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Regarding New Complex Analytical Solutions for the Nonlinear Partial Vakhnenko-Parkes Differential Equation via Bernoulli Sub-Equation Function Method

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Abstract: In this research, a structure of the Bernoulli sub-equation function method is proposed. The nonlinear partial Vakhnenko-Parkes differential equation which is another name the reduced Ostrovsky equation has been taken into consideration. Then, analytical solutions such as rational function solution, exponential function solution, hyperbolic function solution, complex trigonometric function solution and periodic wave solution have been obtained by the same method. All necessary calculations while obtaining the analytical solutions have been accomplished through using commercial wolfram software Mathematica 9.

Keywords: The Bernoulli Sub-Equation Function Method, Nonlinear Partial Vakhnenko-Parkes Differential Equation, The Reduced Ostrovsky Equation, Rational Function Solution, Exponential Function Solution, Hyperbolic Function Solution, Complex Trigonometric Function Solution

1. Introduction

Especially, the last two decade have witnessed significant developments of various methods such as Böcklund transformation method, inverse scattering method, the exp-function method, the modified simple equation method, various trial equation methods, Sumudu transform method, the tanh function method, the sine–cosine method, the tanh–sech method, homogeneous balance method, the Darboux transformation, Variable separation approach, Variational iteration method and so on [1-10] for obtaining exact solutions to the nonlinear evolution equations.

In this paper, we have applied the Bernoulli sub-equation function method (BSEFM) [11] to the nonlinear partial Vakhnenko-Parkes equation (NPVPE) [12-24] defined by

$$uu_{xxt}-u_xu_{xt}+u^2u_t=0. \tag{1}$$

2. General Structure of Approach

In this sub-section, an approach to the NPVPE will be given. In order to apply this method to the NPVPE, we consider the following steps [11].

Step 1. We consider the partial differential equation in two variables such as x,t and a dependent variable u

$$P(u_x,u_t,u_{xt},u_{xx},\cdots)=0, \tag{2}$$

and take the wave transformation

$$u(x,t)=u(\eta),\ \eta=x-ct, \tag{3}$$

where $c\neq 0$. Substituting Eq.(3) in Eq.(2), it gives us the following nonlinear ordinary differential equation;

$$N(u,u',u'',u''',\cdots)=0. \tag{4}$$

Step 2. Take trial equation as follows:

$$\begin{aligned}u(\eta)&=\sum_{i=0}^{n}a_iF^i(\eta),\\&=a_0+a_1F(\eta)+a_2F^2(\eta)+\cdots+a_nF^n(\eta),\end{aligned} \tag{5}$$

in which

$$F'(\eta)=bF(\eta)+dF^{M}(\eta), \tag{6}$$

where $b\neq 0,\ d\neq 0,\ M\in\mathbb{R}-\{0,1,2\}$ and $F(\eta)$ is Bernoulli differential polynomial. Substituting above relations in Eq.(4), we obtain an equation of polynomial $\Omega(F(\eta))$ of $F(\eta)$:

$$\Omega(F(\eta))=\rho_s F(\eta)^s+\cdots+\rho_1 F(\eta)+\rho_0=0. \tag{7}$$

According to the balance principle, we can get values of n and M.

Step 3. Let's consider the coefficients of $\Omega(F(\eta))$ all be zero, we will obtain an algebraic equations system:

$$\rho_i=0,\ i=0,\cdots,s. \tag{8}$$

Solving this system, we will determine the values of $a_0,\cdots,a_n$.

Step 4. When we solve nonlinear Bernoulli differential equation Eq.(6) by using methods known, we obtain following two situations according to b and d;

$$F(\eta)=\left[\frac{-d}{b}+\frac{c}{e^{b(M-1)\eta}}\right]^{\frac{1}{1-M}},\ b\neq d, \tag{9}$$

$$F(\eta)=\left[\frac{(c-1)+(c+1)\tanh\left(\dfrac{b(1-M)\eta}{2}\right)}{1-\tanh\left(\dfrac{b(1-M)\eta}{2}\right)}\right]^{\frac{1}{1-M}}, \tag{10}$$

where $b=d,\ c\in\mathbb{R}$. Using a complete discrimination system for polynomial to classify the roots of $F(\eta)$, we solve Eq.(5) with the help of Mathematica 9 programming and classify the exact solutions to Eq.(5). For a better interpretations of results obtained in this way, we can plot two and three dimensional surfaces of analytical solutions obtained by taking into consideration suitable parameter.

3. Implementation of Proposed Method

In this section, we have obtained some new analytical solutions such as rational function, exponential function, hyperbolic function, complex trigonometric function and periodic wave solutions of NPVPE by using BSEFM.

Application First of all, if we perform travelling wave transformation to NPVPE in the following manner;

$$u(x,t)=u(\eta),\ \eta=x-ct, \tag{11}$$

$$\frac{\partial u}{\partial x}=u',\ \frac{\partial u}{\partial t}=-cu',\ \frac{\partial^2 u}{\partial x\partial t}=-cu'',\ \frac{\partial^3 u}{\partial x^2\partial t}=-cu''', \tag{12}$$

where c is a constant and not zero, we get the nonlinear ordinary differential equation as following;

$$uu'''-u'u''+u^2u'=0. \tag{13}$$

Integrating Eq.(13) and considering the constant of integration to be zero, we rewrite the Eq.(13) as following;

$$3uu''-3(u')^2+u^3=0, \tag{14}$$

when we reconsider to Eq.(5) and Eq.(6) for balance principle, we obtain following relationship for n and M;

$$n+2=2M. \tag{15}$$

This resolution procedure is applied and we obtain results as follows:

Case 1: If we take $M=3$ and $n=4$ for Eq. (15), then, we write following equations;

$$u=a_0+a_1F+a_2F^2+a_3F^3+a_4F^4, \tag{16}$$

$$\begin{aligned}u'&=a_1bF+a_1dF^M+2a_2bF^2+2a_2dF^{1+M}\\&+3a_3bF^3+3a_3dF^{2+M}+4a_4bF^4\\&+4a_4dF^{3+M},\end{aligned} \tag{17}$$

and

$$\begin{aligned}u''&=a_1bF'+a_1dMF^{M-1}F'+4a_2bFF'+16a_4bF^3F'\\&+2a_2d(M+1)F^MF'+9a_3bF^2F'\\&+3(2+M)a_3dF^{1+M}F'+4a_4d(3+M)F^{2+M}F',\end{aligned} \tag{18}$$

where $F'=bF+dF^3,\ b\neq 0,\ d\neq 0.$ When we use Eqs.(16,17,18) in the Eq.(14), we get a system of algebraic equations for Eq.(14). Therefore, we attain a system of algebraic equations from the coefficients of polynomial of $\Omega(F(\eta))$. By solving this algebraic equation system for Eq.(16) with the help of Mathematica programming 9, we obtain the following coefficients;

Case 1a. For $b\neq d$, it can be considered that the following coefficients;

$$\begin{aligned}&a_0=a_1=0,a_2=-24bd,\ a_3=0,\\&a_4=-24d^2,b=b,\ c=c,d=d.\end{aligned} \tag{19}$$

Substituting Eq.(19) in Eq.(16) along with Eq.(3), we obtain the exponential function solution to the NPVPE in the following form;

$$u_1(x,t)=-\frac{24cdb^3e^{2b(x-ct)}}{\left(bc-de^{2b(x-ct)}\right)^2}. \tag{20}$$

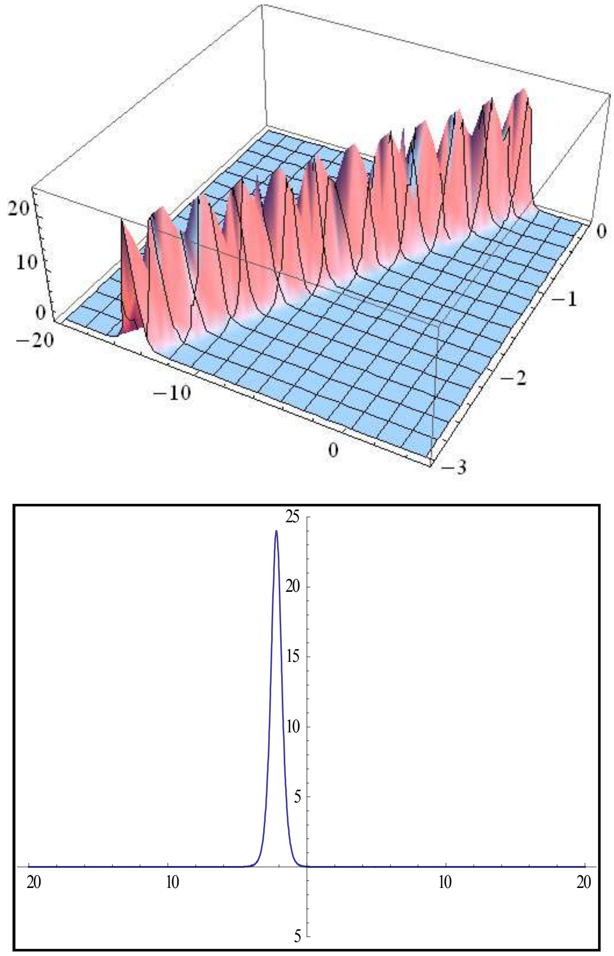

Figure 1. *The 3D and 2D surfaces of the analytical solution Eq.(20) being exponential function solution by considering the values* $b=2,\ d=-3,\ c=5,\ -20<x<5,\ -3<t<0,$ *for 3D graphics and* $b=2,\ d=-3,\ c=5,\ t=-0.5,\ -20<x<20$ *for 2D surfaces.*

Case 1b. If it is taken as $b=d$ for Eq.(19), it can be obtained that the following coefficients;

$$\begin{aligned} &a_0=a_1=0, a_2=-24bd,\ a_3=0,\\ &a_4=-24d^2, b=d,\ c=c. \end{aligned} \tag{21}$$

Substituting Eq.(21) values in Eq.(16) along with Eq.(3), we obtain another exponential function solution to the nonlinear partial Vakhnenko-Parkes equation in the following manner;

$$u_2(x,t)=-\frac{24cd^2e^{2d(x+ct)}}{\left(ce^{2cdt}-e^{2dx}\right)^2}. \tag{22}$$

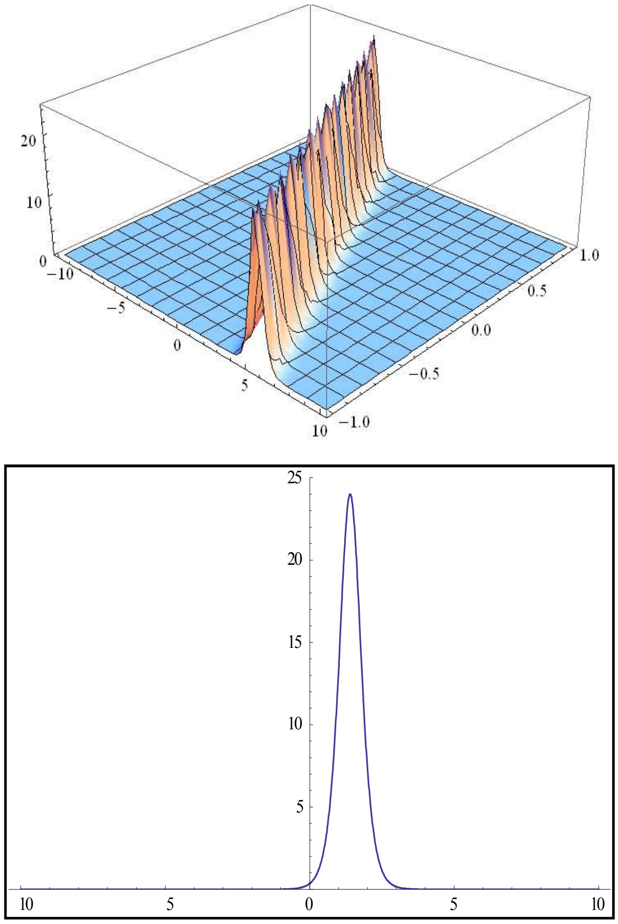

Figure 2. *The 3D and 2D surfaces of the analytical solution Eq.(22) being another exponential function solution by considering the values* $b=d=2,\ c=-5,\ -10<x<10,\ -1<t<1,$ *for 3D graphics and* $b=d=2,\ c=-5,\ t=-0.2,\ -10<x<10$ *for 2D surfaces.*

Case 1c. For $b\neq d$, it can be considered that the following coefficients

$$\begin{aligned} &a_0=a_1=0, a_2=a_2,\ a_3=0,\\ &a_4=-24d^2,\ b=\frac{-a_2}{24d},\ c=c,\ d=d. \end{aligned} \tag{23}$$

Substituting Eq.(23) coefficients in Eq.(16) along with Eq.(3),we obtain the new hyperbolic function solution for NPVPE in the following manner;

$$\begin{aligned} u_3(x,t)=ca_2^3&\left(24d^2+\frac{ca_2+ca_2\tanh\left(\frac{a_2(x-ct)}{24d}\right)}{1-\tanh\left(\frac{a_2(x-ct)}{24d}\right)}\right)^{-2}\\ &\times\left(\frac{1+\tanh\left(\frac{a_2(x-ct)}{24d}\right)}{1-\tanh\left(\frac{a_2(x-ct)}{24d}\right)}\right). \end{aligned} \tag{24}$$

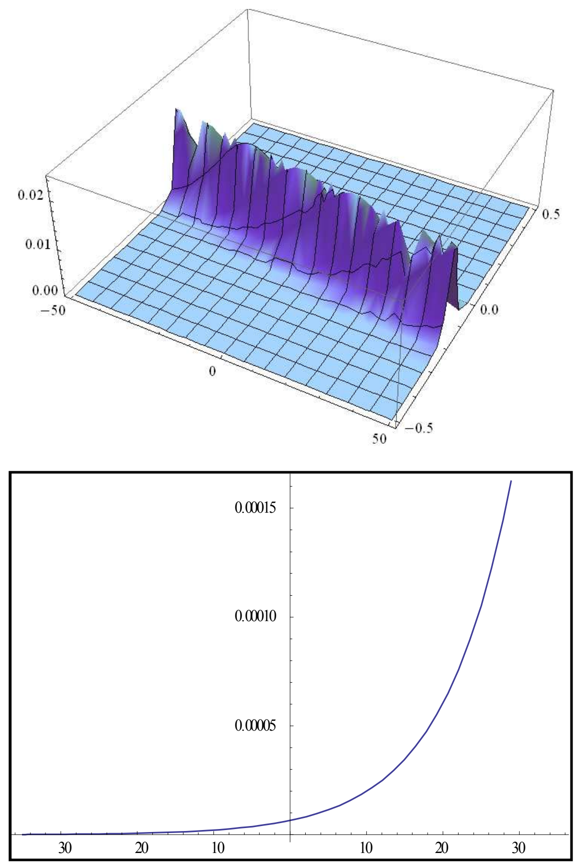

Figure 3. *The 3D and 2D surfaces of the analytical solution Eq.(24) being new hyperbolic function solution by considering the values* $d=-3,\ c=-522, a_2=-4,\ -50<x<50,\ \ -0.5<t<0.5,$ *for 3D graphics and* $d=-3,\ c=-522,\ a_2=-4,\ t=-0.2,\ -10<x<10$ *for 2D surfaces.*

Case 1d. If it is chosen as $b=d$ for Eq.(23), it can be obtained that the following coefficients;

$$\begin{aligned} &a_0=a_1=0, a_2=a_2,\ a_3=0, \\ &a_4=-24d^2,\ b=d=\frac{i\sqrt{a_2}}{24},\ c=c. \end{aligned} \tag{25}$$

Substituting Eq.(25) in Eq.(16) along with Eq.(3), we obtain the new trigonometric complex function solution of the nonlinear partial Vakhnenko-Parkes equation in the following manner;

$$u_4(x,t)=\frac{-ca_2\sec\left(\dfrac{\sqrt{a_2}\,(x-ct)}{2\sqrt{6}}\right)^2}{\left(i(-1+c)+(1+c)\tan\left(\dfrac{\sqrt{a_2}\,(x-ct)}{2\sqrt{6}}\right)\right)^2}. \tag{26}$$

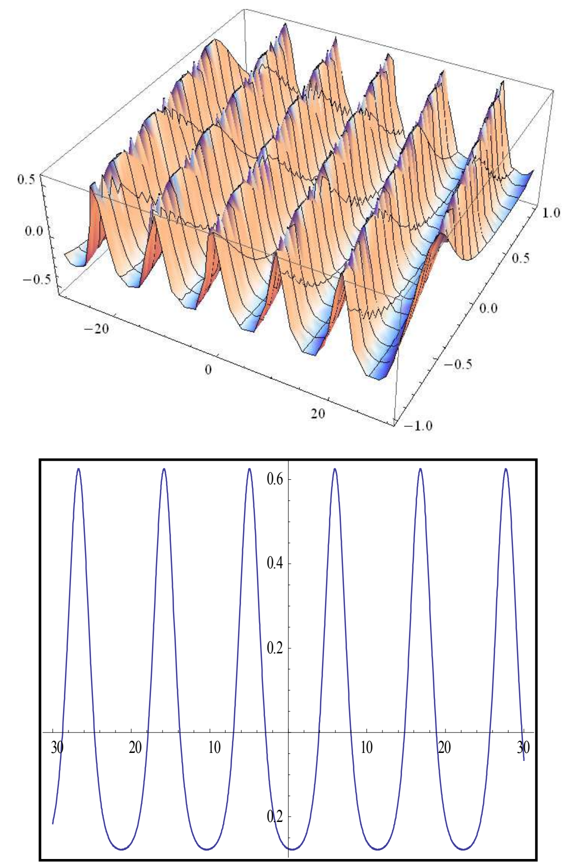

Figure 4. *The 3D and 2D surfaces of real part of the analytical solution Eq.(26) by considering the values* $c=-5,\ a_2=2,\ -30<x<30,\ \ -1<t<1$ *for 3D graphics and* $t=-0.1$ *for 2D surfaces.*

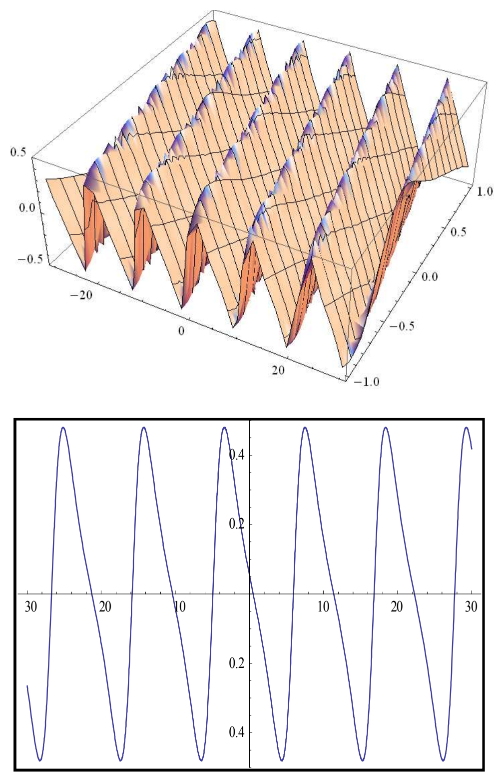

Figure 5. *The 3D and 2D surfaces of imaginary part of the analytical solution Eq.(26) by considering the values* $c=-5,\ a_2=2,\ -30<x<30, -1<t<1,$ *for 3D graphics and* $t=-0.1$ *for 2D surfaces.*

Case 1e. For $b \neq d$, it can be considered that the following coefficients

$$\begin{aligned} &a_0 = a_1 = 0, a_2 = 2bi\sqrt{6a_4}, \ a_3 = 0, \\ &a_4 = a_4, \ b = b, d = \frac{-i\sqrt{a_4}}{2\sqrt{6}}, \ c = c. \end{aligned} \tag{27}$$

Setting Eq.(27) in Eq.(16) along with Eq.(3), we obtain the new complex exponential function solution to the NPVPE as following;

$$u_5(x,t) = -\frac{-288cb^3 i\sqrt{6a_4}e^{2b(x-ct)}}{\left(-12bci + \sqrt{6a_4}e^{2b(x-ct)}\right)^2}. \tag{28}$$

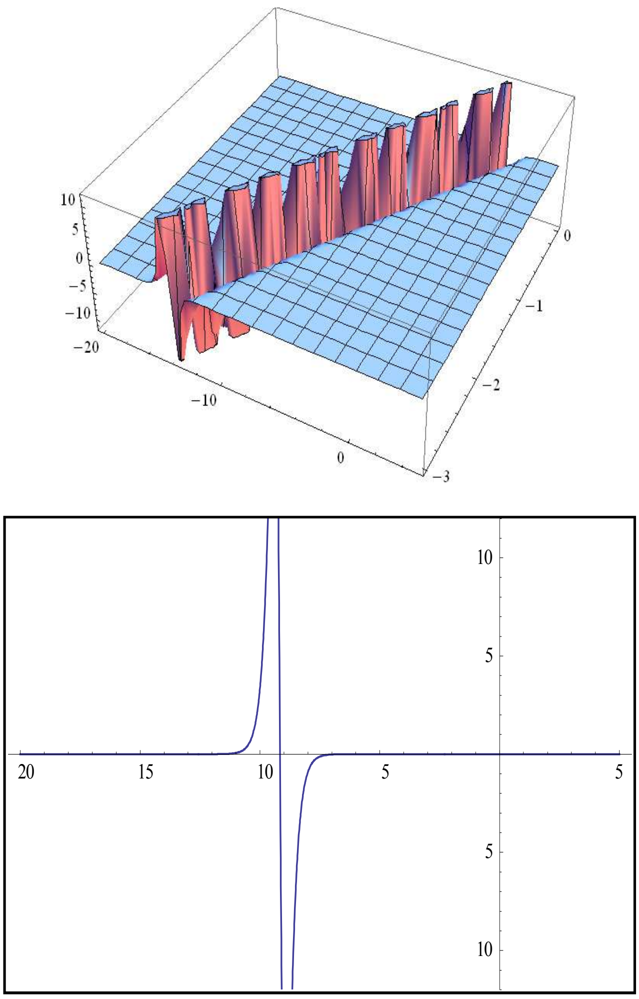

Figure 6. *The 3D and 2D surfaces of imaginary part of the analytical solution Eq.(28) by considering the values* $b=2,\ c=5,\ a_4=3,\ -20<x<5,\ -3<t<0$ *for 3D graphics and* $t=-2$ *for 2D surfaces.*

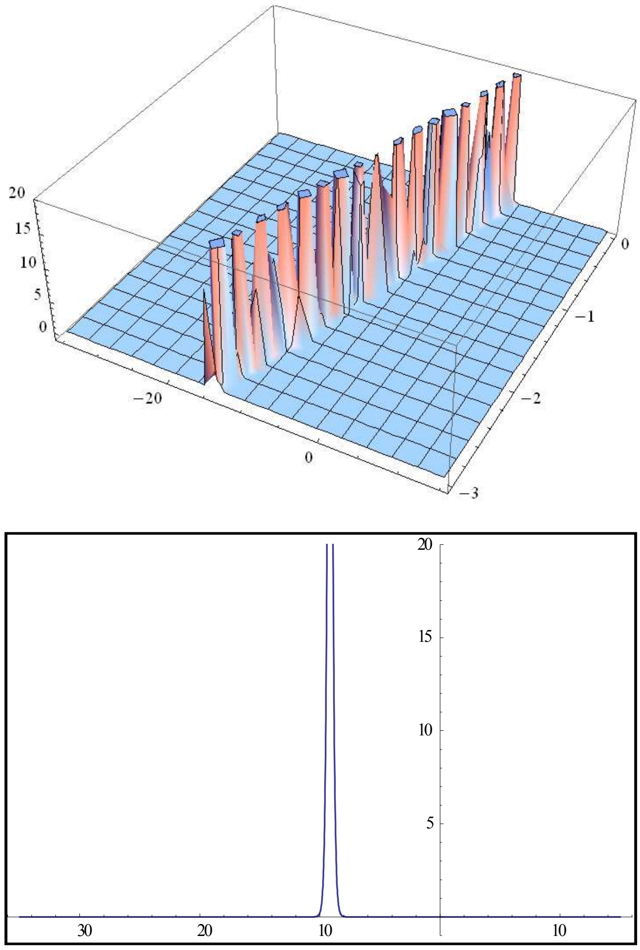

Figure 7. *The 3D and 2D surfaces of real part of the analytical solution Eq.(28) by considering the values* $b=2,\ c=5,\ a_4=3,\ -20<x<5,\ -3<t<0$ *for 3D graphics and* $t=-2$ *for 2D surfaces.*

Case 1f. If it is gotten $b=d$ for Eq.(27), it can be obtained that the following coefficients;

$$\begin{aligned} &a_0 = a_1 = 0, a_2 = 2bi\sqrt{6a_4}, \ a_3 = 0, \\ &a_4 = a_4, \ b = d = \frac{-i\sqrt{a_4}}{2\sqrt{6}}, \ c = c. \end{aligned} \tag{29}$$

Substituting Eq.(29) coefficients in Eq.(16) along with Eq.(3), we obtain the another complex trigonometric function solution for the nonlinear partial Vakhnenko-Parkes equation as following;

$$u_6(x,t) = \frac{-ca_4 \sec\left(\frac{(x-ct)\sqrt{a_4}}{2\sqrt{6}}\right)^2}{\left(-i(-1+c)+(1+c)\tan\left(\frac{(x-ct)\sqrt{a_4}}{2\sqrt{6}}\right)\right)^2}. \tag{30}$$

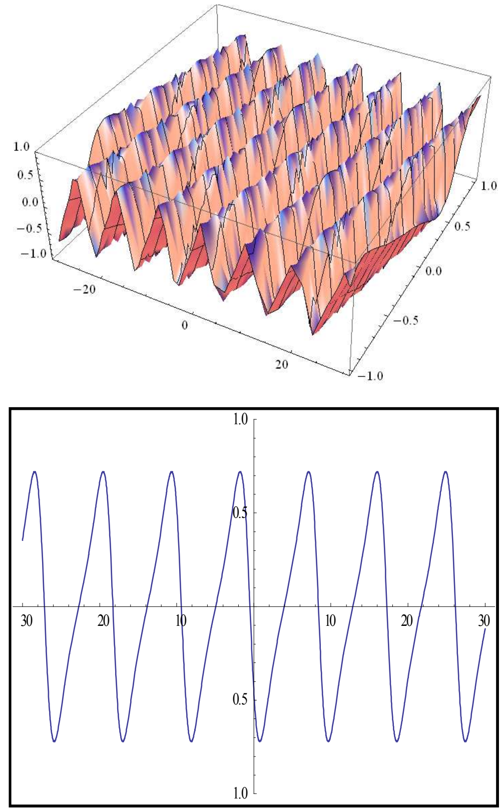

Figure 8. *The 3D and 2D surfaces of imaginary part of the analytical solution Eq.(30) by considering the values* $c=5,\ a_4=3,\ -30<x<30,\ \ -1<t<1$ *for 3D graphics and* $t=-0.1$ *for 2D surfaces.*

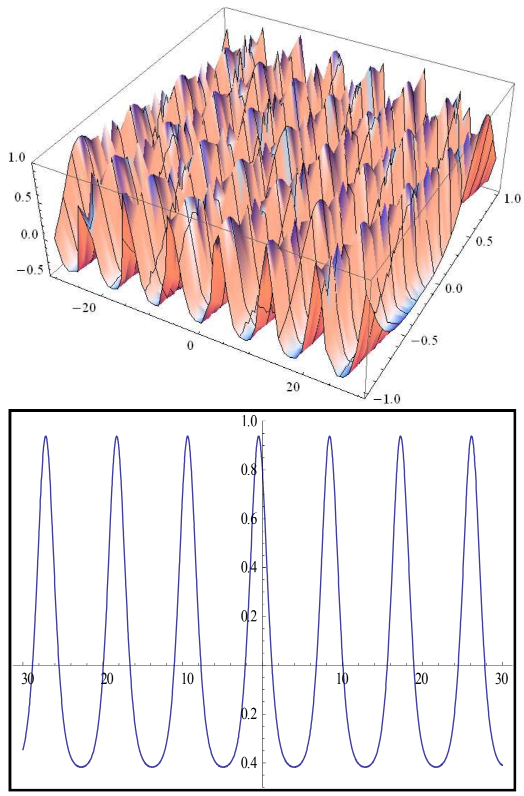

Figure 9. *The 3D and 2D surfaces of real part of the analytical solution Eq.(30) by considering the values* $c=5,\ a_4=3,\ -30<x<30,\ \ -1<t<1$ *for 3D graphics and* $t=-0.1$ *for 2D surfaces.*

Case 1g. For $b\neq d$, it can be considered that the following coefficients;

$$\begin{aligned} &a_0=a_1=0, a_2=-2bi\sqrt{6a_4},\ a_3=0,\\ &a_4=a_4,\ \ b=b, d=\frac{i\sqrt{a_4}}{2\sqrt{6}},\ c=c. \end{aligned} \tag{31}$$

Setting Eq.(31) values in Eq.(16) along with Eq.(3), we obtain the another complex exponential function solution to the NPVPE in the following manner;

$$u_7(x,t)=\frac{288cb^3i\sqrt{6a_4}e^{2b(x-ct)}}{\left(12bci+\sqrt{6a_4}e^{2b(x-ct)}\right)^2}. \tag{32}$$

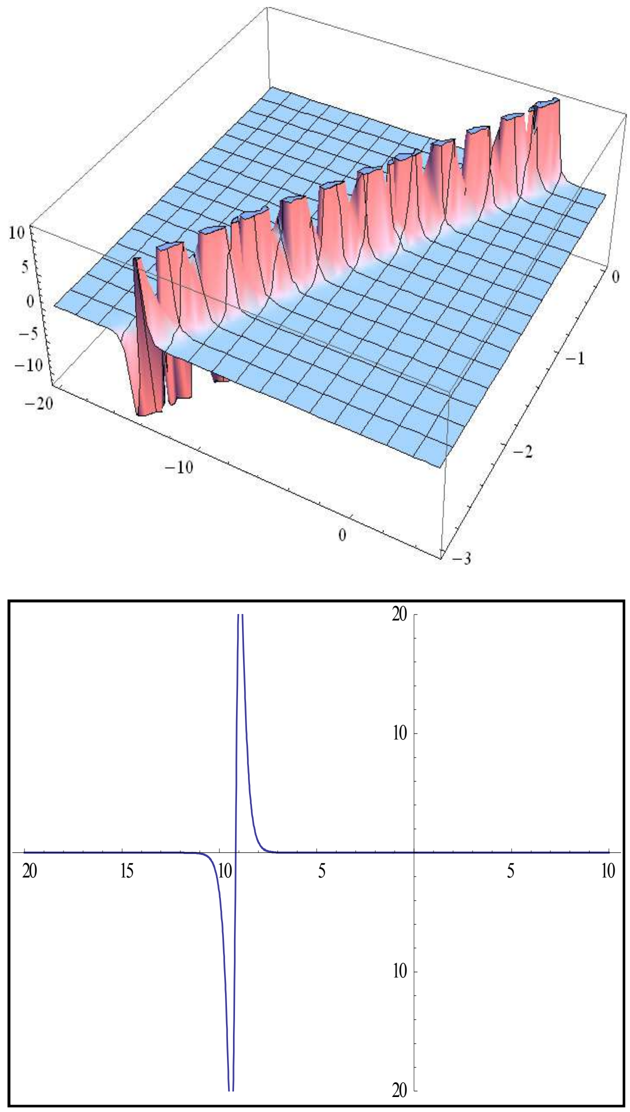

Figure 10. *The 3D and 2D surfaces of imaginary part of the analytical solution Eq.(32) by considering the values* $b=2,\ c=5,\ \ a_4=3,\ -20<x<5,$ $-3<t<0$ *for 3D graphics and* $t=-2$ *for 2D surfaces.*

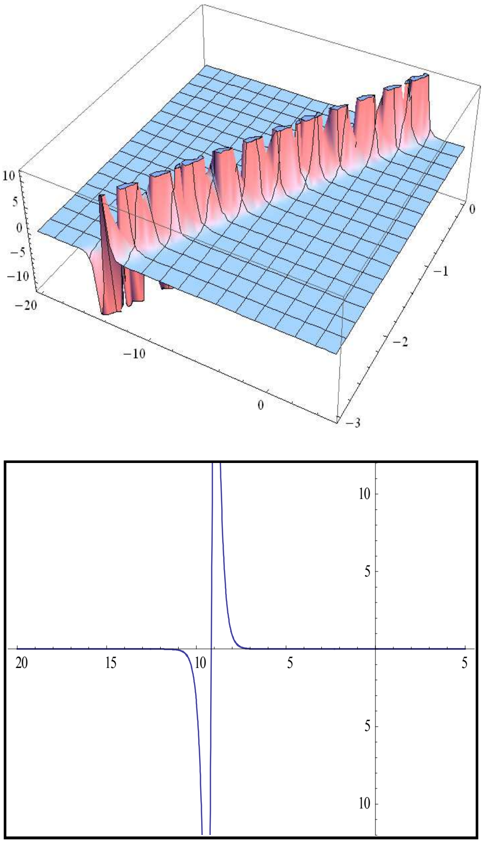

Figure 11. *The 3D and 2D surfaces of real part of the analytical solution Eq.(32) by considering the values* $b=2, c=5, a_4=3, -20<x<5, -3<t<0$ *for 3D graphics and* $t=-2$ *for 2D surfaces.*

Case 1h. When it is taken into consideration as $b=d$ for Eq.(31), it can be obtained that the following coefficients;

$$a_0=a_1=0, a_2=-2bi\sqrt{6a_4}, a_3=0,$$
$$a_4=a_4, b=d=\frac{i\sqrt{a_4}}{2\sqrt{6}}, c=c. \tag{33}$$

Substituting Eq.(33) values in Eq.(16) along with Eq.(3), we obtain the another complex trigonometric function solution for the nonlinear partial Vakhnenko-Parkes equation in the following form;

$$u_8(x,t)=-ca_4\sec\left(\frac{(x-ct)\sqrt{a_4}}{2\sqrt{6}}\right)^2 \times\left(i(-1+c)+(1+c)\tan\left(\frac{(x-ct)\sqrt{a_4}}{2\sqrt{6}}\right)\right)^{-2}. \tag{34}$$

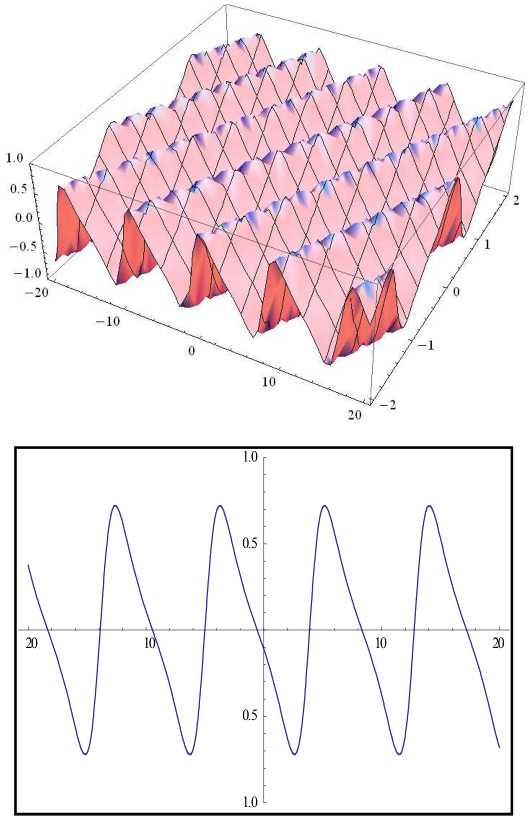

Figure 12. *The 3D and 2D surfaces of imaginary part of the analytical solution Eq.(34) by considering the values* $c=5, a_4=3, -20<x<20, -2<t<2$ *for 3D graphics and* $t=-1$ *for 2D surfaces.*

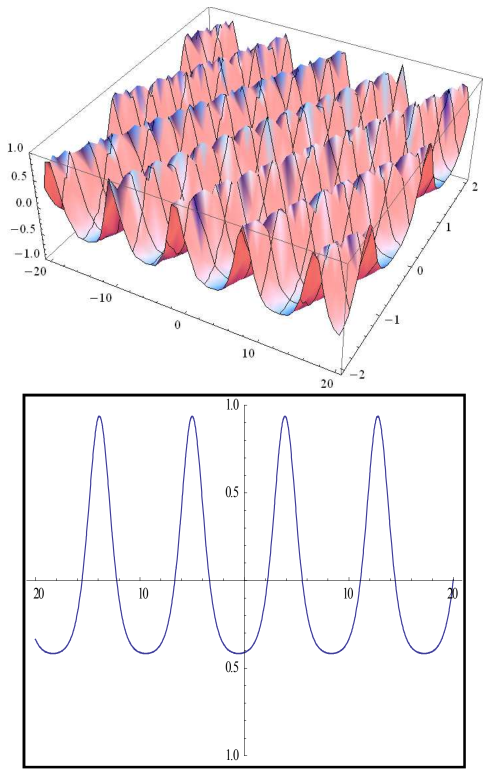

Figure 13. *The 3D and 2D graphs of real part of the analytical solution Eq.(34) by considering the values* $c=5, a_4=3, -20<x<20, -2<t<2$ *for 3D graphics and* $t=-1$ *for 2D surfaces.*

Case 2: If we take $M = 4$ and $n = 6$ for Eq. (15), then, we can write the following equalities;

$$u = a_0 + a_1 F + a_2 F^2 + a_3 F^3 + a_4 F^4 + a_5 F^5 + a_6 F^6, \quad (35)$$

and

$$\begin{aligned} u' &= a_1 F' + 2a_2 FF' + 3a_3 F^2 F' + 4a_4 F^3 F' + 5a_5 F^4 F' \\ &+ 6a_6 F^5 F', \quad (36) \\ &\vdots \end{aligned}$$

where $F' = bF + dF^4$, $b \neq 0$, $d \neq 0$. When we put Eqs.(35,36) in Eq.(14), we obtain a system of algebraic equations for Eq.(14). Therefore, we attain a system of algebraic equations from these coefficients of polynomial of Q. By solving this algebraic equations system with the help of wolfram Mathematica programming 9, we can obtain the following coefficients;

$$\begin{aligned} &a_0 = a_1 = a_2 = 0,\ a_3 = a_3,\ a_4 = a_5 = 0, \\ &a_6 = -54d^2,\ b = \frac{-a_3}{54d},\ c = c, d = d, \end{aligned} \quad (37)$$

in which $b \neq d$. Putting Eq.(37) values in Eq.(35) along with Eq.(3), we get the another new exponential rational function solution for the NPVPE in the following manner;

$$u_9(x,t) = \frac{c a_3^3 e^{\frac{a_3(x-ct)}{18d}}}{\left(54d^2 + c a_3 e^{\frac{a_3(x-ct)}{18d}}\right)^2}. \quad (38)$$

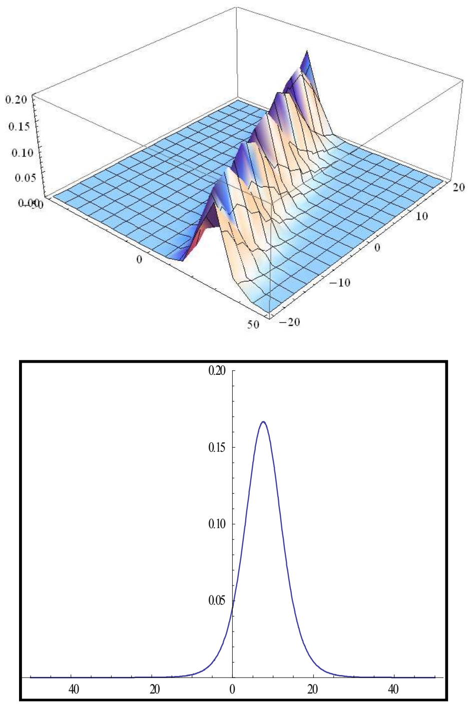

Figure 14. *The 3D and 2D surfaces of the analytical solution Eq.(38) being a new exponential function solution by considering the values* $c = d = -1,\ a_3 = -6, -50 < x < 50,\ -20 < t < 20,$ *for 3D graphics and* $t = -1$ *for 2D surfaces.*

Remark-1. The solutions $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8$ and u_9 obtained via BSEFM are the analytical solutions for Eq.(1). When we compare these analytical solutions with other analytical solutions obtained by using different approaches in literature [2],some of these solutions are complex exponential function solutions and new analytical solutions for Eq.(1).

Afterwards, Figure.1, Figure.2, Figure.3, Figure.4, Figure.5, Figure.6, Figure.7, Figure.8, Figure.9, Figure.10, Figure.11, Figure.12, Figure.13 and Figure.14 have been plotted by the same computer program.

To the best of our knowledge, these complex exponential function solutions have not been submitted to literature in advance. The analytical solutions and figures obtained in this paper give us a different physical interpretation for the nonlinear Vakhnenko-Parkes differential equation.

4. Conclusions

BSEFM has been exerted to the nonlinear Vakhnenko-Parkes differential equation. Afterwards, this method has provided us some new analytical solutions such as rational function solution, exponential function solution, and complex trigonometric function solution, periodic wave solution and so on. All analytical solutions obtained by BSEFM in the paper have been controlled whether they are verified to the nonlinear Vakhnenko-Parkes equation with the aid of commercial software Wolfram Mathematica programming 9 and all new solutions has verified to this equation.

It has been observed that solutions gained via the method proposed are in good agreement with already presented informations. Therefore, we think that this method can also be conducted to other nonlinear evaluation equations.

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