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Advanced Mathematical Formulas to Calculate Prime Numbers

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Abstract: Prime numbers are the core of mathematics and specifically of number theory. The application of prime numbers in modern science, especially in computer science, is very wide. The importance of prime numbers has increased especially in the field of information technology, i.e., in data security algorithms. It is easy to generate the product of two prime numbers but extremely difficult and a laborious to decompose prime factors combined together. The RSA system in cryptography uses prime numbers widely to calculate the public and the private keys. Diffie-Hellman Key Exchange in cryptography uses prime numbers in a similar way and in computing hash codes also we use Prime numbers. Since prime numbers can only divisible by 1 and themselves, they are not factored any further like whole numbers. Their appearance within the infinite string of numbers in random fasion that devising a functional equation to correctly predict them, infinitely, has been belived by many mathematician as impossible task. The problem to calculate prime number using a formula posed for long periods. Though different formulae to calculate prime number were developed by Euler, Fermat and mersenne, the formulae work for limited natural numbers and calculate limited prime numbers. However, on this paper the author wants to show how prime number calculated for all values of integers(x).

Keywords: Prime Numbers, Prime Numbers Formula, Prime Number Forecasting, Prime Number Distribution, Prime Number Calculation

1. Introduction

The Pythagoreans classified positive integers as even, odd, prime and composite around 600 BC. Prime numbers and their properties were first studied extensively by the ancient Greek mathematicians. By the time Euclid's Elements appeared in about 300 BC, several important results about primes had been proved [4]. Soon then discovered the fundamental Theorem of Artimetic, which states that every integer n>1 can be represented as a product of primes in only one way, apart from the order of the factors. Intigers greater than 1 can be either a prime or aproduct of primes which can be prove through mathematical induction. This is to mean that we can calculate all integers greater than 1 as the product of element of a set of all prime numbers. Furthermore, every even number is the sum result of two or more prime numbers. Two numbers are said to be co-prime if their highest common factor is 1. The number 1 is not a prime number since prime numbers have two devisors, 1 and the number itself. The last digit of any prime numbers represented by 1, 3, 7, or 9. The numbers 2, 4, 6 and 8 are excluded to ensure that the number is odd and hence is not divisible by the number 2. Since prime numbers appear in random fashion, they tend to avoid having the same last digit for their immediate prime predecessor. Prime numbers are a set of all numbers which are equally divided by 1 and themselves. Lack of prime number divisor except themselves is still the subject of countless investigation. Another aspect of prime numbers that has confounded mathematicians is their lack of an apparent pattern. They appear within the infinite string of numbers in such random fashion that devising a functional equation to correctly predict them, infinitely, is believed by many to be an impossible task [2, 6]. Though prime number have great importance in information technology such as public-key cryptography, there is no known useful formula that generate all the prime numbers [10]. Nowadays, most computer cryptography works by using prime factoring of large numbers which used to encrypt the file that can be publicly known and available because the encryption works so only the prime factors of that large number can be used to decrypt it again. Prime number are the most fundamental build blocks of all ntural numbers, which are build block of our understanding of the universe. The most widely used applications of prime numbers computing is The RSA encryption system. RSA is a public-key cryptosystem that is widely used for secure data transmission. Prime numbers that are known infinite in number appeared quite irregularly. However, the endeavor of calculating prime number using fomula found unsuccessful for long periods. Instead of seeking an exact formula for the prime, they considered the counting function π (x) and asked for approximations to this function, evidently a new kind of question in number theory [11]. If the form of f(x) function could be obtained exactly we would have a formula to calculate all the prime numbers. Unfortunately until now, it has not been possible to obtain such an exact expression. In 1752 Goldbach showed that no polynomial with integer coefficient can give a prime number for all integer values [5, 10]. The best known prime numbers formula which accredited to Euler in 1772 is n^2+n+41 which give prime numbers for all n=0 to 39 [9]. Another prime number prediction formula, which is 2^r+1 where $r \equiv 2^q$ for *q=0-4,* developed by Fermat and 2^p-1, where p is prime numbers, developed by mersenne [7, 12]. Searching a formula that can calculate prime numbers has become continuous task of researchers. Some like [14] are looking for twin primes others like [16] are looking for large scaled prime numbers. However, these formulae do not calculate all prime numbers and are valid only for some natural numbers. The objective of the paper is to show how prime number can be calculated for all integer values using the formulae by the following theorem.

Theorem 1: Let x be integers, then for all $n \in N$ $x^{2n}+x^n-1$ and $x^{2n}+\left(\frac{1}{2}\right)x^n-1$ are prime numbers except few odd numbers.

Proof. Let $n \in N$ (natural numbers) such that $m_n, R_n \in R$ (real numbers). Let

$$m_1 = 1\ , m_2 = \frac{1}{\left(\frac{1}{m_1 R_1}\right)-1}\ \ ,\ \ m_3 = \frac{1}{\left(\frac{1}{m_2 R_2 - m_1 R_1}\right)-\left(\frac{1}{m_1 R_1}\right)+1}$$

Then it follows that

$$R_1+R_2+R_3=\left(\frac{1}{m_1}\right)(m_1R_1)+\left(\frac{1}{m_2}\right)(m_2R_2)+\left(\frac{1}{m_3}\right)(m_3R_3)$$

This can be expressed as

$$=\left(\frac{1}{m_1}\right)(0+m_1R_1)+\left(\frac{1}{m_2}\right)((m_2R_2-m_1R_1)+m_1R_1)+\left(\frac{1}{m_3}\right)((m_3R_3-m_2R_2+m_1R_1)+(m_2R_2-m_1R_1))$$

Collecting like terms

$$=\left(\frac{1}{m_1}+\frac{1}{m_2}\right)(m_1R_1)+\left(\frac{1}{m_2}+\frac{1}{m_3}\right)(m_2R_2-m_1R_1)+\left(\frac{1}{m_3}\right)(m_3R_3-m_2R_2+m_1R_1)$$

$$=\left(\frac{1}{m_1R_1}\right)(m_1R_1)+\left(\frac{1}{m_2R_2-m_1R_1}\right)(m_2R_2-m_1R_1)+\left(\frac{1}{m_3}\right)(m_3R_3-m_2R_2+m_1R_1)$$

$$=1+1+\left(\frac{1}{m_3}\right)(m_3R_3-m_2R_2+m_1R_1)$$

$$=2+\left(\frac{1}{m_3}\right)(m_3R_3-m_2R_2+m_1R_1)$$

Assume that $R_1=x_1^n, R_2=x_2^n, \frac{1}{m_2R_2-m_1R_1}=\frac{-1}{x_2^n}$ for m_3, then

$$x_1^n+x_2^n+R_3=2+\left(1-(\frac{1}{x_1^n}+\frac{1}{x_2^n})\right)\left(\frac{R_3}{1-(\frac{1}{x_1^n}+\frac{1}{x_2^n})}-\frac{(x_1x_2)^n}{1-x_1^n}+x_1^n\right)$$

Then it reduces to

$$x_1^n+x_2^n=2+\left(1-(\frac{1}{x_1^n}+\frac{1}{x_2^n})\right)\left(\frac{(x_1x_2)^n}{x_1^n-1}+x_1^n\right)$$

$$x_1^n+x_2^n=2+\left((x_1x_2)^n-(x_1^n+x_2^n)\right)\left(\frac{1}{x_1^n-1}+\frac{1}{x_2^n}\right)$$

$$(x_1^n + x_2^n)(\frac{1}{x_1^n - 1} + \frac{1}{x_2^n} + 1) = 2 + (x_1 x_2)^n \left(\frac{1}{x_1^n - 1} + \frac{1}{x_2^n} \right)$$

$$(x_1^n + x_2^n)(\frac{x_1^n + (x_1 x_2)^n - 1}{(x_1^n - 1)(x_2^n)}) = \frac{(x_1 x_2)^n (x_1^n + x_2^n + 1) - 2x_2^n}{(x_1^n - 1)(x_2^n)}$$

So that

$$x_1^n + x_2^n = \frac{(x_1 x_2)^n (x_1^n + x_2^n + 1) - 2x_2^n}{x_1^n + (x_1 x_2)^n - 1} \quad for \ \ x_1 < x_2$$

Let $x_1 = x_2 = x,$ then

$$2x^n = \frac{(x^{2n})(2x^n + 1) - 2x^n}{x^{2n} + x^n - 1}$$

$$\frac{2x^{2n} + x^n - 2}{x^{2n} + x^n - 1} = 2 \quad \text{where } x \neq 1$$

As we can see above $2x^{2n} + x^n - 2$ is twice of $x^{2n} + x^n - 1$.

Therefore, we have the following formulae

$$\frac{1}{2}(2x^{2n} + x^n - 2) = x^{2n} + \left(\frac{1}{2}\right)x^n - 1 \text{ and } x^{2n} + x^n - 1.$$

2. Discussion

The formulae yield only prime numbers for all integer value (x) where $n \in N$ except few odd numbers. The formula to calculate prime number by Euler valid only for values starting from 0 up to 39 [15]. Fermat and mersenne formulae to calculate prime number works only for the limited natural numbers and calculate limited prime numbers [8, 15]. A lot of formulae to calculate prime number developed even though they work only for the first few non-negative values [1, 11]. Using two methods i.e. sequence and decomposition of primes which are based on co-prime and decomposition properties of a prime number, new prime numbers can be generated through co-prime and decomposition properties. It has been proven that the prime numbers can be generated from co-primes and the decomposition of prime numbers [13] even though the method work to produce limited prime number. However, the prime number calculation formulae developed by this paper, with the exception of few odd numbers, calculate prime numbers for all integer values and for all natural numbers exponents. The formulae developed by this paper have great contribution for mathematics specifically and for science in general. The study of prime numbers and their properties has attracted mathematicians for several centuries because of the use of prime numbers in different field [5]. The formulae do have a new outlook in breaking many security systems based on primes. The importance of prime numbers has increased especially in the field of information technology, i.e, in data security algorithms. The RSA system in cryptography uses prime numbers widely to calculate the public and the private keys. The strength of this system relies upon the difficulty of factoring large numbers - specifically the difficulty associated with the finding of the specific pair of prime numbers selected to create a large integer called the modulus. Diffie-Hellman Key Exchange in cryptography uses prime numbers in a similar way. It uses a large prime number as a common modulus based on which two entities, say A and B can communicate securely using their private, undisclosed keys. A hash code is a number code for every object that is created by a program. Hash codes are required for quick retrieval/storing of complex objects from/in a hash table. Hash codes need to be reasonably unique for each object so that correctness is maintained. Prime numbers are used in computing hash codes for this reason [15]. Since the formulae work for all integer values, mathematician should further investigate to which exponent natural number as the formulae perfectly work to calculate prime number. The limitation of the formulae is that they calculate rarely odd number.

3. Conclusion

The use of prime numbers in computer science is very wide. The importance of prime number has been surging especially in data security algorithms. It is easy to generate the product of two prime numbers but extremely difficult and a laborious to decompose prime factors combined together. The RSA system in cryptography uses prime numbers widely to calculate the public and the private keys. Diffie-Hellman Key Exchange in cryptography uses prime numbers in a similar way and in computing hash codes also we use Prime numbers. Even though many formulas has been developed to calculate prime numbers, none of them has been perfect to calculate all prime numbers. The paper introduced a new formulae to calculate prime numbers for all integer value and for all natural numbers exponents even though they rarely calculate odd numbers. This broach a new outlook in number theory specifically and mathematics in general.

4. Significance Statement

This research discovered new mathematical formulae to calculate prime number based on the given integers. This discovery broaches new outlook in number theory. Because primes are of the highest importance to number theorists and mathematic in general, they are the building blocks of whole numbers whose product can be written by one or more prime number, and important to the world because their odd mathematical properties make them perfect for our current uses. Large prime numbers are used obviously in cryptosystems. This study will help researchers to uncover the critical formulae of prime numbers that may researchers were not able to explore. Thus the new theory on prime numbers formulae to calculate prime number may be arrived at.

Conflict of Interest

The author declared no conflict of interest exists.

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