Optics
2025, Vol. 13, No. 1, pp. 9-14
https://doi.org/10.11648/j.optics.20251301.12

Research Article

Interference Fringe Analysis for Precision Measurement by Using Conoscopic Interferometer

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Abstract

In this paper, we propose a novel fringe phase analysis method that can quickly and accurately perform the interference fringe analysis for precise measurement by using a conoscopic interferometer. In general, interference fringe phase analysis is an important issue for the measurement using laser interference. Although many studies have been carried out on the conoscopic interferometer and its applications, no full-scale study has been carried out on the problem of the phase analysis of the interference fringes and shortening the time in the precision measurement using the conoscopic interferometer. This problem can be solved by an efficient fringe analysis method. In the conoscopic interferometer, the precision measurement was performed by measuring the change in the number of fringes due to the change in distance. This method has low accuracy and slow measurement speed. In this paper, we propose a novel method to quickly analyze the phase of the conoscopic interference fringe by means of the determination of the extremum points and briefly discuss the interference fringe filtering. Since the interference pattern obtained in the conoscopic interferometer is related to the distance, the analysis of the interference pattern gives the distance. The interference pattern can be simply analyzed since the phase difference between two adjacent extrema is 2π. The proposed method allows a simple and rapid analysis of the interferogram of the conoscopic interferometer, thus increasing the accuracy and speed of precision measurements using the conoscopic interferometer. The proposed fringe analysis method can be used not only for the analysis of the conoscopic interference fringe but also for other sorts of fringe analysis.

Keywords

Conoscopic Interferometer, Interference Fringe, Precision Measurement, Crystal, Birefringence, Phase Analysis

1. Introduction

The conoscopic interferometer is an interferometer that uses the birefringence of crystals. Conoscopic interferometry has many applications in precision measurement and crystal studies, and is the main tool in studies of crystal [1-4, 6-8]. General Scheme of Conoscopic Interferometer is shown in Figure 1.

In the conoscopic interferometer, light from a laser is reflected from a beam splitter (BS) and focused on the object surface to be measured through an objective lens. Rays reflected in different directions at the object surface pass through objective lens, light splitter, lens and polarizer, and enter birefringent crystal as polarized light. The conoscopic interferometer uses the interference of ordinary ray and extraordinary ray that are produced when ray passes through the crystal.

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Received: 8 April 2025; **Accepted:** 22 April 2025; **Published:** 20 June 2025

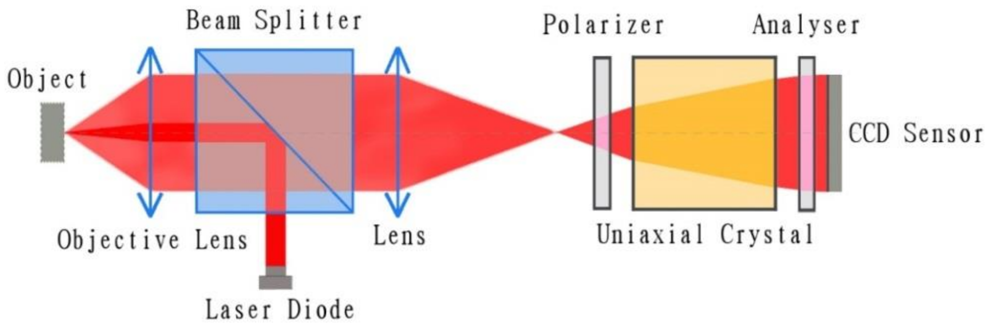

Figure 1. *General Scheme of Conoscopic Interferometer.*

The ordinary ray and extraordinary ray from crystal are polarized rays with different polarization directions, and because the refraction index of the extraordinary ray varies with the direction of the wave normal, the path difference between ordinary ray and extraordinary ray varies with direction.

In conoscopic interferometer, the interference fringe is obtained by interfering ordinary ray and extraordinary ray.

The major advantage of conoscopic interferometer is that requirements of the coherence of the light source and the surrounding environment are not high because two interfering light waves propagate along the same path. Also, since the light is scanned vertically and recorded vertically, the effect of shadows on various measurement methods, including laser triangulation, is not a problem.

The ordinary ray and extraordinary ray interfere in the recording plane after aligning the polarization direction with the analyzer because the polarization direction is different when coming out of crystal.

At that time, the polarization direction of polarizer and analyzer is perpendicular to each other. For precision measurement, the phase of interference fringe pattern must be analyzed exactly. The study of the formation of interference fringe in the conoscopic interferometer has already been studied by several researchers [5, 13, 16-18]. When performing precision measurements using conoscopic interferometer, the measurement speed problem becomes important. This is due to the large amount of time required to measure the three-dimensional shape because this method is performed measurements by scanning method.

Therefore, the measurement algorithm should be as simple as possible to shorten the calculation speed and increase the measurement speed. We have conducted a study to analyze the interference fringes briefly and accurately.

In the present paper, we have studied with uniaxial crystals (calcite $CaCO_3$).

2. Phase Analysis of Interference Fringe Pattern by Means of Extremum Points Determination

2.1. Phase Difference Between Ordinary Ray and Extraordinary Ray Emitted from Crystal

In Figure 1, the light from the laser after focusing on the object to be measured, again passes through the objective lens, the beam splitter, and the converging lens, and then forms a light point source P in front of the polarizer.

The light from the point light source P enters uniaxial crystal in different directions.

The ray that entered on a crystal in different directions is separated by ordinary ray and extraordinary ray and then propagated.

The velocity of ordinary ray is constant and the propagation velocity of the extraordinary ray is different depending on the crystal axis and the direction of propagation of the extraordinary ray. The basic principle of precision measurement using conoscopic interferometer is shown in Figure 2.

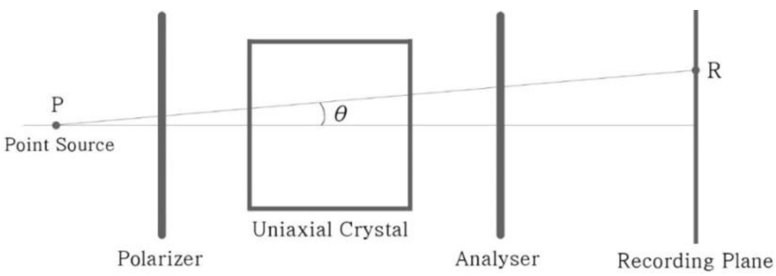

Figure 2. *Schematic diagram of conoscopic interferometer.*

The interference fringe pattern that recorded on the recording plane depends on the phase difference between ordinary ray extraordinary ray emitted from crystal. First, let us find the phase difference equation between the ordinary ray and extraordinary ray from crystal.

The phase difference between ordinary ray and extraordinary ray in parallel plate crystals has been studied in several papers [9, 12, 14]. Here we consider the most popular and simple method.

Light emitted from the polarizer is polarized and then enter parallel plate uniaxial crystal with thickness h. After entering crystal, the ray is divided into two rays propagating with different velocities. These come from a crystal with a constant phase difference δ.

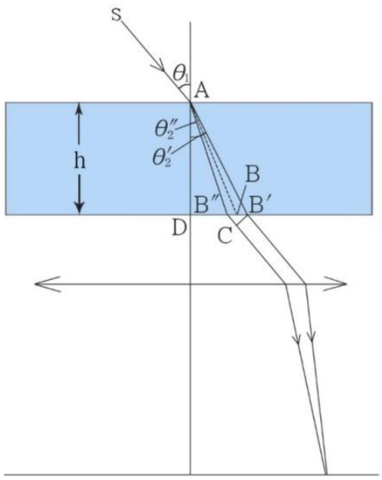

Figure 3. *The optical path of ordinary ray and extraordinary ray emitted from parallel plate crystals.*

It is important to find how the phase difference δ of the two rays varies with the angle of incidence.

As shown in Figure 3, let SA, AB', AB' denote the wave surface normal of the incident and two refracted waves at point A, and let $\theta_1, \theta_2', \theta_2''$ denote angle of incidence and two refraction angles of ordinary ray and extraordinary ray, respectively.

Let λ denotes wavelength of incident light in air, and let $\lambda' = \lambda / n'$ and $\lambda''=\lambda/n''$ denote the wavelength of two refracted waves, respectively, where n' and n'' are the refractive indices of ordinary ray and extraordinary ray in a crystal.

The two refractive rays passing through the crystal plate and the incident ray are parallel to each other, and the phase difference between two refractive rays is the following equation.

$$\delta = 2\pi \left[\frac{AB''}{\lambda''} + \frac{B''C}{\lambda} - \frac{AB'}{\lambda'} \right] \tag{1}$$

where

$$AB' = \frac{h}{\cos\theta_2'}, \quad AB'' = \frac{h}{\cos\theta_2''} \tag{2}$$

From Figure 3 the following relationships are found.

$$B''C = B''B'\sin\theta_1 = h\sin\theta_1(\tan\theta_2' - \tan\theta_2'') \tag{3}$$

From equation (1)-(3) we obtain the following relation.

$$\delta = 2\pi h \left[\frac{1}{\cos\theta_2''}\left(\frac{1}{\lambda''} - \frac{\sin\theta_1 \sin\theta_2''}{\lambda} \right) - \frac{1}{\cos\theta_2'}\left(\frac{1}{\lambda'} - \frac{\sin\theta_1 \sin\theta_2'}{\lambda} \right) \right] \tag{4}$$

Replacing $\sin\theta_1 / \lambda$ as $\sin\theta_2'' / \lambda''$ and $\sin\theta_2' / \lambda'$ from refraction law, we obtain the following relations.

$$\delta = 2\pi h \left[\frac{\cos\theta_2''}{\lambda''} - \frac{\cos\theta_2'}{\lambda'} \right] = \frac{2\pi h}{\lambda}(n''\cos\theta_2'' - n'\cos\theta_2') \tag{5}$$

$n'' - n'$ is given approximately as the following equation.

$$n''\cos\theta_2'' - n'\cos\theta_2' = (n'' - n')\frac{d}{dn}(n\cos\theta_2) = (n'' - n')\left[\cos\theta_2 - n\sin\theta_2 \frac{d\theta_2}{dn} \right] \tag{6}$$

Where n is mean refractive index of n' and n'', θ_2 is mean value of θ_2' and θ_2''. Differentiating refraction law $\sin\theta_1 = n\sin\theta_2$ by assuming θ_1 constant, we have the following equation.

$$0 = \sin\theta_2 + n\cos\theta_2 \frac{d\theta_2}{dn} \tag{7}$$

Consequently, equation (6) can be written.

$$n''\cos\theta_2'' - n'\cos\theta_2' = \frac{1}{\cos\theta_2}(n''-n') \tag{8}$$

Substituting equation (8) into equation (7), we obtain the following.

$$\delta = \frac{2\pi h}{\lambda\cos\theta_2}(n''-n') = \frac{2\pi}{\lambda}\rho(n''-n') \tag{9}$$

where $\rho = AB = \frac{h}{\cos\theta_2}$ means the average value of the geometric path inside the crystal plate of two rays.

In a uniaxial crystal, the following relationship holds between two phase velocities corresponding to the wavefront normal direction, which is the angle θ with the optical axis.

$$\left.\begin{aligned} \upsilon_p'^2 &= \upsilon_0^2 \\ \upsilon_p''^2 &= \upsilon_0^2\cos^2\theta + \upsilon_e^2\sin^2\theta \end{aligned}\right\}$$

where υ_p' is the phase velocity of the ordinary ray in the crystal, υ_p'' is the phase velocity of the extraordinary ray, υ_0 is the velocity of the extraordinary ray in the direction of the optical axis in the crystal, and υ_e is the velocity of the extraordinary ray in the direction perpendicular to the optical axis.

Then we have the following.

$$(\upsilon_p')^2 - (\upsilon_p'')^2 = (\upsilon_0^2 - \upsilon_e^2)\sin^2\theta \tag{10}$$

Using the relationship between refractive index and propagation velocity, the transformation of equation (10) yields

$$\frac{1}{n'^2} - \frac{1}{n''^2} = \left(\frac{1}{n_0^2} - \frac{1}{n_e^2}\right)\sin^2\theta \tag{11}$$

Here, if we consider that the difference of refractive indices is small compared to each other, then equation (11) is approximately equal to

$$n'' - n' = (n_e - n_0)\sin^2\theta \tag{12}$$

where n_0 is the refractive index of the optical axis direction of extraordinary ray, and n_e is the refractive index of the extraordinary ray in the direction perpendicular to the optical axis. Substituting this into equation (9), we obtain the following result.

$$\delta = \frac{2\pi\rho}{\lambda}(n_e - n_0)\sin^2\theta \tag{13}$$

If the incidence angle is small, the path difference between the ordinary ray and extraordinary ray is

$$\delta = \frac{2\pi h}{\lambda\cos\theta_2}(n''-n') = \frac{2\pi h}{\lambda}(n_e - n_0)\theta^2 \tag{14}$$

Hence, the light intensity at the recording plane is

$$I(R,P) = I(P)\left[a + b\cos\left\{\frac{2\pi L\Delta n}{z^2\lambda}\left[x^2+y^2\right]\right\}\right] \tag{15}$$

Under ideal conditions, a and b are 0.5.

Equation (15) holds for the case where the center of the camera lies on the central axis of the optical system.

2.2. Phase Analysis of Interference Fringe by Means of Determination of Extremum Points

To perform precision measurements using a conoscopic interferometer, the fringe phase must be analyzed accurately.

The interference pattern analysis in holography and the interference pattern processing in optical systems have been extensively studied [10, 11, 15]. However, a simple analysis of the conoscopic fringe pattern still has problems.

Analyzing the fringe pattern in a conoscopic interferometer means obtaining the fringe phase term relationship from the fringe pattern. From equation (15), we can see that the distance is known if we know the phase function relation along the x-axis of the fringe.

Then, the phase term relationship of the fringe can be obtained by using only a quarter of the fringe.

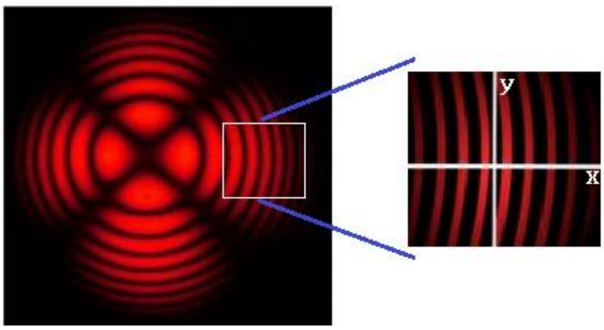

Figure 4. Interference fringe pattern used for phase analysis.

We analyzed the phase on the centerline of the fringe pattern. The speed problem is important for the scanning measurement method. To increase the measurement speed, we used a line camera with a low total number of pixels and a high speed with 2048 pixels camera.

If the center line of the camera matches with the x-axis of the fringe, the phase term in equation (15) can be simply written as

$$\Phi(\mathrm{x}) = \frac{\mathrm{Q}}{\mathrm{z}^2} x^2 \tag{16}$$

Here Q can be called the device factor, which is constant once the optical system is given.

In general, it is not possible to assume that the center of the CCD sensor lies in the optical axis of the optical system.

Thus, equation (16) is generally given by

$$\Phi(\mathrm{x}) = Ax^2 + Bx + C \tag{17}$$

Let x_i denote the coordinates corresponding to the maximal or minimal position of the intensity for a recorded fringe and let $\Phi(x_i)$ denote the corresponding phases, then the phase difference between two adjacent extremum points is 2π, we can write the following equation.

$$\left.\begin{aligned} \Phi(x_1) &= 2\pi + \varphi_0 = A{x_1}^2 + Bx_1 + C \\ \Phi(x_2) &= 4\pi + \varphi_0 = A{x_2}^2 + Bx_2 + C \\ &\vdots \\ \Phi(x_n) &= n * 2\pi + \varphi_0 = A{x_\mathrm{n}}^2 + Bx_\mathrm{n} + C \end{aligned}\right\} \tag{18}$$

We can modify the above equation as follows.

$$\left.\begin{aligned} 2\pi &= A{x_1}^2 + Bx_1 + C' \\ 4\pi &= A{x_2}^2 + Bx_2 + C' \\ &\vdots \\ n * 2\pi &= A{x_\mathrm{n}}^2 + Bx_\mathrm{n} + C' \end{aligned}\right\} \tag{19}$$

Where $C' = C - \varphi_0$.

Thus, solving equation (19) by the least square method, we can obtain the relation between $\Phi(\mathrm{x})$ and x, and from this, we can find the distance.

3. Interference Fringe Filtering

The interference fringe obtained by conoscopic interferometer is added to the noise for several reasons.

Several noises, including speckle noise, are difficult to remove completely although we use precise devices.

The noise removing in interference fringe pattern is a fundamental problem in analysis of the fringe pattern.

A common method for noise removing is to remove high frequency noise using Fourier low-pass filters (FLPF), median filters (MF), etc., and to analyze the fringe frequency or phase.

We used a Fourier low-pass filter to filter the fringe pattern and interference fringe analysis algorithm based on determination of minimum points.

Then the filter transfer function is the following equation.

$$H = \begin{cases} 1 & 1002 < d < 1048 \\ 0.5 & 1000 \le d \le 1002, 1048 \le d \le 1050, \\ 0 & d < 1000, d > 1050 \end{cases} \tag{20}$$

where d is the pixel coordinate in Fourier space.

The parameters of the filter transfer function were determined through experiments.

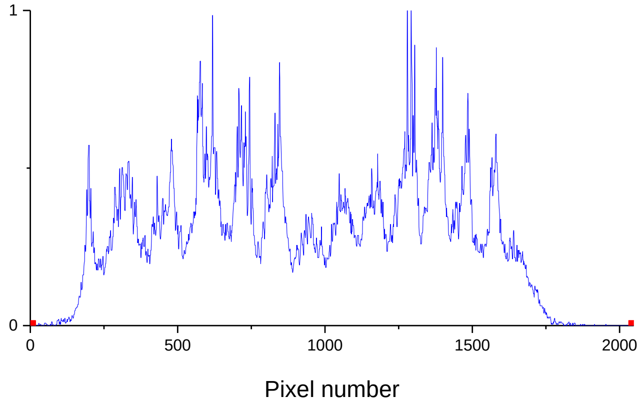

Figure 5. *Interference fringes recorded on sensor.*

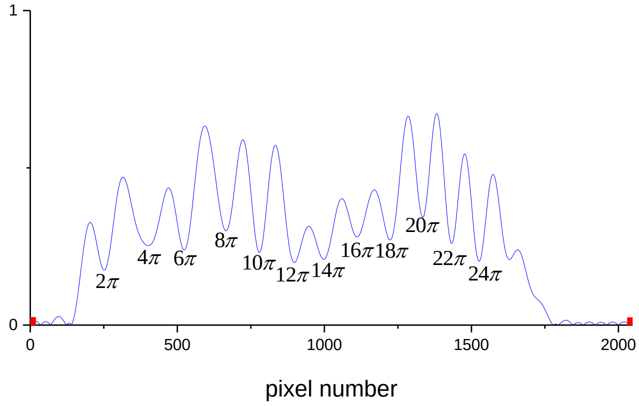

Figure 6. *Filtered fringe patterns and detected minimum points ("o" points mean the detected minimum points.).*

Using the above algorithm, we can quickly and accurately perform the precision measurement of the object surface.

4. Conclusion

In this paper, we studied about fringe filtering and interference fringe phase analysis method for precision measurement using conoscopic interferometer. In the previous work, the method of measuring the number of fringes was used for precision measurement. This method has limitations in measurement speed and accuracy. We have performed Fourier low-pass filtering to remove high frequency noise from the conoscopic fringe pattern. Next, we proposed a novel method for phase analysis of interference fringe by means of determination of extremum points, since the phase difference between neighboring extrema is 2π. The parameters of the Fourier low-pass filter were determined based on the number of fringes used in the measurements. Using this filter, the fringe filtering can be simplified. Moreover, the proposed fringe analysis method is very simple, which can be used to improve the accuracy and speed of measurement using the conoscopic interferometer. This method can be widely used in optical image processing to analyze the fringe pattern. The proposed method of conoscopic interference fringe phase analysis by means of fringe filtering and extremum point determination is not only simple but also applicable to other types of fringe analysis, so the proposed method will provide numerous application prospects in interference measurement.

Abbreviations

BS	Beam Splitter
FLPF	Fourier Low-Pass Filter
MF	Median Filter

Conflicts of Interest

The authors declare no conflicts of interest.

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