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Gravitational Implications of Electromagnetic Charge 4D Spherically Anisotropic Spactime

Liaqat Ali[1], Muhammad Talha[1], Noor Ul Abideen[1], Sajjad Haider[1], Furqan Habib[2,*]

[1]Department of Mathematics, Govt Post Graduate Jehanzeb College Saidu Sharif Swat, Swat, Pakistan
[2]Department of Mathematics, University of Peshawar, Peshawar, Pakistan

Email address:

liaqatali@gmail.com (Liaqat Ali), talhakk128@gmail.com (Muhammad Talha), ulabideennoor2@gmail.com (Noor Ul Abideen), sh7034395@gmail.com (Sajjad Haider), furqanhabib125@gmail.com (Furqan Habib)
[*]Corresponding author

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Abstract: In this study, we examine the behavior of an anisotropic fluid—one where pressures differ in radial and tangential directions—under the influence of gravity and electromagnetic charge in a four-dimensional, spherically symmetric spacetime. We consider both collapsing and expanding scenarios governed by Einsteins field equations, which describe how matter and energy affect the curvature of spacetime. To model a realistic astrophysical setting, we assume the interior of the spacetime is filled with the charged anisotropic fluid, while the exterior is described by the Reissner–Nordström metric, which represents the spacetime outside a charged, non-rotating mass. The two regions are smoothly joined using the *Darmois matching conditions*, ensuring that the geometry and physical quantities remain continuous at the boundary. Our analysis focuses on how the presence of electric charge and pressure anisotropy affects the dynamics of the fluid. Specifically, we investigate the profiles of energy density and pressure during both collapse and expansion. The results show that charge plays a significant role in influencing the fluids behavior, potentially resisting or enhancing the collapse depending on its magnitude. We also explore the evolution of anisotropy and demonstrate its impact through graphical analysis. The energy density, pressure, and anisotropy factor are plotted to visualize how they evolve in the presence of charge. These findings contribute to a deeper understanding of how anisotropic and charged fluids behave in dynamic gravitational settings, and they may have implications for astrophysical objects like charged compact stars or models of early-universe expansion.

Keywords: Gravitational Collapse, Gravitational Expand, Trapped Surfaces, Anisotropy, Charge

1. Introduction

Gravitational collapse is the contraction of an astronomical object due to its inherent gravitational pull, which drives stuff toward the center of gravity [1]. One of the primary mechanisms for the creation of structures in the universe. After enough time, an initial, cocosmos an initial, comparatively smooth dispersion of matter throughout time, following adequate accretion such as stars or black holes, may collapse to a pocket of higher density. The inclination of matter to gravitate toward a single center of mass as in the creation in particular: the sudden collapse of a star near the conclusion of its life cycle. An object in space such as a star or gas cloud, can undergo gravitational collapse due to its own extremely powerful gravity, contracts.

Hawking and Penrose [2, 3] were drawn to spacetime singularities and they discussed the existence and formation of singularities in detail. They presented their theorems and pointed out that singularities in spacetime are formed due to gravitational collapse of giant objects when a trapped surface is created. Two model conjectures were stated by Penrose, weak and strong censorship conjectures. Weak version of the conjecture claims that a singularity can be seen only by an observer living nearby and this singularity could be hidden behind an event horizon. Strong version of the conjecture claims that a singularity after the formation of a gravitational process cannot be seen by a nearby or distant observer. At that time, no mathematical or physical. There was evidence to support or refute Penrose's hypotheses. Penrose's work was contested by some authors [4–6] when they presented counter examples to the Penrose conjectures.

Later, Virbhadra [7] employed gravitational lensing to enhance the articulation of the Penrose conjectures. Oppenheimer and Snyder [8] studied gravitational collapse for dust model. The result of their study was a black hole. Plenty of researchers inspected gravitational collapse with cosmological constant. Sharif and Ahmad [9] used junction conditions to analyze perfect fluid collapse with non zero cosmological constant. Subsequently, the authors expanded it to five dimensions [10]. On the other hand Dabnath et al. [11] examined dust collapse in quasi-spherical geometry with cosmological constant. Further, Sharif and Abbas [12, 13] evaluated perfect fluid collapse with electromagnetic field in four and five dimensions with cosmological constant. Sharif and Abbas [14] also used matching conditions and studied perfect fluid collapse in Friedmans model with charge. Sharif and Yousaf [15] explored charged perfect fluid collapse. They deduced that electromagnetic field reduces the collapsing process. Guha and Banerji [16] applied Darmois junction condition using charged anisotropic fluid for cylindrical collapse. Ahmad and Malik [17] investigated the anisotropic fluid collapse with cosmological constant. This work was further discussed by Khan et al. [18] in five dimensional anisotropic fluid in the presence of cosmological constant. Further Ahmad et al. [19] explored the collapsing process of anisotropic fluid along with heat flux. Prisco et al. [20] studied cylindrical shearfree model with anisotropic fluid.

A primary area of interest for researchers continues to be the study of collapsing objects in the presence of an electromagnetic field in the background. For spherical collapse in the electromagnetic background, Sharif and Bhatti [21] applied their findings. The findings of their investigation indicate that the electromagnetic field causes a star's internal pressure to decrease, hence expediting the process of stellar collapse. Friedmann model with electromagnetic charge and complete fluid collapse were researched by Sharif and Abbas [22]. Darmois junction conditions were used by Guha and Banerji [23] to study cylindrical collapse of charge with anisotropic fluid. Charged cylindrical collapse of anisotropic fluid model was the focus of Sharif and Fatima's [24] work. General relativity was used by Maurya and Gupta [25] to study charged fluid to anisotropic fluid distribution. Khan and colleagues [26] have studied the ultimate fate of charged anisotropic fluid collapse.

Abbas [27] recently investigated how an electromagnetic field affected the expansion and collapse of an anisotropic gravitational source in a four-dimensional spacetime. Numerous scholars examined the collapsing situation in general relativity and certain modified theories concerning higher dimensional spacetimes. Nyonyi and colleagues [28] investigated shear-free relativistic models in higher-dimensions when charge is present and heat flux is present. Collapsing solution in higher dimensions was studied by Shah et al [29]. In f(R) gravity, Sharif and Atiq [30] studied higher dimensional charged collapse. Collapsing and expanding solution with cosmological constant in higher dimensions was studied by Khan et al. [31].

The explanation above makes it clear that the gravitational collapse of huge objects is a phenomenon that warrants more investigation. Furthermore, we observe that this process is aided by the charge in field equations. In a similar vein, both the fluid's form and the presence of charge have a significant influence on the gravitational collapse summons. Researching spherical gravitational collapse with charge and anisotropic fluid will thus be beneficial. It will assist us in ascertaining the collaborate impacts of charge and anisotropic fluid on the collapsing phase.

2. Field Equations

We investigates the solutions of the Einstein Field Equation for an anisotropic fluid in a four-dimensional, spherically symmetric spacetime with an electromagnetic charge, focusing on both collapsing and expanding scenarios. We analyze the spherically symmetric spacetime represented by

$$ds^2 = A^2(t,r)dt^2 - H^2(t,r)dr^2 - L^2(t,r)d\theta^2 - L^2\sin^2\theta(t,r)d\phi^2 \tag{1}$$

The Einstein Field Equation for the spacetime (1) are determined as

$$G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} \tag{2}$$

we get,

$$\begin{aligned} G_{12} &= \frac{2\dot{L}A'}{LA} + \frac{2L'\dot{H}}{LH} - \frac{2\dot{L}'}{L} \\ G_{11} &= \frac{2\dot{H}\dot{L}}{HL} + \frac{H'L'A^2}{LH^3} + \frac{\dot{L}^2}{L^2} - \frac{2L''A^2}{LH^2} - \frac{\dot{L}^2A^2}{L^2H^2} + \frac{A^2}{L^2} \end{aligned}$$

$$\begin{aligned}
G_{22} &= \frac{L'H'}{LH} - \frac{2\ddot{L}H^2}{LA^2} + \frac{2\dot{L}\dot{A}H^2}{LA^3} + \frac{2A'L'}{LA} - \frac{\dot{L}^2H^2}{A^2L^2} + \frac{L'^2}{L^2} - \frac{H^2}{L^2} \\
G_{33} &= \frac{A'L'L}{AH} + \frac{\dot{A}\dot{A}L}{A^3} + \frac{LL'\prime}{H^2} - \frac{\dot{H}\dot{L}L}{HA^2} + \frac{A'\prime L^2}{AH^2} - \frac{A'H'L^2}{AH^3} - \frac{\ddot{H}L^2}{HA^2} - \frac{2L\ddot{L}}{A^2} + \frac{\dot{A}\dot{H}L^2}{HA^3} \\
G_{44} &= \sin^2(\theta)G_{33}
\end{aligned} \tag{3}$$

The Energy momentum Tensor for Anisotropic fluid in the presence of Electromagnetic field with Einstein filed Equation is defined as:

$$G_{ab} = R_{ab} - \frac{1}{2}Rg_{ab} = k(T_{ab} + T^e_{ab}). \tag{4}$$

Where $k = 8\pi$ and T_{ab} is anisotropic fluid determined as

$$T_{ab} = (\rho + p_\top)\omega_a\omega_b - p_r g_{ab} + (p_r - p_\top)x_a x_b \tag{5}$$

and also T^e_{ab} is Electromagnetic Tensor determined as

$$T^e_{ab} = \frac{1}{H}(-F^\gamma_\delta F_{b\gamma} + \frac{1}{4}F^{\gamma\lambda}F_{\gamma\lambda}g_{ab}) \tag{6}$$

Where $H = \frac{3\pi^{\frac{3}{2}}}{\Gamma\frac{5}{2}}\ \rho$, p_r and $p_\top$ signify, in that order, the radial, tangential, and energy density. The equation for the Maxwell field is [34]

$$F_{cd} = \phi_{d,c} - \phi_{c,d}, \quad F^{cd}_c = HJ^c \tag{7}$$

The Potential and Current in 4 dimensions are given by Φ^c and J^c, respectively. For the within-metric (3.1), the constituents w^α, w_α, x^α, and x_α supplied by

$$\omega^a = [\frac{1}{A}, 0, 0, 0], \quad \omega_a = [A, 0, 0, 0],$$
$$X^a = [0, \frac{1}{H}, 0, 0], \quad X_a = [0, -H, 0, 0]$$

As of right now, the electromagnetic field's non-zero components of the energy-momentum tensor are:

$$T_{ab} = (T_a b + T^e_a b)k$$

we get

$$\begin{aligned}
T_{11} &= 8\pi(A^2\rho + \frac{A^2E^2}{2H}) \\
T_{22} &= 8\pi(H^2P_r - \frac{H^2E^2}{2H}) \\
T_{33} &= 8\pi(L^2P_T + \frac{L^2E^2}{2H}) \\
T_{44} &= 8\pi(L^2P_t + \frac{L^2E^2}{2H})\sin^2\theta
\end{aligned} \tag{8}$$

where $E = \frac{q}{L^2}$ and $k = 8\pi$ The expansion scalar Θ for the spherically symmetric spacetime (1) is given by:

$$\Theta = \omega^\alpha_\alpha = \omega^\alpha_\alpha + \Gamma^e_e\alpha X^e = \frac{1}{H}(\frac{\dot{H}}{H} + \frac{2\dot{L}}{L}) \tag{9}$$

The dimensionless measure of anisotropy is defined as [35]

$$\Delta a = 1 - \frac{P_t}{p_r} \tag{10}$$

The EFEs defined as (4) for the spacetime (1) becomes

$$\begin{aligned}
G_{11} &= \frac{1}{A^2}(-\frac{2B''}{B} - \frac{B'^2}{B^2} + \frac{2A'B'}{AB}) + \frac{1}{H^2}(\frac{\dot{B}^2}{B^2} + \frac{2\dot{A}\dot{B}}{AB}) + \frac{1}{B^2} = 8\pi(\rho + \frac{q^2\Gamma\frac{5}{2}}{6B^4\pi^{\frac{3}{2}}}) \\
G_{12} &= -\frac{2\dot{B}'}{B} + \frac{2H'\dot{B}}{HB} + \frac{2\dot{A}B'}{AB} = 0 \\
G_{22} &= \frac{1}{H^2}(-\frac{2\ddot{B}}{B} + \frac{2\dot{H}\dot{B}}{HB} - \frac{\dot{B}^2}{B^2}) + \frac{1}{A^2}(\frac{B'^2}{B^2} + \frac{2H'B'}{BH}) - \frac{1}{B^2} = 8\pi(\rho_r + \frac{q^2\Gamma\frac{5}{2}}{6B^4\pi^{\frac{3}{2}}}) \\
G_{33} &= \frac{1}{H^2}(-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{H}\dot{A}}{AH} - \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{H}\dot{B}}{BH}) \\
&+ \frac{1}{A^2}(\frac{H''}{H} + \frac{B''}{B} - \frac{H'M'}{HA} + \frac{H'B'}{HB} - \frac{A'H'}{AH}) = 8\pi(\rho_t + \frac{q^2\Gamma\frac{5}{2}}{6B^4\pi^{\frac{3}{2}}}) \\
G_{44} &= G_{22} = 8\pi(\rho_t + \frac{q^2\Gamma\frac{5}{2}}{6B^4\pi^{\frac{3}{2}}})
\end{aligned} \tag{11}$$

It is determined that the 4-dimensional Misner-Sharp mass [5] and the shape it takes for the spacetime (1) is as follows

$$m(t,r) = \frac{L}{2}[1 + g^{ab}L_aL_b + \frac{q^2}{L^2}] = \frac{L}{2}(1 + \frac{\dot{L}^2}{A^2} - \frac{L'^2}{H^2} + \frac{q^2}{L^2}) \tag{12}$$

The solution to the field equations with the G_{12} component is as follows.

$$A = \frac{\dot{L}}{L^\alpha} \quad , H = L^\alpha \tag{13}$$

Upon replacing (13) in (9), the scalar expansion now becomes

$$\Theta = (\alpha + 2)L^{\alpha - 1} \tag{14}$$

In this study, we examine the Einstein field equations' collapsing solution. A collapsed resolution for the Einstein field equations will be shown, $\Theta < 0$, for $\alpha < -2$ Regions are crumbling. Equation (13) combined with the field equations produces

$$\begin{aligned} 8\pi\rho &= -\frac{1}{B^{2\alpha}}[\frac{2B''}{B} + (1-2\alpha)\frac{B^{2\prime}}{B^2}] + (1+2\alpha)B^{2\alpha-2} + \frac{1}{B^2} - \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\ 8\pi P_r &= \frac{1}{B^{2\alpha}}[(1-2\alpha)\frac{B'^2}{B^2} + \frac{\overset{\vdots}{B}^2}{B}\frac{B'}{B}] - (1+2\alpha)B^{2\alpha-2} - \frac{1}{B^2} + \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\ 8\pi P_t &= \frac{1}{B^{2\alpha}}[\frac{\dot{B}''}{\dot{B}} + (1-3\alpha)\frac{\dot{B}'}{\dot{B}}\frac{B'}{B} + (1-\alpha)\frac{B''}{B} + (2\alpha^2 \\ &+ 3\alpha - 4\alpha)\frac{B'^2}{B^2}] - (2\alpha^2 - 3\alpha + 4\alpha)B^{2\alpha-2} - \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \end{aligned} \tag{15}$$

Substituting (13) in (12), the Misner-Sharp mass becomes

$$\frac{2m}{B} - 1 - \frac{q^2}{B^2} = B^{2\alpha} - \frac{B'^2}{B^{2\alpha}} \tag{16}$$

From (16) it can be develop that $B' = B^{2\alpha}$ there exist trapped surfaces at $B = 2m$ Thus $B' = B^{2\alpha}$ is in this instance the trapped surface state. Following Glass[35] two trapping scalars provided by are obtained.

$$\kappa_1 = \frac{B^\alpha}{B} + \frac{B'}{B^{\alpha+1}}, \;\; \kappa_2 = \frac{B^\alpha}{B} - \frac{B'}{B^{\alpha+1}} \tag{17}$$

A trapped surface will arise if κ_1 and κ_2 have the same signs. The trapped scalars now take on the following shape when the trapped surface condition is applied.

$$\kappa_1 = 2B^{2\alpha-1}, \;\; \kappa_2 = 0 \tag{18}$$

At $B = 2m$, a trapped surface forms during the gravitational collapse because κ_1 and κ_2 are neither negative. The imprisoned surface's state possesses the integral

$$B_{trap}^{1-2\alpha} = (1-2\alpha)r + L(t), \tag{19}$$

where an arbitrary function on integration is represented by $L(t)$. Applying the trapped surface condition to (15) now, we get

$$\begin{aligned} 8\pi\rho &= B_{trap}^{-2} - \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\ 8\pi P_r &= -B_{trap}^{2} + \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\ 8\pi P_t &= -\frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}}. \end{aligned} \tag{20}$$

3. Collapseing and Eapansion Solution

3.1. Gravitational Collapse for $\alpha = -\frac{5}{2}$

A collapsing system requires a negative rate of growth. Hence, α needs to be smaller than -2. We therefore choose $\alpha = -\frac{5}{2}$, With $\alpha = -\frac{5}{2}$ the trapping condition $B' = B^{2\alpha}$ becomes $B' = B^{-5}$, which further yields

$$B = [6r + f(t)]^{\frac{1}{6}} \tag{21}$$

where a function of integration that is arbitrary is represented by $f(t)$. The density and pressures equations (15) for $\alpha = -\frac{5}{2}$ becomes

$$
\begin{aligned}
8\pi\rho &= \frac{1}{B^2} - 4B^{-7} - B^5[\frac{2B''}{B} + 6\frac{B'^2}{B^2}] - \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\
8\pi P_r &= 4B^{-7} + B^5[6\frac{B'^2}{B^2} + \frac{2\dot{B}'B'}{\dot{B}B}] - \frac{1}{B^2} + \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\
8\pi P_t &= -10B^{-7} + B^5[\frac{\dot{B}''}{\dot{B}} + (\frac{17}{2})\frac{\dot{B}'B'}{\dot{B}B} + (\frac{7}{2})\frac{B''}{B} + 15\frac{B'^2}{B^2}] - \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}}
\end{aligned}
\tag{22}
$$

Substitute (21) in (22), The density and pressures in this case yields

$$
8\pi\rho = k^5 4[6r + f(t)]^{-\frac{7}{6}}(1 - k^{-12}) + k^{-2}[6r + f(t)]^{-\frac{1}{3}} - \frac{4q^2\Gamma\frac{5}{2}[6r + f(t)]^{-\frac{2}{3}}}{3k^4\sqrt{\pi}}
$$

$$
8\pi P_r = -4k^5(1 - k^{12})[6r + f(t)]^{-\frac{7}{6}} - k^{-2}[6r + f(t)]^{-\frac{1}{3}} + \frac{4q^2\Gamma\frac{5}{2}[6r + f(t)]^{-\frac{2}{3}}}{3k^4\sqrt{\pi}}
$$

$$
8\pi P_t = 10k^5(1 - k^{-12})[6r + f(t)]^{-\frac{7}{6}} - \frac{4q^2\Gamma\frac{5}{2}[6r + f(t)]^{-\frac{2}{3}}}{3k^4\sqrt{\pi}} \tag{23}
$$

The Misner-sharp mass function given by (16), becomes

$$
m = \frac{B}{2}(1 + (1 - k^2)B^{-5} + \frac{q^2}{b^2}) \tag{24}
$$

Equation (10) provides the dimensionless measure of anisotropy, which has the following form.

$$
\Delta a = 1 + \frac{80\frac{3\pi^{\frac{3}{2}}}{\Gamma(\frac{5}{2})}k^7(1 - k^{-12})[X]^{-\frac{3}{2}} - 32\pi q^2 k^-2[X]^{-1}}{32\frac{3\pi^{\frac{3}{2}}}{\Gamma(\frac{5}{2})}k^7(1 - k^{-12})[X]^{-\frac{3}{2}} + 8\frac{3\pi^{\frac{3}{2}}}{\Gamma(\frac{5}{2})} - 32\pi q^2 k^-2[X]^{-1}}. \tag{25}
$$

Where $X = 6r + f(t)$

3.2. Gravitational Expansion for $\sigma = \frac{3}{2}$

For expansion ,the rate of expansion must be positive from (14), when $\alpha = \frac{3}{2}$ then expansion scalar become $\Theta = \frac{7}{2}\sqrt{B}$. For expansion $\Theta > 0$ when $\alpha > 0$, also we choose

$$
B = (r^2 - r_0^2)^{-1} + g_1(t) \tag{26}
$$

For $\alpha = \frac{3}{2}$, where $g_1(t)$ and r_0 are arbitrary functions and constants, respectively, the density and pressure equations (15) become

$$
\begin{aligned}
8\pi\rho &= 4B - \frac{1}{B^3}[\frac{2B''}{B} - 2\frac{B'^2}{B^2}] + \frac{1}{B^2} - \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\
8\pi\rho_r &= \frac{1}{B^3}[-2\frac{B'^2}{B^2} + \frac{2\dot{B}'}{\dot{B}}\frac{B'}{B}] - 4B - \frac{1}{B^2} + \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}} \\
8\pi P_t &= \frac{1}{B^3}[\frac{\dot{B}''}{\dot{B}} - \frac{7\dot{B}'B'}{2\dot{B}B} - \frac{1B''}{2B} + 3\frac{B'^2}{B^2}] - 4B - \frac{4q^2\Gamma\frac{5}{2}}{3B^4\sqrt{\pi}}
\end{aligned}
\tag{27}
$$

with $F(t, r) = 1 + (r^2 - r_0^2)g_1(t)$and $B = \frac{F}{(r^2 - r_0^2)}$,In this instance the pressures and densities are

$$
8\pi\rho = \frac{4F}{(r^2 - r_0^2)} + \frac{4(r_0^2 - 3r^3)(r^2 - r_0^2)}{F^4} - \frac{8r^2(r^2 - r_0^2)}{F^5} + \left[\frac{r^2 - r_0^2}{F}\right]^2 - \frac{4q^2\Gamma(\frac{5}{2})(r^2 - r_0^2)^4}{3F^4\sqrt{\pi}}
$$

$$8\pi p_r = \frac{-8r^2(r^2-r_0^2)}{F^5} - \frac{4F}{(r^2-r_0^2)} - \left[\frac{r^2-r_0^2}{F}\right]^2 + \frac{4q^2\Gamma(\frac{5}{2})(r^2-r_0^2)^4}{3F^4\sqrt{\pi}}$$

$$8\pi p_t = \frac{12r^2(r^2-r_0^2)}{F^5} + \frac{(r_0^2-3r^3)(r^2-r_0^2)}{F^4} - \frac{6F}{(r^2-r_0^2)} - \frac{4q^2\Gamma(\frac{5}{2})(r^2-r_0^2)^4}{3F^4\sqrt{\pi}} \tag{28}$$

Now making use of (28) into (10) we arrive at the following expression

$$\Delta a = 1 + \left[\frac{\frac{12r^2(Y)}{F^5} + \frac{(r_0^2-3r^3)(r^2-r_0^2)}{F^4} - \frac{6F}{(Y)} - \frac{4q^2\Gamma(\frac{5}{2})(Y)^4}{3F^4\sqrt{\pi}}}{\frac{-8r^2(Y)}{F^5} - \frac{4F}{(Y)} - \left(\frac{Y}{F}\right)^2 + \frac{4q^2\Gamma(\frac{5}{2})(Y)^4}{3F^4\sqrt{\pi}}}\right]. \tag{29}$$

where $Y = r^2 - r_0^2$

4. Matching Conditions

The smooth matching of the inner and exterior components is the focus of this section. $(-)$ and $(+)$, respectively, denote the quantities in the inner and outer sections. We examine a 4-dimensional Schwarzschild line element in the outer part and a general line element given in Eq. (1) in the inner part. [36, 37],

$$ds_+^2 = \left(1 - \frac{2M}{R}\right)dT^2 - \frac{1}{(1-\frac{2M}{R})} - R^2 d\Omega_2^2, \tag{30}$$

Here, M stands for a star's outermost mass. We use the formulation created by Darmois [38] for the smooth matching of inner and outer sections. The definition of the first fundamental form's continuity over hypersurface Σ is

$$ds^2 = ds_-^2 = ds_+^2, \tag{31}$$

The following relations are obtained by applying this requirement.

$$N(r_\Sigma, t) = R_\Sigma, \tag{32}$$

$$\frac{dt}{d\tau} = \frac{1}{A}, \tag{33}$$

$$\frac{dT}{d\tau} = \left[F(R) - \frac{1}{F(R)}\frac{dR_\Sigma}{dT}\right]^{-\frac{1}{2}}, \tag{34}$$

where τ is the appropriate time and $F(R) = 1 - \frac{2M}{R}$. The definition of the second fundamental form's continuity over hypersurface Σ is

$$[K_{\alpha\beta}] = K^+_{\alpha\beta} - K^-_{\alpha\beta} = 0, \quad (\alpha,\beta) = 0,2,3. \tag{35}$$

The non-vanishing components of the extrinsic curvature for metric (1) are as follows:

$$K^-_{00} = -\left(\frac{AA'}{H}\right)_\Sigma, \tag{36}$$

$$K^-_{22} = \csc^2\varphi_1 K^-_{33} = \left(\frac{LL'}{H}\right)_\Sigma. \tag{37}$$

For metric (30), the non-vanishing components of the extrinsic curvature now have the following shape.

$$K^+_{00} = \left[\frac{dR}{d\tau}\frac{d^2T}{d\tau^2} - \frac{d^2R}{d\tau^2}\frac{dT}{d\tau} + \frac{3M}{(R-2M)R}\frac{dT}{d\tau}(\frac{dR}{dT})^2 - \frac{(R-2M)M}{R^2R}(\frac{dT}{d\tau})^3\right]_\Sigma, \tag{38}$$

$$K^+_{22} = \csc^2\varphi_1 K^+_{33} = -R\frac{dT}{d\tau}\left(\frac{-R+2M}{R}\right)_\Sigma. \tag{39}$$

Form Eqs. (35)-(39), We obtain the following relationships:

$$-\left(\frac{AA'}{H}\right)_\Sigma = \left[\frac{dR}{d\tau}\frac{d^2T}{d\tau^2} - \frac{d^2R}{d\tau^2}\frac{dT}{d\tau} + \frac{3M}{(R-2M)R}\frac{dT}{d\tau}(\frac{dR}{dT})^2 - \frac{(R-2M)M}{R^2R}(\frac{dT}{d\tau})^3\right]_\Sigma, \tag{40}$$

$$\left(\frac{LL'}{H}\right)_\Sigma = -R\frac{dT}{d\tau}\left(\frac{-R+2M}{R}\right)_\Sigma. \tag{41}$$

Using the above relations along with Eqs. (32), (33) and (34), we obtain

$$M =_\Sigma m(t,r). \tag{42}$$

The entire prerequisite for the smooth matching of inner and outer spacetimes is the relation mentioned above, which is found in Eq. (42).

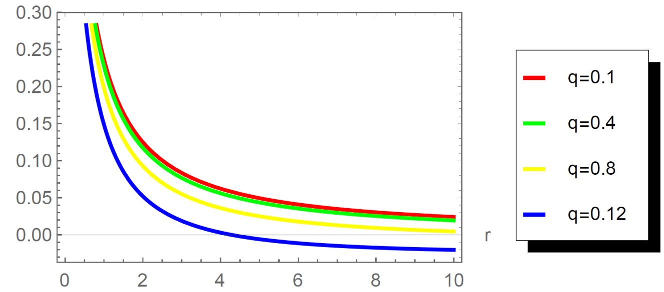

Figure 1. *Density "ρ" w.r.t "r" and "q" when $k = 1.75,\ f(t) = 2$.*

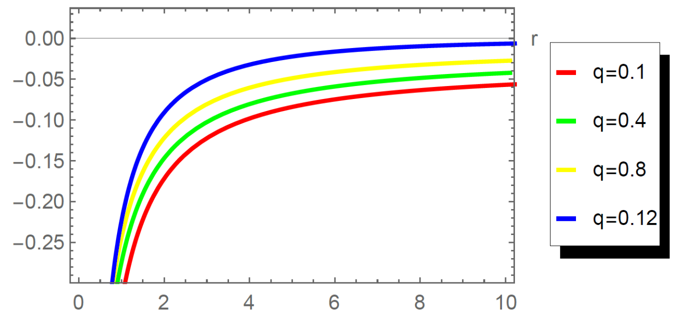

Figure 2. *Radial Pressure "P_r" w.r.t ""r" and "q" when $k = 1.75,\ f(t) = 2$.*

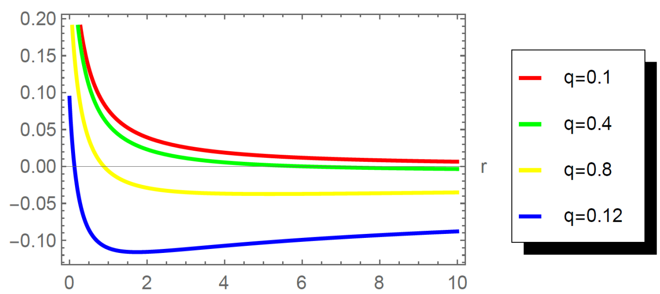

Figure 3. *Tangential Pressure "Pt" w.r.t ""r" and "q" when $k = 1.75,\ f(t) = 2$.*

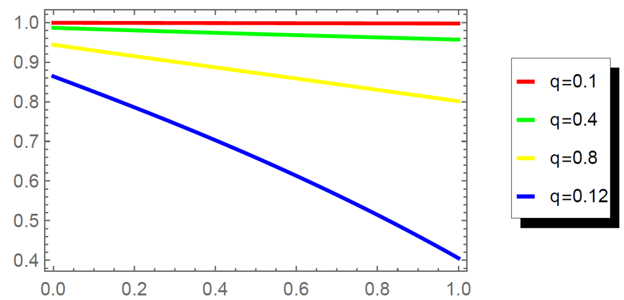

Figure 4. *Anisoptopy "Δ_a" w.r.t ""r" and "q" when $k = 1.75,\ f(t) = 2$.*

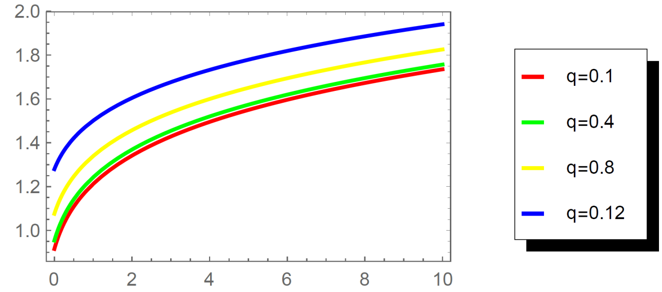

Figure 5. *Misner Sharp mass "m_r" w.r.t "r" and "q" when $k = 1.75,\ f(t) = 2$.*

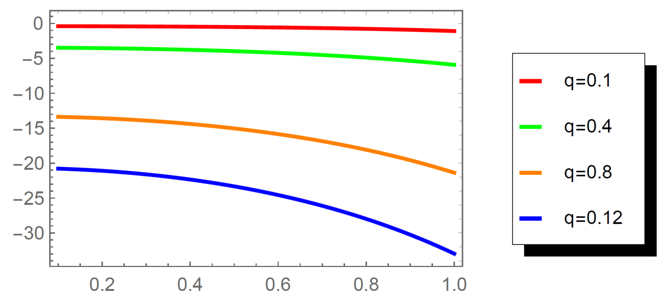

Figure 6. *Density "ρ" w.r.t ""r" and "q" when $k = 1.75,\ f(t) = 2$.*

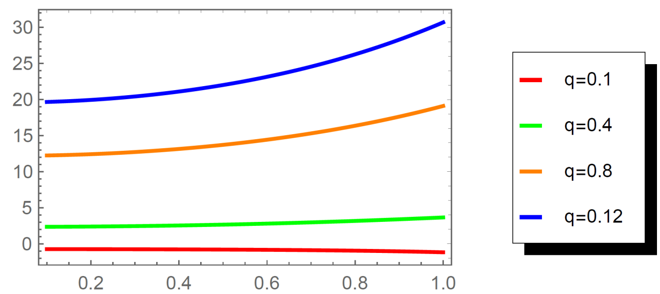

Figure 7. *Radial Pressure "P_r" w.r.t ""r" and "q" when $k = 1.75,\ f(t) = 2$.*

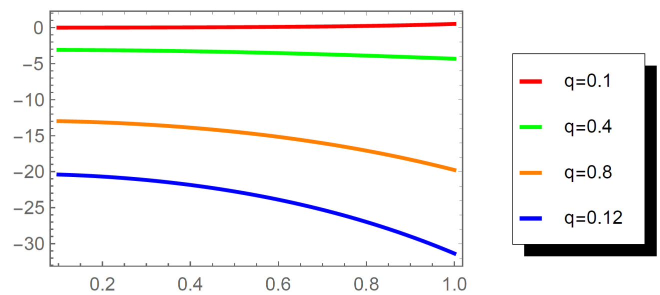

Figure 8. *Tangential Pressure "Pt" w.r.t ""r" and "q" when $k = 1.75,\ f(t) = 2$.*

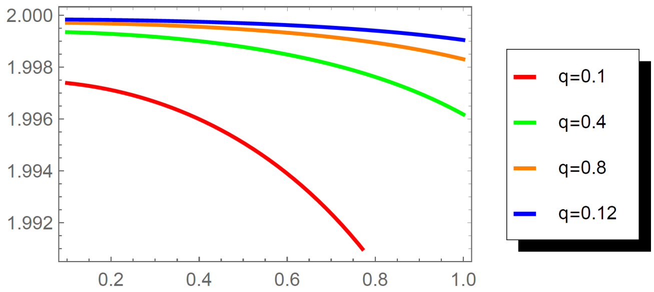

Figure 9. *Anisoptopy "Δ_a" w.r.t ""r" and "q" when $k = 1.75$, $f(t) = 2$.*

5. Conclusion

In this study, we have shown that, for an anisotropic fluid with spherical symmetry, the electromagnetic field has a considerable effect on the collapsing solution of the Einstein Field Equation in four-dimensional spacetime. Furthermore, we have taken into account the spherically symmetric metric in the four-dimensional interior area of the anisotropic fluid with an electromagnetic charge. By analyzing the four-dimensional anisotropic fluid throughout the collapse process, we were able to provide an analytical explanation for the physical features, such as the profiles of charge, density, and pressure. It was discovered that the energy density is a function of electromagnetic charge E and radius r, and that it is always positive in both lower and higher dimensions. We went over each of these physical attributes' graphical findings one by one for validation. A detailed discussion of the interior solution for anisotropic fluids has been given, which is relevant to the modeling of charged anisotropic stars in the process of collapse and expansion. We calculated the trapping requirements for a fluid sphere collapsing and expanding with charge by utilizing an auxiliary version of the metric functions. Depending on the kind of scalar expansion, the resultant solutions are categorized as collapsing or expanding. The mass function, anisotropic parameter, radial and transverse pressures, and matter density were all computed. As q rises, the density falls for the collapse solution where $\alpha = -\frac{5}{2}$, as Fig. 1 illustrates. From the center to the star's surface, the radial pressure P_r and matter density ρ fall from their greatest values in the center. It was observed that the anisotropy is directed outward when $P_r > P_t$, indicating $\Delta_a > 0$, as shown graphically in Figs. 2 and 3. Fig. 4 shows that anisotropy decreases with increasing r and q. The Misner-Sharp mass is positive and increases with increasing r and q, as shown in Fig. 5. Additionally, when $\alpha = \frac{3}{2}$ and the expansion scalar Θ is positive, the gravitating source expands. In this instance, as Fig. 6 illustrates, the matter density falls. As illustrated in Figs. 7, 8, and 9, the anisotropic parameter and the radial and transverse pressures behave in the opposite way from that of the gravitational collapse scenario. The main assumption is that $pr = -\rho$ is evident from the density and radial pressure equation. This demonstrates unequivocally that the ratio of density to radial pressure is -1, which is equivalent to dark energy. According to Herrera et al. [40], there might be a singularity for negative radial pressure. Additionally, the singularity will be covered if the ratio $\frac{pr}{\rho} > -\frac{1}{3}$, and it will be naked if the $\frac{pr}{\rho} \leq -\frac{1}{3}$. The ratio $\frac{pr}{\rho} < -\frac{1}{3}$ in our situation suggests the possibility of a naked singularity.

ORCID

0009-0000-1693-2441 (Liaqat Ali)
0009-0008-6179-3047 (Muhammad Talha)
0009-0000-0927-1278 (Noor Ul Abideen)
0009-0006-3449-3693 (Sajjad Haidera)
0000-0002-0745-5476 (Furqan Habib)

Abbreviations

CCC	Cosmic Censorship Conjectures
$EFEs$	Einstein Field Equations
G_{ab}	Einstein Tensor
T_{ab}	Energy Momentum Tensor
Θ	Expansion Scalar
Δ_a	Anisotropy
ρ	Energy Density
p_r	Radial Pressure
$p_\top$	Tangential Pressure
Δ	Anisotropy
$m(t,r)$	Misner-Sharp Mass
e	Charge

Author Contributions

Liaqat Ali: Played a role in research article, generating new problem ideas, devising problem-solving methods, and reviewing literature.

Muhammad Talha: Applied general techniques, contributed to solving the main problem and Responsible for writing, reviewing, editing, and submitting the initial draft.

Noor Ul Abideen: Applied general techniques, contributed to solving the main problem and Responsible for writing, reviewing, editing, and submitting the initial draft.

Sajjad Haider: Applied general techniques, contributed to solving the main problem and Responsible for writing, reviewing, editing, and submitting the initial draft.

Furqan Habib: Provided the core problem idea and contributed to draft review.

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Conflicts of Interest

There is no competing interest by the authors to declare.

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