to be accurate for a number of macroscopic flows, it remains an assumption that is not based on physical principles. In many practical applications, the particle adjacent to a solid surface no longer takes the velocity of the surface. The particle at the surface has a finite tangential velocity. It slips along the surface. The flow regime is called a slip-flow regime, and this effect cannot be neglected. The study of magneto-micropolar fluid flows in the slip-flow regimes with heat transfer has important engineering applications, e.g., in power generators, refrigeration coils, transmission lines, electric transformers, and heating elements. Khandelwal et al. [9] studied the effects of permeability variation on the MHD unsteady flow of polar fluid through a porous medium in a slip-flow regime over an infinite porous flat plate. Sharma and Chaudhary [10] studied the effect of variable suction on transient free convective viscous incompressible flows past a vertical plate in a slip-flow regime. Hayat et al. [11] presented the analytical solutions of the equations of motion and energy of a second grade fluid for the developed flow over a stretching sheet with slip condition. Asghar et al. [12] studied the rotating flow of a third grade fluid past a porous plate with partial slip effects. Khan [13] presented the exact analytical solutions for three basic fluid flow problems in a porous medium when the no-slip condition is no longer valid. Asghar et al. [14] obtained the exact analytical solutions for general periodic flows of a second grade fluid in the presence of partial slip and a porous medium.

The combined heat and mass transfer problems with chemical reactions are of importance in many processes, and therefore have received a considerable amount of attention in recent years. In processes, such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, the heat and mass transfer occurs simultaneously. Chemical reactions can be codified as either homogeneous or heterogeneous processes. A homogeneous reaction is one that occurs uniformly through a given phase. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. A reaction is said to be the first order if the rate of reaction is directly proportional to the concentration itself. In many chemical engineering processes, a chemical reaction between a foreign mass and the fluid does occur. These processes take place in numerous industrial applications, such as the polymer production, the manufacturing of ceramics or glassware, the food processing [15] and so on. Muthucumarswamy and Ganesan [16] studied the first order homogeneous chemical reaction on the flow past an infinite vertical plate. Kandasamy et al. [17] discussed the heat and mass transfer effect along a wedge with a heat source and concentration in the presence of suction/injection taking into account the chemical reaction of the first order. Bakr [18, 24] presented an analysis on MHD micropolar fluid in presence of heat generation/ absorption and a chemical reaction.

Here we return to the free-convection problem though now we consider the case of Newtonian heating where the rate of heat transfer from the boundary is proportional to the local surface temperature. Heat-transfer characteristics are dependent on the thermal boundary conditions. In general, there are three common heating processes, namely a prescribed surface-temperature distribution, a prescribed surface-heat-flux distribution, and conjugate conditions, whereby heat transfer through a bounding surface of finite thickness and finite heat capacity is specified. The interface temperature is not known a priori but depends on the intrinsic properties of the system, namely, the thermal conductivities of the fluid and solid. In Newtonian heating, the rate of heat transfer from the bounding surface with a finite heat capacity is proportional to the local surface temperature, and it is usually termed conjugate convective flow. This situation occurs in many important engineering devices, for examples: (a) in heat exchangers, where conduction in the solid tube wall is greatly influenced by convection in the fluid flowing past it; (b) in conjugate heat transfer around fins, where conduction within the fin and convection in the fluid surrounding it must be simultaneously analyzed in order to obtain the vital design information; (c) in a convective-flow set-up, where the bounding surfaces absorb heat from solar radiation. Therefore, we can conclude that the conventional assumption of the absence of an inter-relation between coupled conduction–convection effects is not always realistic, and this inter-relation must be considered when evaluating conjugate heat-transfer processes in many practical engineering applications; see [19] for example. Alternatively, this set-up can model the heat transfer when there is a weak exothermic catalytic reaction taking place on the surface generating heat at a rate proportional to the surface temperature. This is a reasonable assumption when the difference between the surface temperatures arising from the reaction and the ambient temperature are small, which is the situation envisaged here. The free-convection flow on a vertical surface resulting from Newtonian heating has been treated in [20]. Convective boundary layer flows in fluid-saturated porous media driven by Newtonian heating have also received some attention; see [21] for example.

The aim of our paper is to study the effect of thermal radiation on the oscillatory free convective flow of the micropolar fluid through a porous medium with Newtonian heating and chemical reaction in the presence of slip velocity at the surface. Examples of the physical situation presented are: (i) heat removal of nuclear fuel debris buried in the deep sea-bed and (ii) heat recovery from geothermal systems. The flow takes place near a hot vertical plate bounding the porous region which is filled with water containing soluble and insoluble chemical materials. Such a fluid is modeled as the micropolar fluid. The flow is due to the buoyancy forces generated by the temperature gradient. It is assumed that the plate is embedded in a uniform porous medium and oscillates in time with a constant frequency in the presence of a transverse magnetic field. The governing equations are solved analytically using perturbation technique. Numerical results are reported for various values of the physical parameters of interest.

The paper is organized as follows. The Section 2 deals with the mathematical formulation of the problems. Section 3 contains the closed form solutions of velocity, temperature concentration etc. Numerical results and discussion are presented in Section 4. The conclusions have been summarized in Section 5

2. Mathematical Analysis

We consider the two-dimensional flow of a micropolar fluid past a semi-infinite vertical plate embedded in a porous medium in the slip-flow regime. The x-axis is taken along the porous plate in the upward direction and the y-axis normal to it. Due to the semi-infinite plane surface assumption, the flow variables are the functions of y and t only. It is assumed that the free convection is generated by Newtonian heating, i.e. the heat transfer from the plate is proportional to the local surface temperature. Initially, for time $t^* \le 0$, both the plate and adjacent fluid are assumed to be at the same temperature T_∞^* and concentration C_∞^*. At time $t^* > 0$, the concentration level at the plate is raised to $\left(C_w^* \ne C_\infty^*\right)$ or a solute is supplied at a constant rate (constant mass flux) and the heat transfer from the plate is proportional to the local surface temperature.

To derive the basic equations for the problem under consideration, the following assumptions are made: (i) The flow is unsteady and laminar, and the magnetic field is applied perpendicularly to the plate. (ii) variations in fluid properties are limited to density variations which the affect the buoyancy term,. (iii) The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field can be neglected. (iv) The effect of the viscous dissipation is negligible in the energy equation. (v) There is a first order chemical reaction between the diffusing species and the fluid. (vi) It is also assumed that there is no applied voltage, which implies the absence of an electric field.

Under these assumptions, the governing equations of the problem become:

$$\frac{\partial u^*}{\partial t^*} = (\upsilon + \upsilon_r)\frac{\partial^2 u^*}{\partial y^{*2}} + g_0\beta(T^* - T_\infty^*) + g_0\beta^*(C^* - C_\infty^*) - \left(\sigma\frac{B_0^2}{\rho} + \frac{\nu}{K^*}\right)u^* - \upsilon_r\frac{\partial \omega^*}{\partial y^*} \tag{1}$$

$$\frac{\partial \omega^*}{\partial t^*} = \frac{\gamma}{\rho\lambda^*}\frac{\partial^2 \omega^*}{\partial y^{*2}} \tag{2}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho C_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p}\frac{\partial q_r^*}{\partial y^*} \tag{3}$$

$$\frac{\partial C^*}{\partial t^*} = D_m\frac{\partial^2 C^*}{\partial y^{*2}} - K_r(C^* - C_\infty^*) \tag{4}$$

$$t^* \le 0:\ u^* = 0,\quad \omega^* = 0,\quad T^* = T_\infty^*,\qquad C^* = C_\infty^* \qquad or\quad y^* \ge 0$$

$$t^* > 0: \begin{cases} u^* = L^*\dfrac{\partial u^*}{\partial y^*}, \omega^* = -\dfrac{1}{2}\dfrac{\partial u^*}{\partial y^*}, \dfrac{\partial T^*}{\partial y^*} = -\dfrac{h}{k}T^*, C^* = C_w^*\ (or)\dfrac{\partial C^*}{\partial y^*} = -\dfrac{j''}{D_m} & at\ \ y^* = 0 \\ u^* \to 0,\ T^* \to T_\infty^*,\qquad \omega^* \to 0,\qquad C^* \to C_\infty^* & as\ y^* \to \infty \end{cases} \tag{5}$$

where u^* is the velocity in the x^*- direction, t^* the time, υ is the kinematics viscosity, υ_r is the microrotation viscosity, g_0 is the acceleration of gravity, β and β^* are the coefficient of volume expansion and volume expansion with concentration, T^* and C^* the temperature and the mass concentration of the fluid near the plate, σ is the electrical conductivity of the fluid, B_0 is the magnetic induction, ρ is the density of the fluid, K^* is the permeability of the porous medium, k is the thermal conductivity, ω^* is the angular velocity, γ is the material property of the fluid, λ^* is the micro-inertia density, C_p is the specific heat at constant pressure, q_r^* is the radiative heat flux, D_m is the molecular diffusivity, h is the heat transfer coefficient and K_r the chemical reaction. Parameter L^* is defined as $L^* = \left(\frac{2-m_1}{m_1}\right)L$, where L is the molecular mean free bath and m_1 is the tangential momentum accommodation coefficient. The quantity in the right-hand side of Eq. (3) represents the radiative heat flux in the y^*-direction. In order to simplify the physical problem, the optically thick radiation limit is considered in the present analysis. Thus the radiative heat fluxterm is simplified by using the Rosseland diffusion approximation (Siegel and Howell [22]) for an optically thick fluid according to,

$$q_r^* = -\frac{4\sigma_1}{3K_R}\frac{\partial T^{*4}}{\partial y^*} \tag{6}$$

where σ_1 is the Stefan- Boltzman constant, K_R the Rosseland mean absorption coefficient. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then Eq. (6) can be linearized by expanding T^{*4} into the Taylor series about T_∞^* and neglecting higher order terms [13], we have,

$$T^{*4} \cong 4T_\infty^{*3}T^* - 3T_\infty^{*4} \tag{7}$$

Using Eqs. (6) and (7), Eq. (3) becomes

$$\frac{\partial T^*}{\partial t^*} = \left(\frac{k}{\rho C_p} + \frac{16\,\sigma_1\, T_\infty^{*3}}{3\rho\, C_p\, K_R}\right)\frac{\partial^2 T^*}{\partial y^{*2}} \tag{8}$$

Introducing the following dimensionless variables

$$\begin{gathered}
y = \frac{y^* h}{k},\quad u = \frac{u^* k}{h\upsilon Gr},\quad t = \frac{t^*\upsilon h^2}{k^2},\quad T = \frac{T^* - T_\infty^*}{T_\infty^*},\quad \Pr = \frac{\mu c_p}{k},\\
Gr = \frac{g\beta k^3\, T_\infty^*}{\upsilon^2 h^3},\quad Sc = \frac{\upsilon}{D_m},\quad C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*}(UWC),\quad C = \frac{C^* - C_\infty^*}{\left(\frac{j'' k}{hD}\right)}(UMF),\\
Gm = \frac{g_0\beta^*(C_w^* - C_\infty^*)k^3}{\upsilon^2 h^3}(UWC),\quad Gm = \frac{g_0\beta^* j'' k^4}{\upsilon^2 D h^4}(UMF),\quad M = \frac{\sigma B_0^2 k^3}{\mu h^3\, Gr},\\
R = \frac{kK_R}{4\sigma_1 T_\infty^{*3}},\quad \omega = \frac{k^2}{\upsilon Gr\, h^2}\omega^*,\quad \Delta = \frac{\upsilon_r}{\upsilon},\quad \lambda = \frac{\rho\lambda^*\upsilon}{\gamma},\ N = \frac{Gm}{Gr}\quad K = \frac{h^3\, Gr\upsilon}{k^2}.
\end{gathered} \tag{9}$$

Eqs. (1), (2), (4) and (8) become

$$\frac{\partial u}{\partial t} = (1+\Delta)\frac{\partial^2 u}{\partial y^2} + T + NC - (M + \frac{1}{K})u - 2\Delta\frac{\partial \omega}{\partial y} \tag{10}$$

$$\frac{\partial \omega}{\partial t} = \frac{1}{\lambda}\frac{\partial^2 \omega}{\partial y^2} \tag{11}$$

$$\frac{\partial T}{\partial t} = \left(\frac{3R+4}{3\,R\Pr}\right)\frac{\partial^2 T}{\partial y^2} \tag{12}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - RC \tag{13}$$

The boundary conditions (5) can be written in non-dimensional forms as:

$$\begin{aligned}
&u = 0,\quad \omega = 0,\quad T = 0,\quad C = 0 && for\quad t \le 0\\
&\left.\begin{aligned}
&u = h\frac{\partial u}{\partial y},\ \omega = -\frac{1}{2}\frac{\partial u}{\partial y},\ \frac{\partial T}{\partial y} = -1 - T,\ C = 1\ or\ \frac{\partial C}{\partial y} = -1 \quad at\ \ y = 0\\
&u \to 0,\quad \omega \to 0,\quad T \to 0,\quad C \to 0 \qquad as\ y \to \infty
\end{aligned}\right\} && for\ \ t > 0
\end{aligned} \tag{14}$$

3. Method of Solution

In this section we present the analytical solution for Eqs. (10) and (13) with boundary conditions (14), To solve the nonlinear system (10)–(13) with the boundary conditions (14), we assume that

$$u(y,t) = U(y)\, e^{nt},\quad \omega(y,t) = \omega(y)e^{nt},\ T(y,t) = \Theta(y)\, e^{nt},\quad C(y,t) = \Phi(y)\, e^{nt} \tag{15}$$

Substituting Egs. (15) into the Eqs. (10)-(14) and comparing the harmonic and non-harmonic terms, we get:

$$(1+\Delta)U'' - (n + M + \frac{1}{K})U = -\Theta - N\Phi - 2\Delta\omega' \tag{16}$$

$$\omega'' - n\lambda\omega = 0 \tag{17}$$

$$\Theta'' - \frac{3R\Pr n}{3R+4}\,\Theta = 0 \tag{18}$$

$$\Phi - Sc(n+\gamma)\Phi = 0 \tag{19}$$

Here primes denote differentiation with respect to y. However, this expansion of the solution is meaningful only if the reduced equations are ordinary differential equations of the independent variable y. In addition, the corresponding boundary conditions can be written as

$$t>0:\begin{cases} U=hU',\ \omega=-\frac{1}{2}U',\ \Theta'+\Theta=e^{-nt},\Phi=e^{-nt}\ (or)\ \Phi'=-e^{-nt}\ at\ \ y=0 \\ U\to\infty,\quad \omega\to\infty,\quad \Theta\to\infty,\quad \Phi\to\infty \quad as\ y\to\infty \end{cases} \tag{20}$$

Solving Eqs. (16)-(19) subject to the boundary conditions (20) we obtain

$$u=(A_4e^{-N_2y}+A_5e^{-N_3y}+A_6A_7e^{-N_4y}+A_8e^{-N_5y})e^{nt} \tag{21}$$

$$\omega=A_7e^{nt-N_4y} \tag{22}$$

$$T=A_2e^{nt-N_2y} \tag{23}$$

$$\varphi=\begin{cases} A_1e^{nt-N_3y}, & (UWC) \\ A_{11}e^{nt-N_3y}, & (UMF) \end{cases} \tag{24}$$

where the constants are given in Appendix.

The physical quantities of engineering interest are skin-friction coefficient, couple stress coefficient, Nusselt number and Sherwood number. The local skin friction coefficient Cf is given as

$$C_f=\frac{\tau_w^*}{\rho\, U_r^2}=(C_{fx}+C_{fy})/\rho\, U_r^2=[1+\Delta(1+\frac{i}{2})]u(0) \tag{25}$$

The couple stress coefficient at the wall C_w is given by

$$C_w=\left.\frac{\partial\omega}{\partial y}\right|_{y=0}=\omega'(0) \tag{26}$$

In addition, the rate of heat transfer at the surface of wall in terms of Nusselt number Nu, can be written as:

$$\mathrm{Nu}=x\left.\frac{(\partial T/\partial z)}{T_\infty-T_w}\right|_{z=0}\quad;\qquad Nu=-\mathrm{Re}_x\,T'(0) \tag{27}$$

where $\mathrm{Re}_x=U_rx/\upsilon$ is the Reynolds number.

4. Results and Discussion

System of equations (16) to (19) subject to the boundary conditions (20) are highly coupled and solved analytically. In order to understand the physical solution, the numerical values of concentration, transverse velocity, angular velocity and temperature are presented.

4.1. Effect of Viscosity Ratio Parameter

In Figure 1, the effect of Δ on the translational velocity u and angular velocity ω for a stationary porous plate is shown. It is observed that, as the viscosity ratio parameter Δ is increased, u is decreases and ω is increased. Comparison of the velocities and angular velocities for the UWC and UMF cases show that, in the UMF case a higher velocity and a lower angular velocity than that of UWC case and the difference increases with increasing the value of Δ.

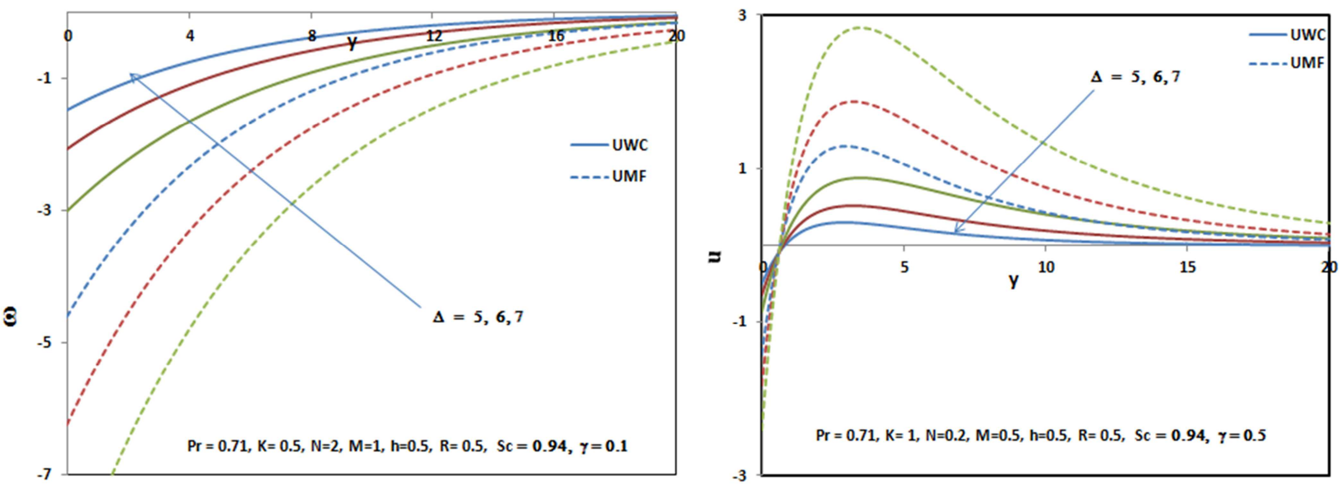

Fig. 1. *Velocity and angular velocity profiles for different.*

4.2. Effect of Slip H

Figure 2 shows the translational velocity and microrotation distribution across the boundary layer for different values of the slip parameter h. It can be seen that, the translational velocity distribution across the boundary layer is increased and the angular velocity is decreased as the slip parameter h is increased. Comparison of the velocities and angular velocities for the UWC and UMF cases indicates that, the velocity is greater and the angular velocity is smaller in case UWC than that of UMF at the same value of h.

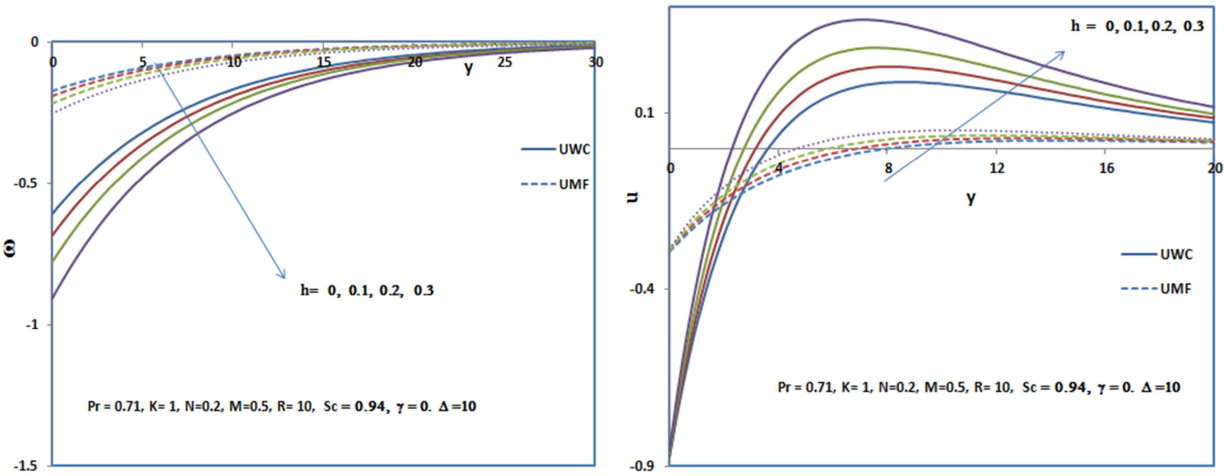

Fig. 2. *Velocity and angular velocity profiles for different h.*

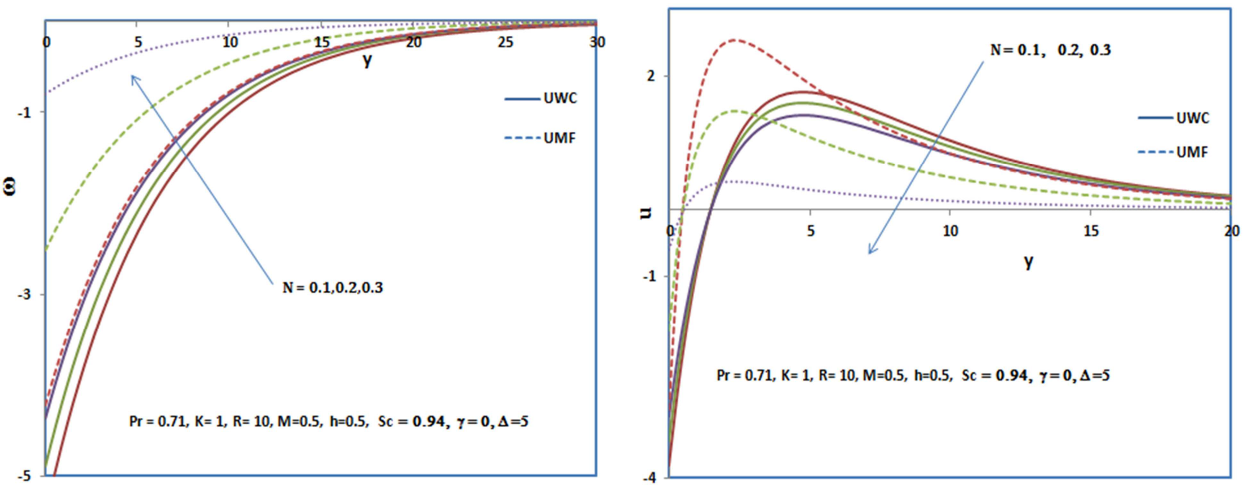

Fig. 3. *Velocity and angular velocity profiles for different N.*

4.3. Effect of the Relative Buoyancy Parameter N

The effect of the relative buoyancy parameter N on the velocity and angular velocity variables is shown in Figure 3. In this figure $N = 0$ corresponds to the situation in which the natural convection arises from the thermal buoyancy force only and there is no contribution from the species diffusion, and $N > 0$ means that the buoyancy force from the species diffusion assists the buoyancy force. It is observed that, the velocity decreases and the angular velocity increases as N increases. Comparison of the velocities and angular velocities for the UWC and UMF cases show that, in the UWC case, a higher velocity and lower angular velocity than that of UMF case and the difference increases with increasing values of N.

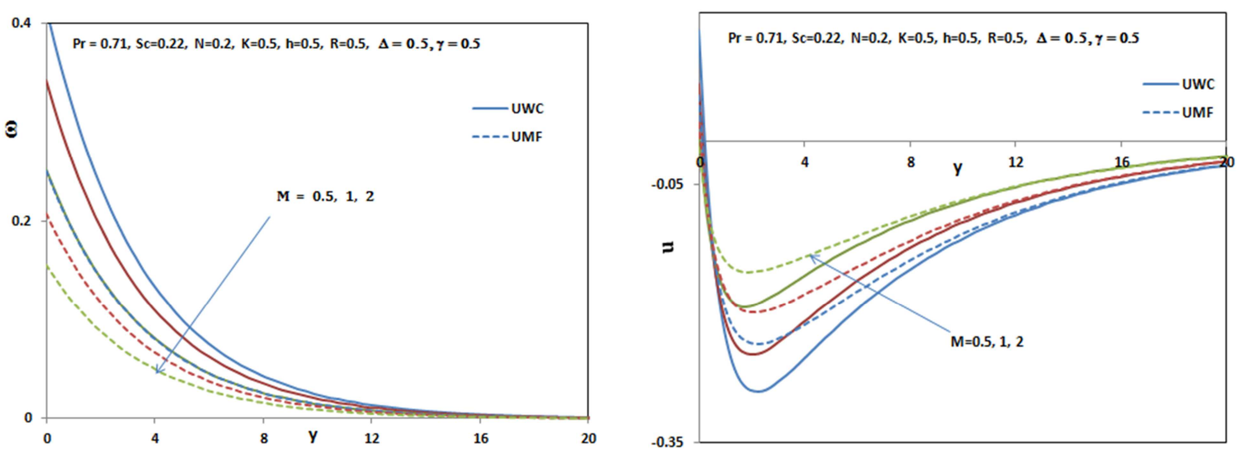

Fig. 4. *Velocity and angular velocity profiles for different M.*

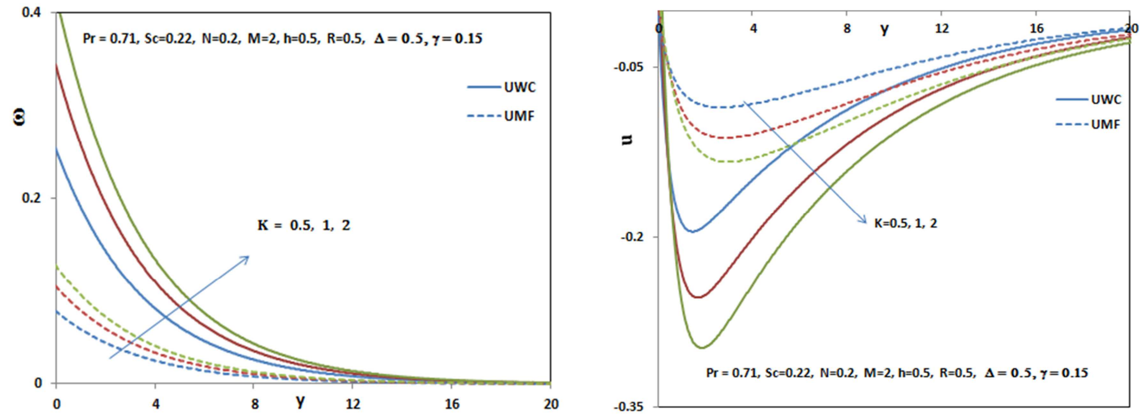

Fig. 5. *Velocity and angular velocity profiles for different K.*

4.4. Effect of Magnetic Field Parameter M

Figure 4 shows the pattern of the translational velocity and angular velocity for different values of the magnetic field parameter M. It is observed that, adjacent to the surface of the plate, the translational velocity increases as M increases. Furthermore, the angular velocity decreases as M increases.

4.5. Effect of the Permeability Parameter K

For different values of the permeability parameter K, the translational velocity and angular velocity are plotted in Figure 5. It is obvious that, as K is increasing, the velocity is decreasing and the angular velocity is increasing. This leads to enhancement of the momentum boundary layer thickness.

4.6. Effect of Prandtl Number and the Thermal Radiation Parameter Pr and R

From Figures 6 and 8, we observe that, an increase in suction velocity an increase in Prandtl number Pr leads to an increase in the velocity, angular velocity and temperature profile. For different values of radiation parameter R, the velocity, angular velocity and temperature profiles are plotted in Fig. 7. Here we find that, as the value of R increases the velocity and temperature increases but leads to a decrease in the angular velocity profile.

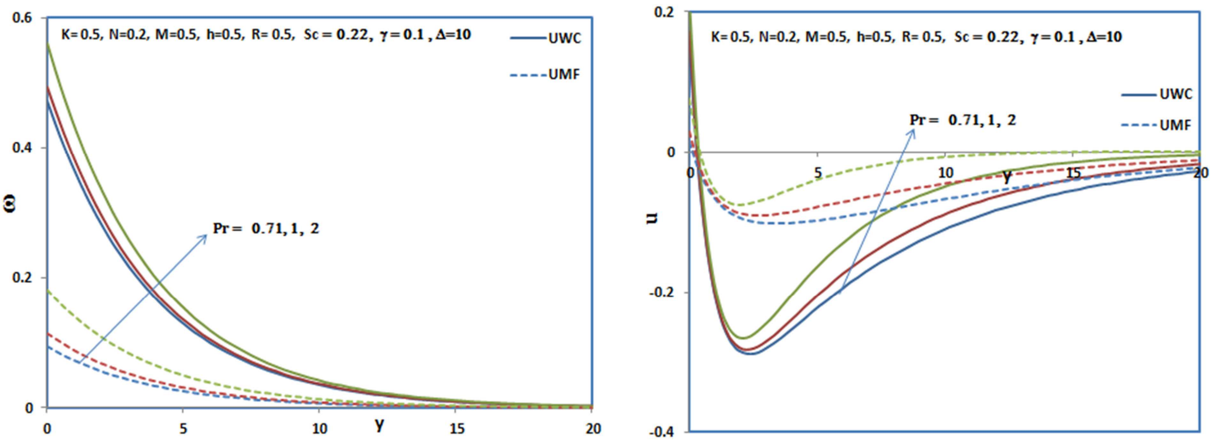

Fig. 6. *Velocity and angular velocity profiles for different Pr.*

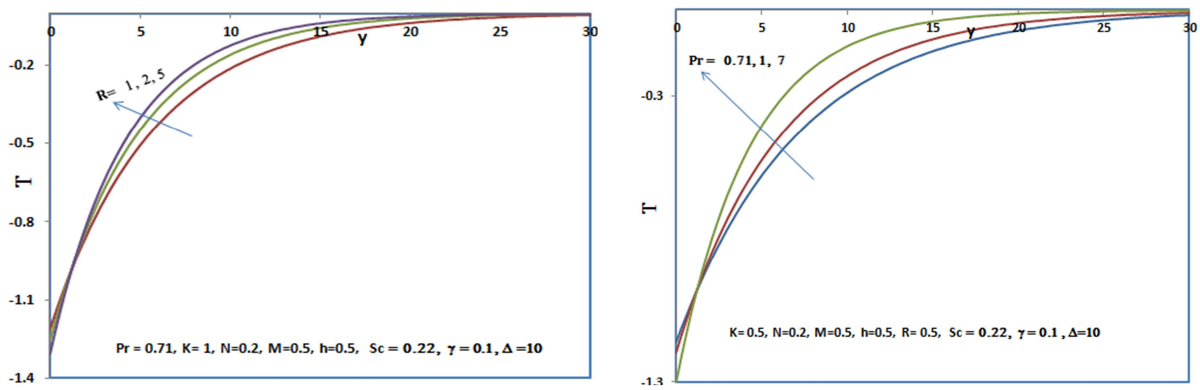

Fig. 7. *Temperature profiles for different Pr and R.*

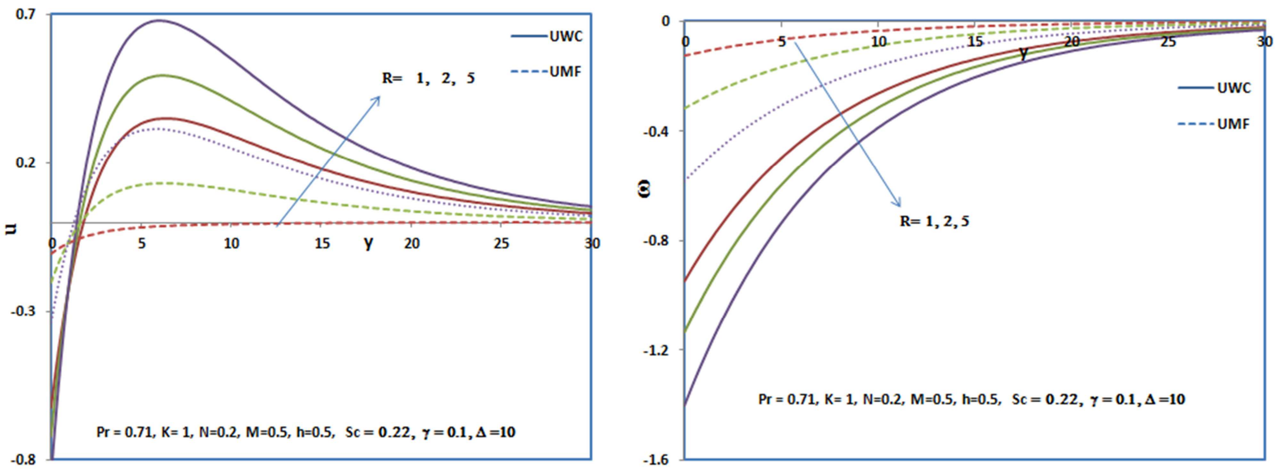

Fig. 8. *Velocity and angular velocity profiles for different R.*

4.7. Effect of Chemical Reaction Parameter γ and Diffusion Parameter Sc

The influences of chemical reaction parameter γ and diffusion parameter Sc on the velocity, angular velocity and concentration profiles across the boundary layer are presented in Figures 9, 10 and 11. We see that, the velocity and concentration distribution across the boundary layer decrease with increasing of γ and Sc. Also, an increasing in the chemical reaction parameter γ will increase the angular velocity profile while, an increase in diffusion parameter Sc leads to a decrease in the angular velocity profile.

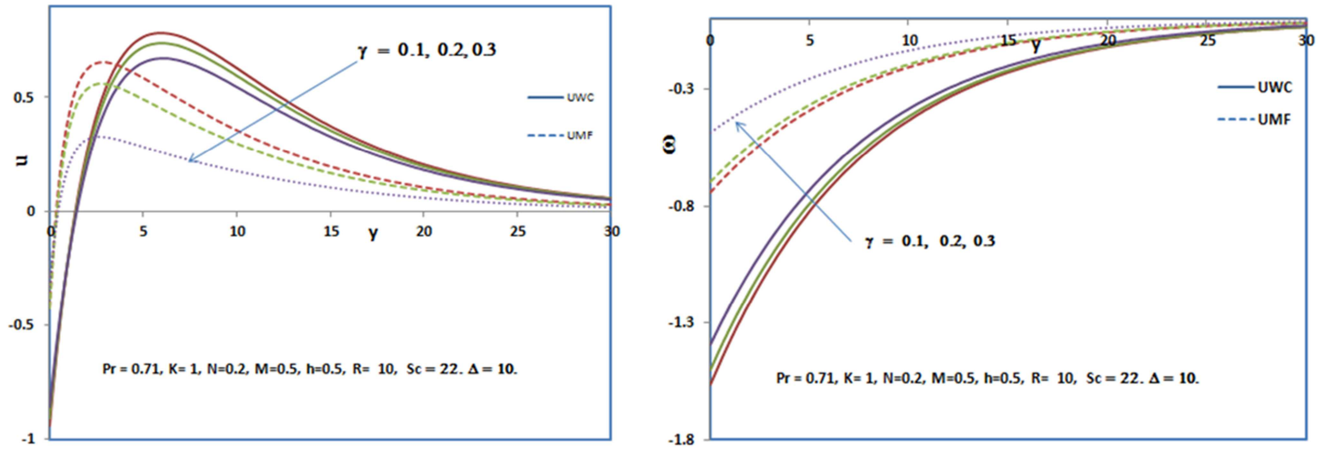

Fig. 9. *Velocity and angular velocity profiles for different .*

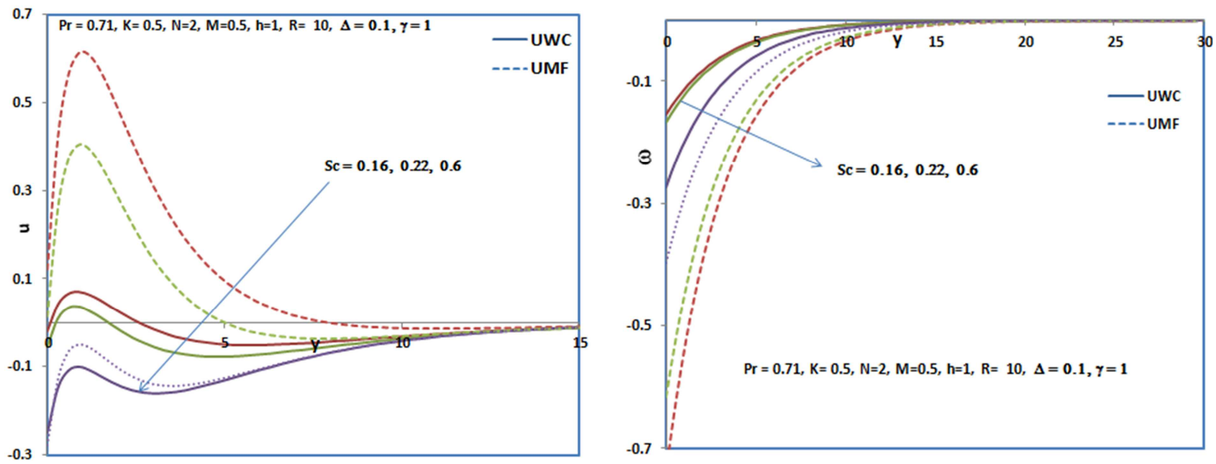

Fig. 10. *Velocity and angular velocity profiles for different Sc.*

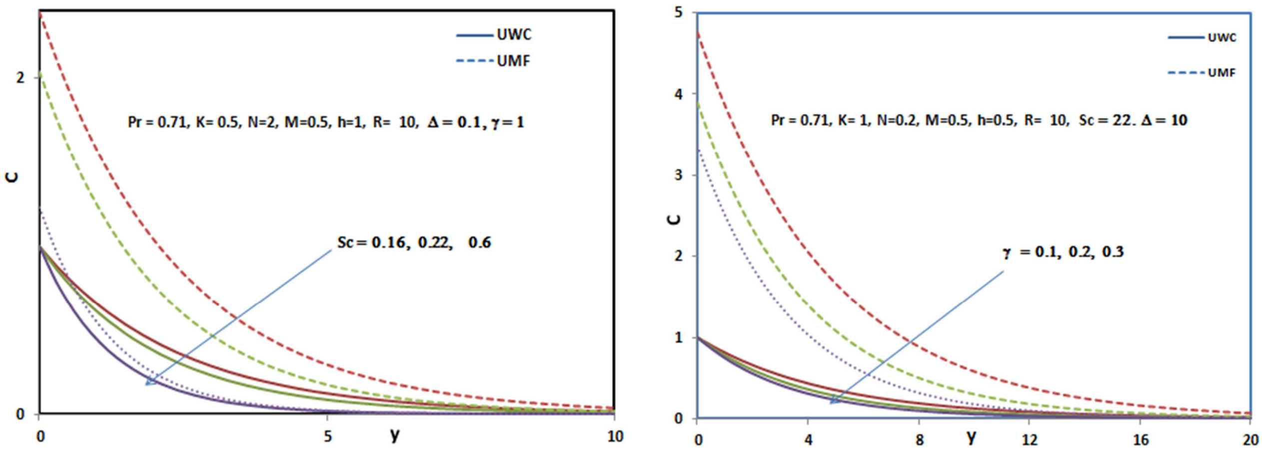

Fig. 11. *Concentration profiles for different and Sc.*

5. Conclusions

The unsteady MHD free convection heat and mass transfer flow of an incompressible, micropolar fluid along a semi-infinite vertical plate with Newtonian heating in the presence of the slip flow regime has been analyzed. The governing equations are solved analytically by using perturbation technique. The results are discussed through graphs. The effects of radiation parameter, viscosity ratio parameter, buoyancy ratio, Schmidt number, porous medium, magnetic field, Prandtl number, and chemical reaction on the velocity, angular velocity, temperature and concentration have been studied in detail. The study reveals that these parameters have significant influence on the velocity, angular velocity, temperature and concentration. It is found that, the velocity in the UWC case is slightly higher than the UMF case at the slip flow regime, buoyancy ratio, radiation parameter and chemical reaction and the velocity is slightly higher in the case UMF than that of UWC at an viscosity ratio parameter, Schmidt number, porous medium, magnetic field and Prandtl number, but an opposite trend is observed at an angular velocity. It is expected that the outcomes of the present study will serve as foundation for more complex and realistic cases of free convection flows resulting from the combined heat and mass transfer along a vertical surface with Newtonian heating.

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Appendix

$N_1^2 = M + n + \dfrac{1}{K}$	$A_1 = 1,\quad A_{11} = \dfrac{1}{N_3}$	$N_5 = \dfrac{N_1}{1+\Delta},$	$A_2 = \dfrac{1}{N_2 - 1}$
$N_2^2 = \dfrac{3n\Pr R}{3R+4}$	$A_4 = \dfrac{-A_2}{(1+\Delta)N_2^2 - N_1^2}$	$N_4^2 = n\,\lambda$	

$$A_5 = \frac{-N A_1}{(1+\Delta)N_3^2 - N_1^2}$$

$$A_7 = \frac{(N_2A_4 + N_3A_5)(1-hN_5) + N_5A_4(1-hN_2) + N_5A_5(1-hN_3)}{(2 - N_4A_6)(1-hN_5) - N_5A_6(1-hN_4)}$$

$$A_6 = \frac{2\Delta N_4}{(1+\Delta)N_4^2 - N_1^2}$$

$$A_8 = \frac{1}{1-hN_5}\left(A_4(hN_2 - 1) + A_5(hN_3 - 1) + A_6A_7(hN_4 - 1)\right)$$

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