

Research Article

# Relativity Theory for All Velocities $v < c$ , $v = c$ , $v > c$ Based on Single Transformation Law for 4-Vectors and Tensors

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## Abstract

Relativity theory based on a single transformation law (STL) for 4-vectors and tensors under universal rotation matrix (URM) on a unit circle satisfies principle of relativity, conservation laws and new symmetry valid for  $v < c$ ,  $v = c$  and  $v > c$ . This model can be called as relativity theory for all velocities (RTAV). The framework of relativity theory under universal Lorentz transformation matrix (ULTM) on a unit hyperbola gave form invariance of spacetime laws of physics for  $v < c$  only. URM is the inverse of ULTM. In this model, time and space components remain equal at  $v = c$  and reciprocate at  $v < c$  and  $v > c$  such that spacetime as a whole remains same for all observers. We consider the transformation of electrodynamic laws consisting of electromagnetic field (EMF), Maxwell's equations (ME) and conservation law in tensor components form. STL under URM gives rise to new symmetry of EMF, ME and conservation law along their diagonals. These terms constitute structure of zero-point electrodynamics (ZPE). Matrix method and Einstein's summation convention method (ESCM) are employed. Both methods agree up to the transformation of EMF, ME but differ in the transformation of conservation law. Usual electrodynamics remains same for all observers without being affected by ZPE. Conservation law in matrix method holds as usual. In ESCM, zero-point conservation appears as 4D EM wave while conservation law itself becomes 7D EM wave. In quantum theory zero-point energy violates conservation law whereas in our model ZPE is necessary to validate form invariance of spacetime laws and conservation law. This model of relativity is equally valid for noninertial frame.

## Keywords

Unified 4-Vectors and Tensors, Universal Rotation Matrix, Relativity for All velocities, Zero-Point Electrodynamics, Relativistic Conservation Laws

## 1. Introduction

Principle of relativity doesn't put any constraint on upper limit of speed and geometry of spacetime. The basic re-

quirement of principle of relativity is that all the spacetime laws of physics must remain same for all observers in their

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original form after transformation. Only STL for 4-vectors and tensors under 4 by 4 universal physical identity matrix can provide universality of spacetime laws. Physical identity matrix is more general concept as mathematical identity matrix becomes its special case. The role of physical identity matrix is to keep the spacetime laws in their original form.

Noether's symmetry principle in the context of spacetime physics demands form invariance of spacetime laws in their original form after transformation, conservation law corresponding to new symmetry. We do need such a model of relativity theory that is not only valid for the whole spectrum of velocities. In the present era of high-Q technology, scientists are looking for faster than light (FTL) based theory to explain warp drive like phenomena that is a special case of RTAV. We must have at least a consistent theoretical framework that can accommodate objects moving faster than light. The new framework of RTAV must obey principle of relativity, conservation law and symmetry principle. Our present model fulfills these requirements.

Where did the idea of RTAV come from? The idea of RTAV emerged from two sources. Firstly, from the form invariance of 4-velocity under URM that gave the relation  $(c', v') = (c, v)$  and secondly, from the squared Lorentz factor in URM  $\gamma^2 = 1/(1 + \frac{v^2}{c^2})$ . In URM on a unit circle, the terms  $1/(1 + \frac{v^2}{c^2})$  and  $\frac{v^2}{c^2}/(1 + \frac{v^2}{c^2})$  clearly indicates the existence of FTL. As for as the case of mass and energy is concerned, 4D momentum remains same for all observers under URM that is  $(\frac{E'}{c}, P^i) = (\frac{E}{c}, P^i)$ . Since 4-velocity remains same for all observers without affecting the constancy of speed of light so we should not worry about it. Experiments on FTL are in pipeline so we need to be patient. Existing statement of principle of relativity doesn't differentiate between spacetime laws of physics and 3-dimensional laws. It has been shown in [1-4] that form invariance of spacetime laws same for all observers require STL for 4-vectors and tensors along with universal transformation matrix. This mechanism contains form invariance of individual 4-vectors and tensors as well as inner product of 4-vectors and tensors. In SR, length contraction and time dilation are not 4D concepts. According to SR, time and space are interwoven so length contraction term must contain time component and time dilation term must contain space component such that combination of length contraction and time dilation as a whole must remain same for all observers to satisfy EPR.

We can state the principle of relativity in the present context as: All the spacetime laws of physics based on STL for 4-vectors and tensors under URM behave such that time and space components remain equal at  $v = c$  and reciprocate at  $v > c$  and  $v < c$  but the spacetime as a whole remains same for all observers.

The reciprocation or undulation of time and space components implies warp drive mechanism. In other words, at  $v > c$ , in the transformation of time component: Time com-

ponent decreases and space component increases while in the transformation of space component: Time component increases and space component decreases. This undulation of time and components happens at a regular pattern. Similar pattern occurs for  $v < c$  but in opposite sense. At  $v = c$ , both components remain equal. In the light of above observation, our model will play as a guidance system for theoretical and experimental physicists.

Hyperbolic geometry doesn't provide the possibility for FTL but URM on a unit circular geometry is found a valid candidate for FTL. URM on unit circle is the reciprocal of ULTM on a unit hyperbola. Squared Lorentz factor in ULTM becomes infinite as  $\gamma^2 = \infty$  at  $v = c$  on a unit hyperbola whereas on a unit circle  $\gamma^2 = 0.5$  at  $v = c$ . The unavailability of ULTM and STL for 4-vectors and tensors have been the two causes of inconsistency of special relativity with the principle of relativity and resolved with applications in [1]. On the other hand, special relativity being questioned even today in the contemporary literature [6-9]. FTL is becoming popular [10-15] due to the need of warp drives [16-22]. We shall focus mainly on RTAV electrodynamics under URM on a unit circle.

In this paper, we present the relationship between matrices on a unit hyperbolic and unit circle where they behave in reciprocal manner in Table 1. Both matrices are universal and behave as physical identity matrices. Furthermore, STL for 4-vectors and tensors with URM on a unit circle predicts structure of zero-point electrodynamics (ZPE) within usual electrodynamics. There is only one model that contains ZPE terms along the diagonal of EMF [5] as an intermediate step of similarity transformation of EMF. Quantum theory predicts zero-point energy that violates conservation law whereas in our RTAV model of electrodynamics, ZPE is necessary to validate form invariance and conservation law. Two very important concepts i.e. unification of Planck's constant and vector angular momentum as 4-angular momentum  $L^\mu$  and Work and torque as components of 4-torque  $\tau^\mu$  will be useful in the unification of relativity with quantum theory. Relativistic conservation laws in terms of equation of continuity for 4-current density, 4-Momentum, 4-Angular momentum, 4-Torque are presented which have no counter in the contemporary literature on relativity and quantum theory.

*Notations Used in Paper:* Notations in this model are adopted according to modern approach of relativity. Greek alphabets  $\mu, \nu, \alpha, \beta, \dots$  runs from 0 to 3 and Latin letters  $i, j, k, \dots$  from 1 to 3. Comma (,) denote partial differentiation e. g.  $E_{,0} = \frac{\partial E}{\partial t}$  Partial derivative of electric field w. r. t. time,  $E_{,1} = \frac{\partial E}{\partial x}$  Partial derivative of electric field w. r. t. x-axis,  $E_{,2} = \frac{\partial E}{\partial y}$  Partial derivative of electric field w. r. t. y-axis,  $E_{,3} = \frac{\partial E}{\partial z}$  Partial derivative of electric field w. r. t. z-axis,  $F^{\mu\nu}_{, \nu}$  means 4-dimensional or spacetime partial derivative of EMF tensor. 4-dimensional Coordinates  $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z) = (ct, x^i)$  with  $x^0 = ct$  and  $x^i = (x, y, z)$ . Time component  $ct$  is

scalar while space components  $x^i$  is vector such that  $x^\mu$  is the unification of time and space. The dimensions of all components are that of length.

### 1.1. Necessary Requirement for Form Invariance of Spacetime Laws of Physics

In order to get form invariance of spacetime laws of physics in their original form after transformation, validity of conservation laws and symmetry, we do need two basic requirements

- i. Universal transformation matrix
  - ii. A single transformation laws for 4-vectors and tensors
- What problem gave rise to these requirements?

While studying Maxwell's equations in tensor form where divergence of electromagnetic field tensor is equal to 4-current density. Maxwell's equations in tensor form are the unification of Gauss's law and Ampere's law. According to relativity theory, transformation of Gauss's law must appear as the mixture of Gauss's law and Ampere's law and similar for the transformation of Ampere's law such that transformation of Maxwell's equations as a whole must remain same for all observers. It is only possible if and only if the both conditions are satisfied.

Maxwell's Equations in tensor form are given as

$$F^{\mu\nu}_{,\nu} = J^\mu \tag{1}$$

If both sides of above equation are transformed under different transformation laws, then we never get the same results.

$$F^{\mu'\nu'}_{,\nu'} = R^{\mu'}_{\alpha} R^{\nu'}_{\beta} F^{\alpha\beta}_{,\nu'} J^{\mu'} = R^{\mu'}_{\alpha} J^{\alpha} \tag{2}$$

Means

$$F^{\mu'\nu'}_{,\nu'} \neq J^{\mu'} \tag{3}$$

On the other hand, if the both sides of equation are transformed under the same transformation law, then the results are always in complete agreement.

$$F^{\mu'\nu'}_{,\nu'} = R^{\mu'}_{\alpha} F^{\alpha\nu'}_{,\nu'} J^{\mu'} = R^{\mu'}_{\alpha} J^{\alpha} \tag{4}$$

implies

$$F^{\mu'\nu'}_{,\nu'} = J^{\mu'} \tag{5}$$

This also implies the validity of transformation of electromagnetic field and conservation law

$$F^{\mu'\nu'} = R^{\mu'}_{\alpha} F^{\alpha\nu'} \tag{6}$$

$$F^{\mu'\nu'}_{,\nu'} = R^{\mu'}_{\alpha} F^{\alpha\nu'}_{,\nu'} J^{\mu'} = R^{\mu'}_{\alpha} J^{\alpha}_{,\mu'} \tag{7}$$

Where  $R^{\mu}_{\nu}$  is universal rotation matrix on a unit circle.

These are entirely new results completely consistent with the principle of relativity.

### 1.2. Discovery of Universal Rotation Matrix on a Unit Circle

It was the most exciting moment when URM on a unit circle emerged as inverse of ULTM on a unit hyperbola. This discovery not only established a relation between unit circle and unit hyperbola but also gave rise the possibility of RTAV theory. The reciprocal relation between mathematical equations of hyperbola and circle in terms of velocity is very strange. Universal rotation matrix on a unit circle in 2D and 4D were mentioned in [1], page 9, 10, [2], page 224, Table 5, rotation matrix as universal spacetime exchanger matrix (USEM) in [3], page 28, for record but were not applied due to further investigations.

#### 1.2.1. Universal Rotation Matrix on a Unit Circle

Universal rotation matrix in 2D on a unit circle in terms of velocity is

$$[R^{\mu}_{\nu}] = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$\text{Det } R = \frac{1-\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})}$$

It behaves as a physical identity matrix and possesses continuous symmetry.

Its inverse in terms of velocity is universal rotation matrix on a unit hyperbola

$$(R^{\mu}_{\nu})^{-1} = [L^{\mu}_{\nu}] = \begin{bmatrix} \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$\text{Det } (R^{\mu}_{\nu})^{-1} = \frac{(1+\frac{v^2}{c^2})}{(1-\frac{v^2}{c^2})}$$

The discovery of URM on a unit circle as inverse of ULTM on a unit hyperbola is the key point for FTL theory of relativity.

Their product gives us identity matrix that satisfy the inverse relation between them

$$\begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

$$\text{Det} (R_v^\mu) \times \text{Det} (R_v^\mu)^{-1} = \frac{(1+\frac{v^2}{c^2})}{(1-\frac{v^2}{c^2})} \times \frac{(1-\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})} = 1$$

### 1.2.2. A Comparison of URM on Unit Circle and ULTM on Unit Hyperbola

We have preferred URM on a unit circle over ULTM on a unit hyperbola because URM possesses continues symmetry at all values at  $v < c$ ,  $v = c$  and  $v > c$ , whereas ULTM on hyperbola becomes undefined at  $v = c$  and imaginary at  $v > c$ . In Table 1, a brief comparison is given

**Table 1.** Comparison of Universal Rotation Matrices on Unit Circular and Unit Hyperbolic Geometry.

Universal Rotation Matrix on a Unit Circle

Universal Rotation Matrix

$$[R_v^\mu] = \begin{bmatrix} \text{Cos}^2\theta & \text{Sin}^2\theta & 0 & 0 \\ \text{Sin}^2\theta & \text{Cos}^2\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\det R = \text{Cos}^2\theta - \text{Sin}^2\theta$$

Matrix above possesses continues symmetry as spacetime laws of physics remain same at all angles.

The values of  $\text{Cos}^2\theta$  and  $\text{Sin}^2\theta$  in terms of velocity can be found from the relation

$$\tan^2\theta = \frac{v^2}{c^2}$$

And using

$$\text{Cos}^2\theta + \text{Sin}^2\theta = 1$$

$$\text{Cos}^2\theta = \frac{1}{(1+\frac{v^2}{c^2})}$$

$$\text{Sin}^2\theta = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})}$$

Substituting these values in (1), we get

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Det} R = \frac{(1-\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})}$$

$$\gamma^2 = \frac{1}{(1+\frac{v^2}{c^2})}$$

The inverse of matrix (6) is

$$[R_v^{\mu^{-1}}] = [L_v^\mu]$$

$$[R_v^{\mu^{-1}}] = [L_v^\mu] = \begin{bmatrix} \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Universal Rotation Matrix on a Unit Hyperbola

Universal Lorentz Transformation Matrix

$$[L_v^\mu] = \begin{bmatrix} \text{Cosh}^2\theta & -\text{Sinh}^2\theta & 0 & 0 \\ -\text{Sinh}^2\theta & \text{Cosh}^2\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\det L = \text{Cosh}^2\theta + \text{Sinh}^2\theta$$

Matrix above possesses continues symmetry as spacetime laws of physics remain same at all angles.

The values of  $\text{Cosh}^2\theta$  and  $\text{Sinh}^2\theta$  in terms of velocity can be found from the relation

$$\tanh^2\theta = \frac{v^2}{c^2}$$

And using

$$\text{Cosh}^2\theta - \text{Sinh}^2\theta = 1$$

$$\text{Cohs}^2\theta = \frac{1}{(1-\frac{v^2}{c^2})}$$

$$\text{Sinh}^2\theta = \frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})}$$

Substituting these values in (1), we get

$$[L_v^\mu] = \begin{bmatrix} \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ -\frac{\frac{v^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{Det} L = \frac{(1+\frac{v^2}{c^2})}{(1-\frac{v^2}{c^2})}$$

$$\gamma^2 = \frac{1}{(1-\frac{v^2}{c^2})}$$

The inverse of matrix (6) is

$$[L_v^{\mu^{-1}}] = [R_v^\mu]$$

$$[L_v^{\mu^{-1}}] = [R_v^\mu] = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

at  $v = c$ , gives

$$\gamma^2 = \frac{1}{2} \text{ and } \det R = 0$$

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix above is singular idempotent but it behaves like an identity matrix in the sense that all the spacetime laws of physics in terms of 4-vectors and tensors including their inner products remain same for all observers. It is entirely a new discovery. Matrix is valid for all velocities and gives finite results at  $v < c$ ,  $v = c$  and  $v > c$

at  $v = c$ , gives

$$\gamma^2 = \infty \text{ and } \det L = \infty$$

$$[L_v^\mu] = \begin{bmatrix} \infty & -\infty & 0 & 0 \\ -\infty & \infty & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Matrix above implies All the 4-Vectors and tensors remain same in their original form after transformation only for  $v < c$ .

### 1.2.3. Universal Rotation Matrix in 4D on a Unit Circle

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} \\ \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} \\ \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} \\ \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} \end{bmatrix} \tag{11}$$

$$(R_v^\mu)^{-1} = [L_v^\mu] = \begin{bmatrix} \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} \\ -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} \\ -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} \\ -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} \end{bmatrix} \tag{12}$$

Their 4D version also satisfy the inverse relation

$$\begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} \\ \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} \\ \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} \\ \frac{\frac{v_3^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_1^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{\frac{v_2^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} \end{bmatrix} \begin{bmatrix} \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} \\ -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} \\ -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} \\ -\frac{\frac{v_3^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_1^2}{c^2}}{(1-\frac{v^2}{c^2})} & -\frac{\frac{v_2^2}{c^2}}{(1-\frac{v^2}{c^2})} & \frac{1}{(1-\frac{v^2}{c^2})} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

## 2. Application of 2D URM in the Transformation of 4-Vectors

### 2.1. Relativity of Spacetime Position Coordinates Under URM in 2D

$$x^{\mu'} = R_{\alpha}^{\mu'} x^{\alpha} \tag{14}$$

$$x^{\mu} = (x^0, x^1, x^2, x^3) = (ct, x^1, x^2, x^3)$$

$$\begin{bmatrix} (ct)' \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{bmatrix} = \begin{bmatrix} 1 & \frac{v^2}{c^2} & 0 & 0 \\ (1+\frac{v^2}{c^2}) & (1+\frac{v^2}{c^2}) & 0 & 0 \\ \frac{v^2}{c^2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (ct) \\ x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

$$\begin{bmatrix} (ct)' \\ x^{1'} \\ x^{2'} \\ x^{3'} \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} (ct) + \frac{v^2}{c^2} x^1 \\ \frac{v^2}{c^2} (ct) + \frac{1}{(1+\frac{v^2}{c^2})} x^1 \\ x^2 \\ x^3 \end{bmatrix}$$

Relativity of Time Coordinate

$$(ct)' = \left[ \frac{1}{(1+\frac{v^2}{c^2})} \right] (ct) + \left[ \frac{v^2}{(1+\frac{v^2}{c^2})} \right] x^1 \tag{15}$$

$$(ct)' = \frac{1}{(1+\frac{v^2}{c^2})} [(ct) + \frac{v^2}{c^2} x^1]$$

In the transformation of time coordinate, we get the mixture of time component and space component so time is relative concept.

Relativity of Space Coordinates

$$x^{1'} = \left[ \frac{v^2}{(1+\frac{v^2}{c^2})} \right] (ct) + \left[ \frac{1}{(1+\frac{v^2}{c^2})} \right] x^1 \tag{16}$$

In the transformation of space coordinate, we get the mixture of time component and space component so time is relative concept.

Relativity of time and space coordinates remain equal at  $v = c$  but reciprocate in the case of  $v > c$  and  $v < c$ . The sum of time and space coordinate remains same for all observers.

$$x^{2'} = x^2 \tag{17}$$

$$x^{3'} = x^3 \tag{18}$$

Universality of Spacetime as a Whole

Collecting all the components on left- and right-hand side

$$(ct', x^{1'} + x^{2'} + x^{3'}) = \left[ \frac{(1+\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})} \right] (ct),$$

$$\left[ \frac{(1+\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})} \right] x^1 + x^2 + x^3 \tag{19}$$

$$(ct', x^{1'} + x^{2'} + x^{3'}) = (ct, x^1 + x^2 + x^3)$$

$$(ct', x^{i'}) = (ct, x^i)$$

$$x^{\mu'} = x^{\mu} \tag{20}$$

Spacetime 4-D coordinates remain same for all observers

### 2.2. Relativity of 4-Velocity Under URM in 2D

$$v^{\mu'} = R_{\alpha}^{\mu'} v^{\alpha} \tag{21}$$

$$v^{\mu} = \left( \frac{cdt}{dt}, \frac{dx^1}{dt}, \frac{dx^2}{dt}, \frac{dx^3}{dt} \right)$$

$$\begin{bmatrix} \frac{cdt'}{dt} \\ \frac{dx^{1'}}{dt} \\ \frac{dx^{2'}}{dt} \\ \frac{dx^{3'}}{dt} \end{bmatrix} = \begin{bmatrix} 1 & \frac{v^2}{c^2} & 0 & 0 \\ (1+\frac{v^2}{c^2}) & (1+\frac{v^2}{c^2}) & 0 & 0 \\ \frac{v^2}{c^2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{cdt}{dt} \\ \frac{dx^1}{dt} \\ \frac{dx^2}{dt} \\ \frac{dx^3}{dt} \end{bmatrix}$$

$$\begin{bmatrix} \frac{cdt'}{dt} \\ \frac{dx^{1'}}{dt} \\ \frac{dx^{2'}}{dt} \\ \frac{dx^{3'}}{dt} \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} \frac{cdt}{dt} + \frac{v^2}{(1+\frac{v^2}{c^2})} \frac{dx^1}{dt} \\ \frac{v^2}{(1+\frac{v^2}{c^2})} \frac{cdt}{dt} + \frac{1}{(1+\frac{v^2}{c^2})} \frac{dx^1}{dt} \\ \frac{dx^2}{dt} \\ \frac{dx^3}{dt} \end{bmatrix}$$

Relativity of Time Component of 4-velocity

$$\frac{cdt'}{dt} = \left[ \frac{1}{(1+\frac{v^2}{c^2})} \right] \frac{cdt}{dt} + \left[ \frac{v^2}{(1+\frac{v^2}{c^2})} \right] \frac{dx^1}{dt} \tag{22}$$

$$c' = \left[ \frac{1}{(1+\frac{v^2}{c^2})} \right] c + \left[ \frac{v^2}{(1+\frac{v^2}{c^2})} \right] v^1$$

In the transformation of time coordinate  $c$  or speed of light, we get the mixture of speed of light component and of spatial speed component  $v^1$  so  $c$  or speed of light is relative concept not absolute.

Relativity of Space Components of 4-velocity

$$\frac{dx^{1'}}{dt} = \left[ \frac{\frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} \right] \frac{cdt}{dt} + \left[ \frac{1}{1 + \frac{v^2}{c^2}} \right] \frac{dx^1}{dt} \quad (23)$$

$$v^{1'} = \left[ \frac{\frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} \right] c + \left[ \frac{1}{1 + \frac{v^2}{c^2}} \right] v^1$$

Similarly, In the transformation of spatial component of velocity  $v^1$ , we get the mixture of  $c$  or speed of light and of spatial component  $v^1$  so  $v^1$  is relative concept.

$$\frac{dx^{2'}}{dt} = \frac{dx^2}{dt} \quad (24)$$

$$\frac{dx^{3'}}{dt} = \frac{dx^3}{dt} \quad (25)$$

Universality of 4-Velocity

$$\begin{aligned} & \left( \frac{cdt'}{dt}, \frac{dx^{1'}}{dt} + \frac{dx^{2'}}{dt} + \frac{dx^{3'}}{dt} \right) \\ &= \left( \left[ \frac{1 + \frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} \right] \frac{cdt}{dt}, \left[ \frac{1 + \frac{v^2}{c^2}}{1 + \frac{v^2}{c^2}} \right] \frac{dx^1}{dt} + \frac{dx^2}{dt} + \frac{dx^3}{dt} \right) \end{aligned} \quad (26)$$

$$\begin{aligned} \left( \frac{cdt'}{dt}, \frac{dx^{1'}}{dt} + \frac{dx^{2'}}{dt} + \frac{dx^{3'}}{dt} \right) &= \left( \frac{c\theta dt}{dt}, \frac{dx^1}{dt} + \frac{dx^2}{dt} + \frac{dx^3}{dt} \right) \\ (c', v^{1'} + v^{2'} + v^{3'}) &= (c, v^1 + v^2 + v^3) \\ (c', v^{i'}) &= (c, v^i) \end{aligned} \quad (27)$$

The above result implies FTL

$$v^{\mu'} = v^\mu \quad (28)$$

4-velocity remains same for all observers

### 2.3. Transformation of 4-Momentum $P^\mu$ Under 2D Universal Rotation Matrix

For simplicity, we use 2D URM for transformation otherwise the result remains same under 4D URM on a unit circle.

$$P^{\mu'} = R_\alpha^{\mu'} P^\alpha \quad (29)$$

$$\begin{bmatrix} \frac{E}{c} \\ P^1 \\ P^2 \\ P^3 \end{bmatrix}' = \begin{bmatrix} 1 & \frac{v^2}{c^2} & 0 & 0 \\ \frac{v^2}{c^2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{E}{c} \\ P^1 \\ P^2 \\ P^3 \end{bmatrix}$$

Result:

$$P^{\mu'} = P^\mu \quad (30)$$

Contravariant 4-Momentum remains same for all observers in its original form after transformation. Covariant 4-momentum has the same form for all observers.

At  $v = c$ , energy and momentum are equal or balanced

At  $v > c$  and  $v < c$ , energy and momentum reciprocate. As a whole, Energy-momentum remains same for all observers

### 2.4. Transformation of 4-Angular Momentum $L^\mu$ Under 2D Universal Rotation Matrix

Symmetry principle requires conservation of angular momentum to satisfy invariance of rotation. We have the relation of 4-angular momentum that is a unification of Plank's constant  $h$  and vector angular momentum  $L$ . This relation is our new discovery and have been un-noticed in the contemporary literature on relativity and quantum theory.

For simplicity, we use 2D URM for transformation otherwise the result remains same under 4D URM on a unit circle.

$$L^\alpha = (L^0, L^i) = (h, L) = (r.P, r \times P)$$

$$L^{\mu'} = R_\alpha^{\mu'} L^\alpha \quad (31)$$

$$\begin{bmatrix} h \\ L^1 \\ L^2 \\ L^3 \end{bmatrix}' = \begin{bmatrix} 1 & \frac{v^2}{c^2} & 0 & 0 \\ \frac{v^2}{c^2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h \\ L^1 \\ L^2 \\ L^3 \end{bmatrix}$$

Result:

$$L^{\mu'} = L^\mu \quad (32)$$

Contravariant 4- Angular Momentum remains same for all observers in its original form after transformation. Covariant 4-Angular momentum has the same form for all observers.

At  $v = c$ , Plank's constant and angular momentum are equal or balanced

At  $v > c$  and  $v < c$ , Plank's constant and angular momentum reciprocate. As a whole, Plank's constant and angular momentum remains same for all observers

### 2.5. Transformation of 4-Toeque $\tau^\mu$ Under 2D URM

Symmetry principle requires conservation of angular momentum to satisfy invariance of rotation. We have the relation of 4-angular momentum that is a unification of Plank's constant  $h$  and vector angular momentum  $L$ . This relation is our new discovery and have been un-noticed in the contemporary literature on relativity and quantum theory.

For simplicity, we use 2D URM for transformation other-

wise the result remains same under 4D URM on a unit circle.  
 $\tau^\alpha = (\tau^0, \tau^i) = (W, \tau) = (r.F, r \times F)$

$$\tau^{\mu'} = R_{\alpha}^{\mu'} \tau^\alpha \tag{33}$$

Result:

$$\tau^{\mu'} = \tau^\mu \tag{34}$$

Contravariant 4- torque remains same for all observers in its original form after transformation. Covariant 4-torque has the same form for all observers.

### 2.6. Conservation Laws in Spacetime Physics

Relativistic conservation laws are more general than classical physics. Relativistic conservation laws in terms of 4-vectors are expressed in the language of Lorenz gauge condition  $A_{,\mu}^\mu = \nabla \cdot A + \frac{\partial \phi}{\partial t} = 0$ , where A and  $\phi$  are vector and scalar potential respectively. One of the well-known conservation laws is electromagnetic conservation law in terms of divergence of 4-current density which is also called equation of continuity. In the contemporary literature on relativity, no attention is given on such simple and consistent formulation of conservation laws.

Conservation of electromagnetic Sources in Terms of 4-Current Density

$$J_{,\mu}^\mu = \nabla \cdot J + \frac{\partial \rho}{\partial t} = 0 \tag{35}$$

In tensor notation electromagnetic conservation law is

$$F^{,\mu\nu}_{,\nu\mu} = \nabla \cdot E_{,0} + \nabla \cdot [(\nabla \times B) - E_{,0}] = 0 \tag{36}$$

Conservation of 4-Momentum

$$P_{,\mu}^\mu = \nabla \cdot P + \frac{\partial P^0}{\partial t} = 0 \tag{37}$$

Conservation of 4-Angular Momentum

$$L_{,\mu}^\mu = \nabla \cdot L + \frac{\partial h}{\partial t} = 0 \tag{38}$$

Conservation of 4-Force

$$F_{,\mu}^\mu = \nabla \cdot F + \frac{\partial E}{c \partial t} = 0 \tag{39}$$

Conservation of 4-Torque

$$\tau_{,\mu}^\mu = \nabla \cdot \tau + \frac{\partial W}{\partial t} = 0 \tag{40}$$

### 2.7. Transformation of Conservation Law

All the above conservation laws remain same for all ob-

servers in their original form after transformation under URM. We here only mention one for convenience.

$$P_{,\mu'}^{\mu'} = R_{\alpha}^{\mu'} P_{,\mu}^\alpha = P_{,\mu}^\mu \tag{41}$$

$$\begin{bmatrix} P_{,0'}^{0'} \\ P_{,1'}^{1'} \\ P_{,2'}^{2'} \\ P_{,3'}^{3'} \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{,0}^0 \\ P_{,1}^1 \\ P_{,2}^2 \\ P_{,3}^3 \end{bmatrix}$$

$$P_{,0'}^{0'} = \frac{1}{(1+\frac{v^2}{c^2})} P_{,0}^0 + \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} P_{,1}^1$$

$$P_{,1'}^{1'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} P_{,0}^0 + \frac{1}{(1+\frac{v^2}{c^2})} P_{,1}^1$$

$$P_{,2'}^{2'} = P_{,2}^2$$

$$P_{,3'}^{3'} = P_{,3}^3$$

$$P_{,\mu'}^{\mu'} = P_{,\mu}^\mu \tag{42}$$

Conservation of 4-Linear momentum remains same for all observers. Usual Lorentz transformation doesn't possess this property.

### 2.8. Transformation of Inner Product of 4-current Density $J^{\mu'} J_{\mu'}$

In special relativity theory, it is believed that only inner product of 4-vectors remains same for all observers but in our formulation, 4-vectors and their inner product remain same for all observers.

$$\begin{bmatrix} (J^{0'})^2 \\ -(J^{1'})^2 \\ -(J^{2'})^2 \\ -(J^{3'})^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (J^0)^2 \\ -(J^1)^2 \\ -(J^2)^2 \\ -(J^3)^2 \end{bmatrix} \tag{43}$$

$$(J^{0'})^2 = \frac{1}{(1+\frac{v^2}{c^2})} (J^0)^2 - \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} (J^1)^2$$

$$-(J^{1'})^2 = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} (J^0)^2 - \frac{1}{(1+\frac{v^2}{c^2})} (J^1)^2$$

$$-(J^{2'})^2 = -(J^2)^2$$

$$-(J^{3'})^2 = -(J^3)^2$$

Collecting all the terms on left- and right-hand sides

$$[(J^{0'})^2, - (J^{1'})^2 - (J^{2'})^2 - (J^{3'})^2] = \left[ \frac{(1 + \frac{v^2}{c^2})}{(1 - \frac{v^2}{c^2})} (J^0)^2, \right. \\ \left. - \frac{(1 + \frac{v^2}{c^2})}{(1 - \frac{v^2}{c^2})} (J^1)^2 - (J^2)^2 - (J^3)^2 \right]$$

$$[(J^{0'})^2, - (J^{1'})^2 - (J^{2'})^2 - (J^{3'})^2] = (J^0)^2, - (J^1)^2 - (J^2)^2 - (J^3)^2$$

$$[(J^{0'})^2, - (J^{i'})^2] = [(J^0)^2, - (J^i)^2]$$

Result:

$$(J^{i'})^2 = (J^i)^2 \tag{44}$$

Inner product of 4-current density remains same in single step transformation

### 2.9. Transformation of $\square^2 = \frac{\partial^2}{\partial t^2} - \nabla^2$

$$\begin{bmatrix} \frac{\partial^2}{\partial t'^2} \\ -\frac{\partial^2}{\partial x'^2} \\ -\frac{\partial^2}{\partial y'^2} \\ -\frac{\partial^2}{\partial z'^2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{v^2}{c^2} & 0 & 0 \\ \frac{v^2}{c^2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial^2}{\partial t^2} \\ -\frac{\partial^2}{\partial x^2} \\ -\frac{\partial^2}{\partial y^2} \\ -\frac{\partial^2}{\partial z^2} \end{bmatrix} \tag{45}$$

$$\begin{bmatrix} F^{0'0'} & F^{0'1'} & F^{0'2'} & F^{0'3'} \\ F^{1'0'} & F^{1'1'} & F^{1'2'} & F^{1'3'} \\ F^{2'0'} & F^{2'1'} & F^{2'2'} & F^{2'3'} \\ F^{3'0'} & F^{3'1'} & F^{3'2'} & F^{3'3'} \end{bmatrix} = \begin{bmatrix} 1 & \frac{v^2}{c^2} & 0 & 0 \\ \frac{v^2}{c^2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^3 & -B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'v'} \\ F^{1'v'} \\ F^{2'v'} \\ F^{3'v'} \end{bmatrix} = \begin{bmatrix} -\frac{v^2}{c^2} E^1 & \frac{1}{(1 + \frac{v^2}{c^2})} E^1 & \frac{1}{(1 + \frac{v^2}{c^2})} E^2 + \frac{v^2}{c^2} B^3 & \frac{1}{(1 + \frac{v^2}{c^2})} E^3 - \frac{v^2}{c^2} B^2 \\ -\frac{1}{(1 + \frac{v^2}{c^2})} E^1 & \frac{v^2}{c^2} E^1 & \frac{v^2}{c^2} E^2 + \frac{1}{(1 + \frac{v^2}{c^2})} B^3 & \frac{v^2}{c^2} E^3 - \frac{1}{(1 + \frac{v^2}{c^2})} B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix} \tag{49}$$

Equation (49) is the new symmetry of EMF but remains in original symmetry of EMF after simplification that is very strange prediction. The new symmetry terms or zero-point terms along the diagonal of EMF validate the form invariance of EMF.

Zero-Point Electric Field

Adding and rearranging all terms on left- and right-hand side, we get

$$\frac{\partial^2}{\partial t'^2} - \frac{\partial^2}{\partial x'^2} - \frac{\partial^2}{\partial y'^2} - \frac{\partial^2}{\partial z'^2} = \frac{(1 + \frac{v^2}{c^2})}{(1 - \frac{v^2}{c^2})} \frac{\partial^2}{\partial t^2} - \frac{(1 + \frac{v^2}{c^2})}{(1 - \frac{v^2}{c^2})} \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial^2}{\partial t'^2} - \nabla'^2 = \frac{\partial^2}{\partial t^2} - \nabla^2$$

$$\square'^2 = \square^2 \tag{46}$$

$$\square^2 = \frac{\partial^2}{\partial t^2} - \nabla^2 \tag{47}$$

4-D wave operator or de Alembertian operator remains same in its original form

### 3. Application of 2D URM of Unit Circle in Electrodynamics

Electrodynamics based on single transformation law for 4-vectors and tensor in 4D universal rotation matrix predicts complete structure of zero-point electrodynamics within usual electrodynamics. Zero-point electrodynamics is affected by frame of reference but the results remain finite at  $v < c$ ,  $v = c$  and  $v > c$  that was not possible in our earlier model [4] where zero-point structure became infinite at  $v = c$ . Usual electrodynamics remains same for all observers. Both models obey EPR, conservation law and symmetry principle.

#### 3.1. Transformation of Electromagnetic Field

$$F^{\mu'v'} = R_{\alpha}^{\mu'} F^{\alpha v'} \tag{48}$$

$$F^{0'0'} = -\frac{v^2}{c^2} E^1 \tag{50}$$

4D Electric Field

$$F^{0'v'} = \frac{1}{(1+\frac{v^2}{c^2})} E + \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} [(B^3 - B^2) - E^1] \quad (51)$$

Transformation of electric field gives us the combination of electric and magnetic field

It remains finite at  $v < c$ ,  $v = c$  and  $v > c$ .

*Zero-Point Magnetic Field*

$$F^{i'i'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E^1 \quad (52)$$

*Tensorial Magnetic Field*

$$F^{i'v'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E + \frac{1}{(1+\frac{v^2}{c^2})} \{[(B^3 - B^2) - E^1] \quad (53)$$

Transformation of magnetic field gives us the combination of electric and magnetic field

It remains finite at  $v < c$ ,  $v = c$  and  $v > c$ .

$$\begin{bmatrix} F^{0'v'} \\ F^{1'v'} \\ F^{2'v'} \\ F^{3'v'} \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{v^2}{c^2} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E_{,1}^1 & E_{,2}^2 & E_{,3}^3 \\ -E_{,0}^1 & 0 & B_{,2}^3 & -B_{,3}^2 \\ -E_{,0}^2 & -B_{,1}^3 & 0 & B_{,3}^1 \\ -E_{,0}^3 & B_{,1}^2 & -B_{,2}^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'v'} \\ F^{1'v'} \\ F^{2'v'} \\ F^{3'v'} \end{bmatrix} = \begin{bmatrix} -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,0}^1 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,1}^1 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,2}^2 + \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} B_{,2}^3 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,3}^3 - \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} B_{,3}^2 \\ -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,0}^1 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,1}^1 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,2}^2 + \frac{1}{(1+\frac{v^2}{c^2})} B_{,2}^3 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,2}^2 - \frac{1}{(1+\frac{v^2}{c^2})} B_{,3}^2 \\ -E_{,0}^2 & -B_{,1}^3 & 0 & B_{,3}^1 \\ -E_{,0}^3 & B_{,1}^2 & -B_{,2}^1 & 0 \end{bmatrix} \quad (57)$$

Equation (57) is the new symmetry of Maxwell's equations. The two new symmetry terms or zero-point terms along the diagonal of (57) contribute to validate the form invariance of ME that remain same for all observers.

*Zero-Point Gauss's Law*

$$F^{0'0'} = -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,0}^1 \quad (58)$$

*Gauss's Law*

$$F^{0'v'} = \frac{1}{(1+\frac{v^2}{c^2})} \nabla \cdot E + \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} [(\nabla \times B)^1 - E_{,0}^1] \quad (59)$$

Transformation of Gauss's law gives us the combination of Gauss's law and Ampere's law. It means Gauss's law is relative.

*Zero-Point Electromagnetic Field*

$$F^{\mu'\mu'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} [E^1 - E^1] = 0 \quad (54)$$

It remains finite at  $v < c$ ,  $v = c$  and  $v > c$ .

*Electromagnetic Field*

$$F^{\mu'v'} = E + \{[(B^3 - B^2) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3]\} = 0 \quad (55)$$

The combination of electric and magnetic field as electromagnetic field remains same for all observers in its original form after transformation.

### 3.2. Transformation of Maxwell's Equations

$$F^{\mu'v'} = R_{\alpha}^{\mu'} F^{\alpha v'} \quad (56)$$

*Zero-Point Ampere's Law*

$$F^{1'1'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,1}^1 \quad (60)$$

*Ampere's Law*

$$F^{i'v'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} \nabla \cdot E + \frac{1}{(1+\frac{v^2}{c^2})} [(\nabla \times B)^1 - E_{,0}^1] \quad (61)$$

Transformation of Ampere's law gives us the combination of Ampere's law and Gauss's law. It means Ampere's law's law is relative

*Zero-Point Maxwell's Equations*

$$F^{\mu'\mu'}_{,\nu'} = -\frac{v^2}{c^2} E_{,0}^1 + \frac{v^2}{c^2} E_{,1}^1 \quad (62)$$

Maxwell's Equations as Sum of Gauss's Law and Ampere's Law

$$F^{\mu'\nu'}_{,\nu'} = \frac{(1+\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})} \nabla \cdot E + \frac{(1+\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})} [(\nabla \times B) - E_{,0}] \quad (63)$$

$$F^{\mu'\nu'}_{,\nu'} = [\nabla \cdot E] + [(\nabla \times B) - E_{,0}] \quad (64)$$

$$\begin{bmatrix} F^{0'\nu'}_{,\nu'0'} \\ F^{1'\nu'}_{,\nu'1'} \\ F^{2'\nu'}_{,\nu'2'} \\ F^{3'\nu'}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E_{,10}^1 & E_{,20}^2 & E_{,30}^3 \\ -E_{,01}^1 & 0 & B_{,21}^3 & -B_{,31}^2 \\ -E_{,02}^2 & -B_{,12}^3 & 0 & B_{,32}^1 \\ -E_{,03}^3 & B_{,13}^2 & -B_{,23}^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}_{,\nu'0'} \\ F^{1'\nu'}_{,\nu'1'} \\ F^{2'\nu'}_{,\nu'2'} \\ F^{3'\nu'}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} -\frac{v^2}{c^2} E_{,01}^1 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,10}^1 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,20}^2 + \frac{v^2}{c^2} B_{,21}^3 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,30}^3 - \frac{v^2}{c^2} B_{,31}^2 \\ -\frac{1}{(1+\frac{v^2}{c^2})} E_{,01}^1 & \frac{v^2}{c^2} E_{,01}^1 & \frac{v^2}{c^2} E_{,20}^2 + \frac{1}{(1+\frac{v^2}{c^2})} B_{,21}^3 & \frac{v^2}{c^2} E_{,30}^3 - \frac{1}{(1+\frac{v^2}{c^2})} B_{,31}^2 \\ -E_{,02}^2 & -B_{,12}^3 & 0 & B_{,32}^1 \\ -E_{,03}^3 & B_{,13}^2 & -B_{,23}^1 & 0 \end{bmatrix} \quad (66)$$

Relation (66) is the new symmetry of electromagnetic conservation law. The two new symmetry terms or zero-point terms along the diagonal of (66) are necessary to validate conservation law in its original form after transformation.

Zero-Point Conservation of Gauss's Law

$$F^{0'\nu'}_{,\nu'0'} = -\frac{v^2}{c^2} E_{,01}^1 \quad (67)$$

Zero-Point Conservation of Ampere's Law

$$F^{1'\nu'}_{,\nu'1'} = \frac{v^2}{c^2} E_{,01}^1 \quad (68)$$

Zero-Point Conservation Law

$$F^{\mu'\mu'}_{,\mu'\mu'} = -\frac{v^2}{c^2} E_{,01}^1 + \frac{v^2}{c^2} E_{,01}^1 = 0 \quad (69)$$

Conservation of Gauss's Law

Usual Maxwell's equations in tensor form remain same for all observers in their original form where constancy of speed of light is contained in it by birth.

### 3.3. Transformation of Conservation Law by Matrix Method

Conservation law by matrix method holds as usual

$$F^{\mu'\nu'}_{,\nu'\mu'} = R_{\alpha}^{\mu'} F^{\alpha\nu'}_{,\nu'\mu'} \quad (65)$$

$$F^{0'\nu'}_{,\nu'0'} = \frac{1}{(1+\frac{v^2}{c^2})} \nabla \cdot E_{,0} + \frac{v^2}{c^2} [(B_{,21}^3 - B_{,31}^2) - E_{,01}^1] \quad (70)$$

Conservation of Ampere's Law

$$F^{1'\nu'}_{,\nu'1'} = \frac{v^2}{c^2} \nabla \cdot E_{,0} + \frac{1}{(1+\frac{v^2}{c^2})} [(B_{,21}^3 - B_{,31}^2) - E_{,01}^1] + [(B_{,32}^1 - B_{,12}^3) - E_{,02}^2] + [(B_{,13}^2 - B_{,23}^1) - E_{,03}^3] \quad (71)$$

Total Conservation Law

$$F^{\mu'\nu'}_{,\nu'\mu'} = \frac{(1+\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})} \nabla \cdot E_{,0} + \frac{(1+\frac{v^2}{c^2})}{(1+\frac{v^2}{c^2})} [(B_{,21}^3 - B_{,31}^2) - E_{,01}^1] + [(B_{,32}^1 - B_{,12}^3) - E_{,02}^2] + [(B_{,13}^2 - B_{,23}^1) - E_{,03}^3]$$

$$F^{\mu'\nu'}_{,\nu'\mu'} = \nabla \cdot E_{,0} + [\nabla \cdot (\nabla \times B) - \nabla \cdot E_{,0}]$$

$$F^{\mu'\nu'}_{,\nu'\mu'} = 0 \quad (72)$$

Electromagnetic conservation law remains same for all observers in its original form after transformation.

## 4. Spacetime Laws of Electrodynamics at $v = c$

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (73)$$

The situation at  $v = c$ , is very interesting where the determinant of above matrix becomes zero and it reduces to singular idempotent matrix but still applicable to validate form invariance spacetime laws of physics and conservation laws with new symmetry.

$$R_\alpha^\mu = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (74)$$

This matrix behaves like an identity matrix in the sense that all the spacetime laws of physics in terms of 4-vectors and tensors remain same for all observers. Time and space components are equal but spacetime as a whole remains same for all observers. The motion along the trajectory  $v = c$  is smooth and undisturbed is a wonderful prediction of this model. The trajectory below and above  $v = c$ , time and space components reciprocate each other but spacetime trajectory as a whole remains same for all observers.

### 4.1. Transformation of Electromagnetic Field

$$F^{\mu'\nu'} = R_\alpha^{\mu'} F^{\alpha\nu} \quad (75)$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1 & E^2 & E^3 \\ -E^1 & 0 & B^3 & -B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'} \\ F^{1'\nu'} \\ F^{2'\nu'} \\ F^{3'\nu'} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}E^1 & \frac{1}{2}E^1 & \frac{1}{2}E^2 + \frac{1}{2}B^3 & \frac{1}{2}E^3 - \frac{1}{2}B^2 \\ -\frac{1}{2}E^1 & \frac{1}{2}E^1 & \frac{1}{2}E^2 + \frac{1}{2}B^3 & \frac{1}{2}E^3 - \frac{1}{2}B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix} \quad (76)$$

Equation (76) is the new symmetry of EMF which on simplification gives the original symmetry of EMF that remains same for all observers.

#### Zero-Point Electric Field

$$F^{0'0'} = -\frac{1}{2}E^1 \quad (77)$$

#### 4D Electric Field

##### Zero-Point Magnetic Field

$$F^{i'i'} = \frac{1}{2}E^1 \quad (78)$$

##### Electromagnetic Field

$$F^{\mu'\nu'} = E + \{[(B^3 - B^2) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3]\} = 0 \quad (79)$$

The combination of electric and magnetic field as electromagnetic field remains same for all observers in its original form

### 4.2. Transformation of Maxwell's Equations

$$F^{\mu'\nu'}_{,\nu'} = R_\alpha^{\mu'} F^{\alpha\nu'}_{,\nu'} \quad (80)$$

$$\begin{bmatrix} F^{0'\nu'}_{,\nu'} \\ F^{1'\nu'}_{,\nu'} \\ F^{2'\nu'}_{,\nu'} \\ F^{3'\nu'}_{,\nu'} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1_{,1} & E^2_{,2} & E^3_{,3} \\ -E^1_{,0} & 0 & B^3_{,2} & -B^2_{,3} \\ -E^2_{,0} & -B^3_{,1} & 0 & B^1_{,3} \\ -E^3_{,0} & B^2_{,1} & -B^1_{,2} & 0 \end{bmatrix}$$

$$\begin{bmatrix} F^{0'\nu'}_{,\nu'} \\ F^{1'\nu'}_{,\nu'} \\ F^{2'\nu'}_{,\nu'} \\ F^{3'\nu'}_{,\nu'} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}E^1_{,0} & \frac{1}{2}E^1_{,1} & \frac{1}{2}E^2_{,2} + \frac{1}{2}B^3_{,2} & \frac{1}{2}E^3_{,3} - \frac{1}{2}B^2_{,3} \\ -\frac{1}{2}E^1_{,0} & \frac{1}{2}E^1_{,1} & \frac{1}{2}E^2_{,2} + \frac{1}{2}B^3_{,2} & \frac{1}{2}E^3_{,3} - \frac{1}{2}B^2_{,3} \\ -E^2_{,0} & -B^3_{,1} & 0 & B^1_{,3} \\ -E^3_{,0} & B^2_{,1} & -B^1_{,2} & 0 \end{bmatrix} \quad (81)$$

Equation (81) is the new symmetry of ME at  $v = c$  that reduces to original symmetry of ME.

#### Gauss's Law

$$F^{0'\nu'}_{,\nu'} = \frac{1}{2} \nabla \cdot E + \frac{1}{2} [(\nabla \times B)^1 - E^1_{,0}] \quad (82)$$

Transformation of Gauss's law gives us the combination of Gauss's law and Ampere's law. It means Gauss's law is relative

#### Ampere's Law

$$F^{i'\nu'}_{,\nu'} = \frac{1}{2} \nabla \cdot E + \frac{1}{2} [(\nabla \times B)^1 - E^1_{,0}] \quad (83)$$

Transformation of Ampere's law gives us the combination of Ampere's law and Gauss's law. It means Ampere's law's law is relative

#### Zero-Point Maxwell's Equations

$$F^{\mu'\mu'}_{,\mu'} = -\frac{1}{2}E^1_{,0} + \frac{1}{2}E^1_{,1} \quad (84)$$

Maxwell's Equations as Sum of Gauss's Law and Ampere's Law

$$F^{\mu\nu}_{,\nu'} = \left(\frac{1}{2} + \frac{1}{2}\right) \nabla \cdot E + \left(\frac{1}{2} + \frac{1}{2}\right) [(\nabla \times B) - E_{,0}] \quad (85)$$

$$F^{\mu\nu}_{,\nu'} = [\nabla \cdot E] + [(\nabla \times B) - E_{,0}] \quad (86)$$

Usual Maxwell's equations in tensor form remain same for all observers in their original form

### 4.3. Transformation of Conservation Law by Matrix Method

Conservation law by matrix method holds as usual

$$F^{\mu\nu}_{,\nu'\mu'} = R^{\mu'}_{\alpha} F^{\alpha\nu'}_{,\nu'\mu'} \quad (87)$$

$$\begin{bmatrix} F^{0\nu'}_{,\nu'0'} \\ F^{1\nu'}_{,\nu'1'} \\ F^{2\nu'}_{,\nu'2'} \\ F^{3\nu'}_{,\nu'3'} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & E^1_{,10} & E^2_{,20} & E^3_{,30} \\ -E^1_{,01} & 0 & B^3_{,21} & -B^2_{,31} \\ -E^2_{,02} & -B^3_{,12} & 0 & B^1_{,32} \\ -E^3_{,03} & B^2_{,13} & -B^1_{,23} & 0 \end{bmatrix}$$

Conservation of Gauss's Law

$$F^{0\nu'}_{,\nu'0'} = \frac{1}{2} \nabla \cdot E_{,0} + \frac{1}{2} [(B^3_{,21} - B^2_{,31}) - E^1_{,01}] \quad (88)$$

Conservation of Ampere's Law

$$F^{i\nu'}_{,\nu'i'} = \frac{1}{2} \nabla \cdot E_{,0} + \frac{1}{2} [(B^3_{,21} - B^2_{,31}) - E^1_{,01}] + [(B^1_{,32} - B^3_{,12}) - E^2_{,02}] + [(B^2_{,13} - B^1_{,23}) - E^3_{,03}] \quad (89)$$

Total Conservation Law

$$F^{\mu\nu'}_{,\nu'\mu'} = \left(\frac{1}{2} + \frac{1}{2}\right) \nabla \cdot E_{,0} + \left(\frac{1}{2} + \frac{1}{2}\right) [(B^3_{,21} - B^2_{,31}) - E^1_{,01}] + [(B^1_{,32} - B^3_{,12}) - E^2_{,02}] + [(B^2_{,13} - B^1_{,23}) - E^3_{,03}]$$

$$F^{\mu\nu'}_{,\nu'\mu'} = \nabla \cdot E_{,0} + [\nabla \cdot (\nabla \times B) - \nabla \cdot E_{,0}] = 0 \quad (90)$$

Electromagnetic conservation law remains same for all observers at  $v = c$  in its original form after transformation.

## 5. Results of Electrodynamics Under 4D URM on a Unit Circle

Electrodynamics in tensor form consists of EMF, ME and conservation law. The final results of ZPE and usual electrodynamics are presented here

Zero-Point Electromagnetic Field

$$F^{\mu\mu'}_{,\mu'\mu'} = \frac{v^2}{(1 + \frac{v^2}{c^2})} [E - E] + \frac{v^2}{(1 + \frac{v^2}{c^2})} [B^1 - B^1] + \frac{v^2}{(1 + \frac{v^2}{c^2})} [B^2 - B^2] + \frac{v^2}{(1 + \frac{v^2}{c^2})} [B^3 - B^3] = 0 \quad (91)$$

Zero-Point Maxwell's Equations

$$F^{\mu\mu'}_{,\mu'\mu'} = \frac{v^2}{(1 + \frac{v^2}{c^2})} \{[\nabla \cdot E] + [(\nabla \times B) - E_{,0}]\} \quad (92)$$

Zero-Point Conservation Law by Matrix Method

$$F^{\mu\mu'}_{,\mu'\mu'} = -\frac{v^2}{(1 + \frac{v^2}{c^2})} \nabla \cdot E_{,0} + \frac{v^2}{(1 + \frac{v^2}{c^2})} \nabla \cdot E_{,0} = 0 \quad (93)$$

Equations (91), (92) and (93) constitute ZPE which is frame dependent but remains finite at  $v = c$  and  $v > c$ . They play very important role to validate EPR, conservation law and symmetry principle.

Usual Electromagnetic Field

$$F^{\mu\nu'} = E + \{[(B^3 - B^2) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3]\} = 0 \quad (94)$$

Usual electromagnetic field remains same for all observers in its original form

Usual Maxwell's Equations

$$F^{\mu\nu'}_{,\nu'} = [\nabla \cdot E] + [(\nabla \times B) - E_{,0}] \quad (95)$$

Usual Maxwell's equations in tensor form remain same for all observers in their original form

Usual Conservation Law by Matrix Method

$$F^{\mu\nu'}_{,\nu'\mu'} = 0 \quad (96)$$

Usual conservation law in tensor form remains same for all observers in its original form

Equations (94), (95) and (96) constitute usual structure of electrodynamics that remains same for all observers and obeys EPR, conservation law and symmetry principle.

Conservation Law by Einstein Summation Convention Method

Zero-Point Conservation Law

$$F^{\mu\mu'}_{,\mu'\mu'} = -\square^2 \left(\frac{v^2}{(1 + \frac{v^2}{c^2})}\right) E \quad (97)$$

Total Conservation Law

$$F^{\mu\nu'}_{,\nu'\mu'} = -[\square^2 - \nabla^2] \left(\frac{v^2}{(1 + \frac{v^2}{c^2})}\right) E \quad (98)$$

Note that zero-point conservation law and total conservation law are frame dependent but remain finite at  $v < c$ ,  $v = c$  and  $v > c$ .

## 6. Discussion and Comparison

A model of relativity theory for all velocities RTAV has been developed that can be called as amplified version of relativity as it contains existing relativity as a special case and covers all regions of spacetime diagram. STL for 4-vectors and tensors and URM on a unit circle are the two mathematical tools that have played key role in the development of this model.

*Main Features of the RTAV Model*

### 6.1. Properties of URM on a Unit Circle At $v = 0$ , $v < c$ , $v = c$ and $v > c$

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{(1+\frac{v^2}{c^2})} & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} & \frac{1}{(1+\frac{v^2}{c^2})} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (99)$$

$$\text{Det } R = \frac{1-\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})}$$

Now, we analyze the form of URM at different values of  $v$  and behavior of spacetime laws of physics in each situation

i. At  $v = 0$ , URM matrix (99) reduces to usual identity matrix

$$[\delta_v^\mu] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (100)$$

Here, time and space components remain independent of each other. Gauss's and Ampere's law remain independent of each other.

ii. At  $v < c$ , consider for example  $v = 0.5 c$  then matrix (99) becomes

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{1.25} & \frac{0.25}{1.25} & 0 & 0 \\ \frac{0.25}{1.25} & \frac{1}{1.25} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} & 0 & 0 \\ \frac{1}{5} & \frac{4}{5} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (101)$$

$$[F^{\mu\nu}] = \begin{bmatrix} -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E^1 & \frac{1}{(1+\frac{v^2}{c^2})} E^1 & \frac{1}{(1+\frac{v^2}{c^2})} E^2 + \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} B^3 & \frac{1}{(1+\frac{v^2}{c^2})} E^3 - \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} B^2 \\ -\frac{1}{(1+\frac{v^2}{c^2})} E^1 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E^1 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E^2 + \frac{1}{(1+\frac{v^2}{c^2})} B^3 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E^3 - \frac{1}{(1+\frac{v^2}{c^2})} B^2 \\ -E^2 & -B^3 & 0 & B^1 \\ -E^3 & B^2 & -B^1 & 0 \end{bmatrix} \quad (104)$$

Here,  $\text{Det } R = \frac{3}{5}$  but it behaves like an identity matrix in the sense that all the spacetime laws of physics in terms of 4-vectors and tensors remains same for all observers such that time and space components reciprocate each other but as a whole spacetime remains same for all observers.

iii. At  $v = c$ , matrix (99) becomes singular idempotent

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (102)$$

Its  $\text{det } R = 0$  but it behaves like an identity matrix in the sense that all the spacetime laws of physics in terms of 4-vectors and tensors remains same for all observers such that time and space components remain equal but as a whole spacetime remains same for all observers. In other words, singularity at  $v = c$  doesn't, affect the existence of spacetime laws of physics that is an outstanding prediction of URM on a unit circle represent form invariance of EMF, ME and conservation law that remain same for all observers.

iv. At  $v > c$ , (99) Represents FTL. Consider for example  $v = 2c$  then matrix (99) becomes

$$[R_v^\mu] = \begin{bmatrix} \frac{1}{5} & \frac{4}{5} & 0 & 0 \\ \frac{4}{5} & \frac{1}{5} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (103)$$

Its  $\text{det } R = -\frac{3}{5}$  but it behaves like an identity matrix in the sense that all the spacetime laws of physics in terms of 4-vectors and tensors remains same for all observers such that time and space components reciprocate each other but as a whole spacetime remains same for all observers.

### 6.2. The Nature of Electrodynamics Under URM On a Unit Circle

i. *New Symmetry of Electromagnetic Field*

The transformation of EMF under URM by STL for 4-vectors and tensor method gave rise to new symmetry.

Apparently, the above matrix of EMF under URM seems to violate original symmetry of  $F^{\mu\nu}$  but the final result gives us the original form of EMF that remains same for all observers. Two new symmetry terms or zero-point terms along the diagonal are

$$F^{0'0'} = -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E^1 \quad (105)$$

$$F^{i'i'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E^1 \quad (106)$$

There exists only one model of spacetime electromagnetism by H. A. Atwater [5], page129, Equation (107), where he transformed electromagnetic field under Lorentz transformation matrix by similarity transformation but in two steps. In the first step single time Lorentz matrix is employed due to which new symmetry terms appeared along the diagonal of electromagnetic field like our method but he further moved towards similarity transformation. Since his model doesn't provide form invariance so cannot be persuaded further. Anyhow, his model provided the existence of new symmetry or zero-point terms along the diagonal of electromagnetic field.

Atwater's electromagnetic field equation containing new

$$[F^{\mu'\nu'}] = \begin{bmatrix} -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,0}^1 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,1}^1 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,2}^2 + \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} B_{,2}^3 & \frac{1}{(1+\frac{v^2}{c^2})} E_{,3}^3 - \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} B_{,3}^2 \\ -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,0}^1 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,1}^1 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,2}^2 + \frac{1}{(1+\frac{v^2}{c^2})} B_{,2}^3 & \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,2}^2 - \frac{1}{(1+\frac{v^2}{c^2})} B_{,3}^2 \\ -E_{,0}^2 & -B_{,1}^3 & 0 & B_{,3}^1 \\ -E_{,0}^3 & B_{,1}^2 & -B_{,2}^1 & 0 \end{bmatrix} \quad (110)$$

The above relation represents Maxwell's equations with two new symmetry terms or zero-point terms along the diagonal.

$$F^{0'0',0'} = -\frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,0}^1 \quad (111)$$

$$F^{1'1',1'} = \frac{\frac{v^2}{c^2}}{(1+\frac{v^2}{c^2})} E_{,1}^1 \quad (112)$$

These terms contribute to validate form invariance of Maxwell's equations.

On simplification, (110) gives us original form of Maxwell's equations in tensor form that remains same for all observers as

$$F^{\mu'\nu'} = [\nabla \cdot E] + [(\nabla \times B) - E_{,0}] \quad (113)$$

Atwater is the only author in the whole literature who at-

tempted to transform Maxwell's equations in tensor component form based on similarity transformation technique in Galilean metric page-132, as a candidate for noninertial frame. Atwater's Maxwell's equations are on page-133.

$$F_{\alpha\nu'} = \begin{bmatrix} -\gamma\beta c E^1 & -\gamma E^1 & -E^2 & -E^3 \\ \gamma E^1 & \frac{\gamma\beta}{c} E^1 & B^3 & -B^2 \\ \gamma(E^1 - \beta c B^3) & \gamma(\frac{\beta}{c} E^2 - B^3) & 0 & B^1 \\ \gamma(E^3 + \beta c B^2) & \gamma(\frac{\beta}{c} E^3 + B^2) & -B^1 & 0 \end{bmatrix} \quad (107)$$

tempted to transform Maxwell's equations in tensor component form based on similarity transformation technique in Galilean metric page-132, as a candidate for noninertial frame. Atwater's Maxwell's equations are on page-133.

$$F^{0'0'} = -\gamma\beta c E^1 \quad (108)$$

$$F^{i'i'} = \frac{\gamma\beta}{c} E^1 \quad (109)$$

tempted to transform Maxwell's equations in tensor component form based on similarity transformation technique in Galilean metric page-132, as a candidate for noninertial frame. Atwater's Maxwell's equations are on page-133.

#### ii. New Symmetry of Maxwell's Equations Under URM

Transformation of Maxwell's equations in tensor component form under URM resulted in new symmetry

#### Gauss's Law in Galilean Metric

$$(\nabla \cdot E)' = \frac{\rho}{\epsilon_0} - v(\nabla \times B)_1 \quad (114)$$

#### Components of Ampere's Law in Galilean Metric

$$(1 - \frac{v^2}{c^2})(\nabla \times B)_1' = \mu_0 J_1 + \frac{1}{c^2} \frac{\partial E_1}{\partial t} + \frac{v}{c^2} (\nabla \cdot E - E_{,1,1}) \quad (115)$$

$$(1 - \frac{v^2}{c^2})(\nabla \times B)_2' = \mu_0 J_2 + \frac{1}{c^2} \frac{\partial E_2}{\partial t} + \frac{v}{c^2} \frac{\partial B_3}{\partial t} + \frac{v}{c^2} E_{2,1} - \frac{v^2}{c^2} E_{3,1} \quad (116)$$

$$(1 - \frac{v^2}{c^2})(\nabla \times B)_3' = \mu_0 J_3 + \frac{1}{c^2} \frac{\partial E_3}{\partial t} + \frac{v}{c^2} \frac{\partial B_2}{\partial t} - \frac{v}{c^2} E_{3,1} + \frac{v^2}{c^2} B_{1,2} \quad (117)$$

It is to be noted that similarity transformation technique doesn't contain zero-point terms along the diagonal of EMF, ME and conservation law. They contain zeros along the diagonals. Antisymmetric EMF under similarity transformation remains antisymmetric but not in its original form. Similar is true for conservation law in tensor form.

Our Maxwell's equations in tensor form remain same in their original form but Atwater's set of Maxwell's equations

(114) to (117) do not remain same for all observers. Anyhow, Atwater's work is highly appreciable to introduce new ideas.

We have done complete framework of electrodynamics in 2D and 4D Lorentz transformation by STL method as well as in similarity transformation method given in tables 2 and 3. These results were presented in [4] but there are misprinting of mathematical symbols of operators which lead to confusion in physical meanings.

**Table 2.** Electrodynamics in 2D and 4D Lorentz Transformation (Single Transformation Law Method).

STL Based Electrodynamics in 2D Lorentz Matrix

$$[L_{\nu}^{\mu}] = \begin{bmatrix} \gamma & -\gamma v_1 & 0 & 0 \\ -\gamma v_1 & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L^0_0 = L^1_1 = \gamma, \quad L^2_2 = L^3_3 = 1, \quad L^0_1 = L^1_0 = -\gamma v_1$$

Transformation of Electromagnetic Field

$$F^{\mu\nu'} = L^{\mu}_{\alpha} F^{\alpha\nu'}$$

Zero-Point Term of Electric Field

$$F^{0'0'} = \gamma v_1 E^1$$

4D Electric Field

$$F^{0'\nu'} = \gamma v_1 E^1 + \gamma [E + v_1 (B^2 - B^3)]$$

Zero-Point Term of Magnetic Field

$$F^{i'\nu'} = -\gamma v_1 E$$

Magnetic Field in Tensor Form

$$F^{i'\nu'} = -\gamma v_1 E + \gamma [(B^2 - B^3) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3]$$

Electromagnetic Field

$$F^{\mu\nu'} = \gamma(1 - v_1)E + \gamma(1 - v_1)[(B^2 - B^3) - E^1] + [(B^1 - B^3) - E^2] + [(B^2 - B^1) - E^3]$$

Transformation of Maxwell's Equations

$$F^{\mu\nu'}_{,\nu'} = L^{\mu}_{\alpha} F^{\alpha\nu'}_{,\nu'}$$

Zero-Point Term of Gauss's Law

$$F^{0'0'}_{,0'} = \gamma v_1 E^1_{,0}$$

Zero-Point Term of Ampere's Law

$$F^{1'1'}_{,1'} = -\gamma v_1 E^1_{,1}$$

Zero-Point Term of Maxwell's Equations

$$F^{\mu'\mu'}_{,\mu'} = \gamma v_1 E^1_{,0} - \gamma v_1 E^1_{,1}$$

Gauss's Law

$$F^{0'\nu'}_{,\nu'} = \gamma(\nabla \cdot E) - \gamma v_1 [(\nabla \times B)^1 - E^1_{,0}]$$

Ampere's Law

$$F^{i'\nu'}_{,\nu'} = -\gamma v_1 (\nabla \cdot E) + \gamma [(\nabla \times B)^1 - E^1_{,0}] + [(\nabla \times B)^2 - E^2_{,0}] + [(\nabla \times B)^3 - E^3_{,0}]$$

Maxwell's Equations in Tensor Form

$$F^{\mu\nu'}_{,\nu'} = \gamma(1 - v_1)(\nabla \cdot E) + \gamma(1 - v_1)[(\nabla \times B)^1 - E^1_{,0}] + [(\nabla \times B)^2 - E^2_{,0}] + [(\nabla \times B)^3 - E^3_{,0}]$$

Transformation of Conservation Law

STL Based Electrodynamics in 4D Lorentz Matrix

$$[L_{\nu}^{\mu}] = \begin{bmatrix} \gamma & -\gamma v_1 & -\gamma v_2 & -\gamma v_3 \\ -\gamma v_1 & \gamma & 0 & 0 \\ -\gamma v_2 & 0 & \gamma & 0 \\ -\gamma v_3 & 0 & 0 & \gamma \end{bmatrix}$$

$$L^0_0 = L^1_1 = L^2_2 = L^3_3 = \gamma, \quad L^0_i = L^i_0 = -\gamma v_i$$

Transformation of Electromagnetic Field

$$F^{\mu\nu'} = L^{\mu}_{\alpha} F^{\alpha\nu'}$$

Zero-point Terms of Electric field

$$F^{0'0'} = (\gamma v \cdot E)$$

Zero-point Terms of Magnetic field

$$F^{i'i'} = -(\gamma v \cdot E)$$

Zero-point Terms of Electromagnetic field

$$F^{\mu'\mu'} = (\gamma v \cdot E) - (\gamma v \cdot E)$$

4D electric Field as 4D Lorentz Force

$$F^{0'\nu'} = \gamma\{v \cdot E + [E + (v \times B)]\}$$

Magnetic Field

$$F^{i'\nu'} = -\gamma E - (\gamma v \cdot E)$$

Electromagnetic Field

$$F^{\mu\nu'} = \gamma(v \times B)$$

Transformation of Maxwell's Equations

$$F^{\mu\nu'}_{,\nu'} = L^{\mu}_{\alpha} F^{\alpha\nu'}_{,\nu'}$$

Zero-Point Term of Gauss's Law

$$F^{0'0'}_{,0'} = \gamma v \cdot E_{,0}$$

Zero-Point Term of Ampere's Law

$$F^{i'i'}_{,i'} = -\gamma \nabla \cdot (v \cdot E)$$

Zero-Point Term of Maxwell's Equations

$$F^{\mu'\mu'}_{,\mu'} = \gamma v \cdot E_{,0} - \gamma \nabla \cdot (v \cdot E)$$

Gauss's Law

$$F^{0'\nu'}_{,\nu'} = \gamma\{v \cdot E_{,0} + \nabla \cdot [E + (v \times B)]\}$$

Ampere's Law

$$F^{i'\nu'}_{,\nu'} = -\gamma\{\nabla \cdot (v \cdot E) - [(\nabla \times B) - E_{,0}]\}$$

Maxwell's Equations in Tensor Form

$$F^{\mu\nu'}_{,\nu'} = \gamma\{v \cdot E_{,0} + \nabla \cdot [E + (v \times B)]\} - \gamma\{\nabla \cdot (v \cdot E) - [(\nabla \times B) - E_{,0}]\}$$

Transformation of Conservation Law

$$F^{\mu\nu'}_{,\nu'\mu'} = L^{\mu}_{\alpha} F^{\alpha\nu'}_{,\nu'\mu'}$$

$$F^{\mu\nu}_{,\nu\mu'} = L_{\alpha}^{\mu'} F^{\alpha\nu}_{,\nu\mu'}$$

Zero-Point Term of Conservation of Gauss's Law

$$F^{0'0'}_{,0'0'} = \gamma v_1 E^1_{,00}$$

Zero-Point Term of Conservation of Ampere's Law

$$F^{1'1'}_{,1'1'} = -\gamma v_1 E^1_{,11}$$

Zero-Point Conservation Law

$$F^{\mu\mu'}_{,\mu\mu'} = \square_1^2 (\gamma v_1 E^1)$$

Conservation of Gauss's Law

$$F^{0'\nu'}_{,\nu'0'} = \gamma(\nabla \cdot E_{,0}) + \square_1^2 \gamma v_1 E^1$$

Conservation of Ampere's Law

$$F^{i'\nu'}_{,\nu'i'} = -\gamma v_1 (\nabla \cdot E_{,1}) + \gamma[(\nabla \times B)_{,1}^1 - E^1_{,01}] + [(\nabla \times B)_{,2}^2 - E^2_{,02}] + [(\nabla \times B)_{,3}^3 - E^3_{,03}]$$

Complete Conservation Law

$$F^{\mu\nu'}_{,\nu'\mu'} = [\square_1^2 - \nabla_1^2](\gamma v_1 E^1) + \gamma(\nabla \cdot E_{,0}) + \gamma[(\nabla \times B)_{,1}^1 - E^1_{,01}] + [(\nabla \times B)_{,2}^2 - E^2_{,02}] + [(\nabla \times B)_{,3}^3 - E^3_{,03}]$$

Zero-Point Term of Conservation of Gauss's Law

$$F^{0'0'}_{,0'0'} = \gamma v \cdot E_{,00}$$

Zero-Point Term of Conservation of Ampere's Law

$$F^{i'i'}_{,i'i'} = -\nabla^2(\gamma v \cdot E)$$

Zero-Point Conservation Law

$$F^{\mu\mu'}_{,\mu\mu'} = \square^2(\gamma v \cdot E)$$

Conservation of Gauss's Law

$$F^{0'\nu'}_{,\nu'0'} = \square^2(\gamma v \cdot E) + \gamma(\nabla \cdot E_{,0})$$

Conservation of Ampere's Law

$$F^{i'\nu'}_{,\nu'i'} = -\nabla^2(\gamma v \cdot E) + \gamma v[(\nabla \times B) - E_{,0}]$$

Complete Conservation Law

Conservation Law as 7D Wave of Electrical Power

$$F^{\mu\nu'}_{,\nu'\mu'} = [\square^2 - \nabla^2](\gamma v \cdot E)$$

**Table 3.** Electrodynamics in 2D and 4D Lorentz Matrix (Similarity Transformation Method).

Similarity Based Electrodynamics in 2D Lorentz Matrix

$$[L_{\nu}^{\mu}] = \begin{bmatrix} \gamma & -\gamma v_1 & 0 & 0 \\ -\gamma v_1 & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$L^0_0 = L^1_1 = \gamma, L^2_2 = L^3_3 = 1, L^0_1 = L^1_0 = -\gamma v_1$$

Transformation of Electromagnetic Field

$$F^{\mu\nu'} = L_{\alpha}^{\mu'} L_{\beta}^{\nu} F^{\alpha\beta}$$

Electric Field

$$F^{0'i'} = E^1 + \gamma[E^2 + E^3] - \gamma v_1(B^3 - B^2)$$

Magnetic Field

$$F^{i'\nu'} = -E^1 - \gamma E^2 - \gamma E^3 - \gamma v_1(B^2 - B^3)$$

Electromagnetic Field

$$F^{\mu\nu'} = E^1 + \gamma[E^2 + E^3] - \gamma v_1(B^3 - B^2) - E^1 - \gamma E^2 - \gamma E^3 - \gamma v_1(B^2 - B^3) = 0$$

$$F^{\mu\nu'} = 0$$

Electromagnetic field remains antisymmetric but not in its original form

Transformation of Maxwell's Equations

$$F^{\mu\nu'}_{,\nu'} = L_{\alpha}^{\mu'} L_{\beta}^{\nu} F^{\alpha\beta}_{,\nu'}$$

Gauss's Law

$$F^{0'\nu'}_{,\nu'} = E^1_{,1} + \gamma E^2_{,2} + \gamma E^3_{,3} + \gamma v_1[(B^2_{,3} - B^3_{,2})]$$

Ampere's Law

$$F^{i'\nu'}_{,\nu'} = -E^1_{,0} - \gamma E^2_{,0} - \gamma E^3_{,0} + \gamma v_1[E^2_{,1} + E^3_{,1} - E^2_{,2} - E^3_{,2}] + \gamma[(B^3_{,2} - B^2_{,3})] + \gamma v_1(B^3_{,0} - B^2_{,0}) - \gamma(B^3_{,1} + B^2_{,1}) + B^1_{,3} - B^1_{,2}$$

Maxwell's Equations in Tensor Form

$$F^{\mu\nu'}_{,\nu'} = E^1_{,1} + \gamma E^2_{,2} + \gamma E^3_{,3} + \gamma v_1[(B^2_{,3} - B^3_{,2})] - E^1_{,0} - \gamma E^2_{,0} - \gamma E^3_{,0} + \gamma v_1[E^2_{,1} + E^3_{,1} - E^2_{,2} - E^3_{,2}] + \gamma[(B^3_{,2} -$$

Similarity Based Electrodynamics in 4D Lorentz Matrix

$$[L_{\nu}^{\mu}] = \begin{bmatrix} \gamma & -\gamma v_1 & -\gamma v_2 & -\gamma v_3 \\ -\gamma v_1 & \gamma & 0 & 0 \\ -\gamma v_2 & 0 & \gamma & 0 \\ -\gamma v_3 & 0 & 0 & \gamma \end{bmatrix}$$

$$L^0_0 = L^1_1 = L^2_2 = L^3_3 = \gamma, L^0_i = L^i_0 = -\gamma v_i$$

Transformation of Electromagnetic Field

$$F^{\mu\nu'} = L_{\alpha}^{\mu'} L_{\beta}^{\nu} F^{\alpha\beta}$$

Electric Field

$$F^{0'i'} = \gamma^2\{[E + (v \times B)] - v[v \cdot E]\}$$

Magnetic field

$$F^{i'\nu'} = -\gamma^2\{[E + (v \times B)] - v[v \cdot E]\}$$

Electromagnetic Field

$$F^{\mu\nu'} = \gamma^2\{[E + (v \times B)] - v[v \cdot E]\} - \gamma^2\{[E + (v \times B)] - v[v \cdot E]\} = 0$$

$$F^{\mu\nu'} = 0$$

Electromagnetic field remains antisymmetric but not in its original form

Transformation of Maxwell's Equations

$$F^{\mu\nu'}_{,\nu'} = L_{\alpha}^{\mu'} L_{\beta}^{\nu} F^{\alpha\beta}_{,\nu'}$$

Gauss's Law

$$F^{0'i'}_{,i'} = -\gamma^2 v^2 [\nabla \cdot E] + \gamma^2 \nabla \cdot [E + (v \times B)]$$

Ampere's Law

$$F^{i'\nu'}_{,\nu'} = \gamma^2 v [\nabla \cdot E]_{,0} - \gamma^2 [E + (v \times B)]_{,0} + \gamma^2 \nabla \times [B - (v \times E)]$$

Maxwell's Equations in Tensor Form

$$F^{\mu\nu'}_{,\nu'} = -\gamma^2 v^2 [\nabla \cdot E] + \gamma^2 \nabla \cdot [E + (v \times B)] + \gamma^2 v [\nabla \cdot E]_{,0} - \gamma^2 [E + (v \times B)]_{,0} + \gamma^2 \nabla \times [B - (v \times E)]$$

Transformation of Conservation Law

$$B^2_{,3}) + \gamma v_1(B^3_{,0} - B^2_{,0}) - \gamma(B^3_{,1} + B^2_{,1}) + B^1_{,3} - B^1_{,2}$$

Transformation of Conservation Law

$$F^{\mu\nu}_{,\nu\mu'} = L^{\mu'}_{\alpha} L^{\nu}_{\beta} F^{\alpha\beta}_{,\nu\mu'}$$

$$F^{\mu\nu}_{,\nu\mu'} = E^1_{,10} + \gamma E^2_{,20} - \gamma v_1 B^3_{,20} + \gamma E^3_{,30} + \gamma v_1 B^2_{,30} - E^1_{,01} \gamma B^3_{,21} - \gamma v_1 E^2_{,21} - \gamma v_1 E^3_{,31} - \gamma B^2_{,31} - \gamma E^2_{,02} + \gamma v_1 B^3_{,02} + \gamma v_1 E^2_{,12} - \gamma B^3_{,12} + B^1_{,32} - \gamma E^3_{,03} - \gamma v_1 B^2_{,03} + \gamma v_1 E^3_{,13} - \gamma B^2_{,13} - B^1_{,23} = 0$$

$$F^{\mu\nu}_{,\nu\mu'} = 0$$

Conservation law holds but not in its original form

$$F^{\mu\nu}_{,\nu\mu'} = L^{\mu'}_{\alpha} L^{\nu}_{\beta} F^{\alpha\nu}_{,\nu\mu'}$$

$$F^{\mu\nu}_{,\nu\mu'} = -\gamma^2 v^2 [\nabla \cdot E]_{,0} + \gamma^2 \nabla \cdot [E + (v \times B)]_{,0} + \gamma^2 \{v^2 [\nabla \cdot E]_{,0} - \nabla \cdot [E + (v \times B)]_{,0}\}$$

$$F^{\mu\nu}_{,\nu\mu'} = 0$$

Conservation law holds but not in its original form

### iii. New Symmetry of Electromagnetic Conservation Law in Tensor Form Under URM: Matrix Method

Finally, no one has ever attempted to transform conservation law in tensor components form even by similarity transformation technique.

Transformation of electromagnetic conservation law in tensor component form under URM resulted in new symmetry

$$[F^{\mu\nu}_{,\nu\mu'}] = \begin{bmatrix} -\frac{v^2}{c^2} E^1_{,01} & \frac{1}{(1+\frac{v^2}{c^2})} E^1_{,10} & \frac{1}{(1+\frac{v^2}{c^2})} E^2_{,20} + \frac{v^2}{c^2} B^3_{,21} & \frac{1}{(1+\frac{v^2}{c^2})} E^3_{,30} - \frac{v^2}{c^2} B^2_{,31} \\ -\frac{1}{(1+\frac{v^2}{c^2})} E^1_{,01} & \frac{v^2}{c^2} E^1_{,01} & \frac{v^2}{c^2} E^2_{,20} + \frac{1}{(1+\frac{v^2}{c^2})} B^3_{,21} & \frac{v^2}{c^2} E^3_{,30} - \frac{1}{(1+\frac{v^2}{c^2})} B^2_{,31} \\ -E^2_{,02} & -B^3_{,12} & 0 & B^1_{,32} \\ -E^3_{,03} & B^2_{,13} & -B^1_{,23} & 0 \end{bmatrix} \quad (118)$$

Equation (118) is electromagnetic conservation law with two new symmetry terms.

$$F^{0'0'}_{,0'0'} = -\frac{v^2}{c^2} E^1_{,01} \quad (119)$$

$$F^{1'1'}_{,1'1'} = \frac{v^2}{c^2} E^1_{,01} \quad (120)$$

These terms are necessary to get form invariance of electromagnetic conservation law that remains same for all observers.

$$F^{\mu\nu}_{,\nu\mu'} = \nabla \cdot E_{,0} + [\nabla \cdot (\nabla \times B) - \nabla \cdot E_{,0}] = 0 \quad (121)$$

### iv. New Version of Electromagnetic Conservation Law in Tensor Form Under URM: ESCM Method

$$F^{\mu\nu}_{,\nu\mu'} = R^{\mu'}_{\alpha} F^{\alpha\nu}_{,\nu\mu'} \quad (122)$$

Expanding equation (122) by Einstein summation convention method under 4D URM (11), we have

$$F^{\mu\nu}_{,\nu\mu'} = R^{\mu'}_0 F^{0\nu}_{,\nu\mu'} + R^{\mu'}_1 F^{1\nu}_{,\nu\mu'} + R^{\mu'}_2 F^{2\nu}_{,\nu\mu'} + R^{\mu'}_3 F^{3\nu}_{,\nu\mu'}$$

One can calculate 16 components of conservation law in terms of tensor components. The 4 diagonal

elements  $F^{0'0'}_{,0'0'}$ ,  $F^{1'1'}_{,1'1'}$ ,  $F^{2'2'}_{,2'2'}$ ,  $F^{3'3'}_{,3'3'}$  constitute zero-point conservation law  $F^{\mu\mu'}_{,\mu\mu'}$  as 4D EM wave.  $F^{\mu\mu'}_{,\mu\mu'}$  with remaining 12 components represent complete conservation law  $F^{\mu\nu\nu'}_{,\nu\mu'}$  as 7D EM wave.

Zero-Point Conservation Law as 4D EM Wave

$$F^{\mu\mu'}_{,\mu\mu'} = -\square^2 \left( \frac{v^2}{c^2} \right) E \quad (123)$$

Conservation Law as 7D EM Wave

$$F^{\mu\nu\nu'}_{,\nu\mu'} = -[\square^2 - \nabla^2] \left( \frac{v^2}{c^2} \right) E \quad (124)$$

SR is being questioned in the contemporary literature [6-9] and we have already pointed out the inconsistencies of SR with EPR and a consistent model of relativity is developed [1]. The discovery of URM on a unit circle as the inverse of ULTM on a unit hyperbola provided a definite framework of RTAV relativity theory. It is very important to note that this new framework of RTAV completely obeys EPR, conservation law and symmetry principle. All the spacetime laws of physics in terms of 4-vectors and tensors remain same for all observers in their original form after transformation. Electrodynamics in 4D under URM on a unit circle predicts structure of zero-point electrodynamics within usual electrodynamics. Transformation of EMF, ME and conservation

law based on STL for 4-vectors and tensors by matrix method obeys form invariance as usual. Conservation law by ESCM predicts zero-point conservation law and conservation law itself represent 7D electromagnetic wave. In this model, zero-point electrodynamic laws remain finite at  $v < c$ ,  $v = c$  as well as on  $v > c$  which were undefined or infinity at  $v = c$  in our previous model [4]. References [10-15] talk about FTL but their mathematical framework doesn't provide direct evidence of FTL whereas our approach is very simple and straightforward. Now, we discuss some important results and their comparison in short:

### 6.3. On the Nature of Spacetime Fabric in RTAV

At  $v = c$ , time component and space components remain equal. In the case for  $v < c$ ,  $v > c$ , time and space components reciprocate or undulate but spacetime as a whole remains same for all observers. The undulation of time and space components at a regular pattern predicts the possibility of warp drives needed by NASA and other scientists.

### 6.4. Relativity of 4-Position, 4-Velocity, 4-Momentum etc. Under URM

All the 4-vectors and inner product of 4-vectors remain same for all observers under URM such that time and space components are relative but 4-vectors as a whole remain same.

### 6.5. Relativistic Conservation Laws

It is very strange that in the presence of electromagnetic conservation law in terms of divergence of 4-current density  $J^\mu_{,\mu} = 0$ , other conservation laws are not considered like conservation of 4-momentum  $P^\mu_{,\mu} = 0$ . We have developed conservation laws for 4-vectors viz. equations (35) to (40). These are entirely new results.

## 7. Conclusion

Einstein's relativity theory is limited up to speed of light but our model covers the whole spectrum of velocity ranging  $v < c$ ,  $v = c$  and  $v > c$ . In our model, electromagnetic conservation law in tensor form based on STL for 4-vectors and tensors by Einstein's summation convention method resulted in marvelous form of conservation law where we got zero-point conservation law as 4D EM wave and conservation law itself as 7D EM wave. Here the 7D wave operator is an invariant operator and there is no counter example in the relativity literature. RTAV emerged as the result of reciprocal relation between URM on a unit circle and ULTM on a unit hyperbola. All the spacetime laws of physics based on STL for 4-vectors and tensors under URM on a unit circle remain same for all observers that satisfies EPR, conservation law and symmetry principle. The undulating nature of time and space components at a regular pattern indicates the

possibility of WARP DRIVES. The mathematical framework of the model is very simple and explains every concept without any assumption where new Lorentz factor is less than 1 due to which RTAV is explained. It predicts structure of ZPE that remains finite for  $v < c$ ,  $v = c$  and  $v > c$  within usual electrodynamics. Zero-point structure of electrodynamics can be employed as exotic stuff for WARP DRIVES. Our next paper electrodynamics in an accelerating and rotating frame of reference based on STL for 4-vectors and tensors is nearly ready for submission.

## Abbreviations

STL	Single Transformation Law
RTAV	Relativity Theory for All Velocities
URM	Universal Rotation Matrix
ULTM	Universal Lorentz Transformation Matrix
EMF	Electromagnetic Field
EM Wave	Electromagnetic Wave
ME	Maxwell's Equations
ZPE	Zero-Point Electrodynamics
ESCM	Einstein Summation Convention Method
FTL	Faster Than Light
SR	Special Relativity
EPR	Einstein Principle of Relativity
USEM	Universal Spacetime Exchanger Matrix

## Declaration

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## Author Contributions

**Naveed Hussain:** Conceptualization, Writing – original draft

**Hassnain Abdullah Hussain:** Conceptualization, Formal Analysis

**Ather Qayyum:** Writing – review & editing

**Muhammad Ali Shaukat Rao:** Software

## Conflicts of Interest

The authors declare no conflicts of interest.

## References

- [1] Hussain, N., et al. "Re Birth of Einstein's Relativity via Unified Transformation Law." *International Journal of Physics* 11.1 (2023): 6-16. <https://doi.org/10.12691/ijp-11-4-5>
- [2] Naveed Hussain, Hassnain Abdullah Hussain, Ather Qayyum. "Electrodynamics in Noninertial Metrics Predicts Entirely New Origin of 4D Electromagnetic Wave." *International Journal of Physics* Vol. 11. No. 4 (2023): 193-230. <https://doi.org/10.12691/ijp-11-4-5>

- [3] Naveed Hussain, Hassnain Abdullah Hussain, Ather Qayum. "Spacetime Exchange Method of Derivation of Electromagnetic Laws Based on Symmetry of Universal Lorentz Transformation Matrix" *International Journal of Theoretical and Mathematical Physics* 2024, 14(2): 17-39  
<https://doi.org/10.5923/j.ijtmp.20241402.01>
- [4] Naveed Hussain, Hassnain Abdullah Hussain, Ather Qayum, Muhammad Ali Shaikat Rao. "Electrodynamics in Universal Noninertial Frame Predicts Complete Structure of Zero Point Electrodynamics" *International Journal of Theoretical and Mathematical Physics* 2024, 14(2): 40-65  
<https://doi.org/10.5923/j.ijtmp.20241402.02>
- [5] H. A. Atwater (1974), Introduction to General Relativity, Pergamon Press.
- [6] Tardad, Ulaanbaatar. "Supreme Theory of Everything: Special Theory of Relativity Was Lost from the Beginning." *Journal of Applied Mathematics and Physics* 10.12 (2022): 3665-3675.
- [7] Ziefle, Reiner Georg. "Einstein's relativity falsified: I. The factor  $\gamma$  can take values larger than  $\sqrt{2}$ , which requires a speed of light  $c$  in moving inertial frames." *Physics Essays* 36.2 (2023): 230-241.
- [8] Li, Zifeng. "Special relativity arising from a misunderstanding of experimental results on the constant speed of light." *Physics Essays* 21.2 (2008): 96-102.
- [9] Buenker, Robert J. "The new relativity theory: Absolute simultaneity, modified light speed constancy postulate, uniform scaling of physical properties." *Int. J. App. Math. & Stat. Sci* 10.2 (2021): 151-162.
- [10] Hill, James M., and Barry J. Cox. "Einstein's special relativity beyond the speed of light." *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 468.2148 (2012): 4174-4192.
- [11] Cataldo, Carmine. "Faster than Light: Again, on the Lorentz Transformations." *Applied Physics Research* 8.6 (2016): 17-24.
- [12] Meholic, Gregory V. "A Novel View of Spacetime Permitting Faster-Than-Light Travel." *AIP Conference Proceedings*. Vol. 699. No. 1. American Institute of Physics, 2004.
- [13] Matradji, Matradji, Tutug Dhanardono, and Heri Justiono. "Energy Profile of Faster-than-Light Particles: A New Approach to The Special Theory of Relativity." *IPTEK Journal of Proceedings Series* 2.1 (2016).
- [14] Andrzej Dragan *et al* 2023, Relativity of superluminal observers in 1 + 3 spacetime, *Class. Quantum Grav.* 40 025013  
<https://doi.org/10.1088/1361-6382/acad60>
- [15] Schwartz, Charles. "Toward a quantum theory of tachyon fields." *International Journal of Modern Physics A* 31.09 (2016): 1650041.
- [16] Fell, Shaun DB, and Lavinia Heisenberg. "Positive energy warp drive from hidden geometric structures." *Classical and Quantum Gravity* 38.15 (2021): 155020.
- [17] Natário, José "Warp drive with zero expansion." *Classical and Quantum Gravity* 19.6 (2002): 1157.
- [18] Robertson, Glen A., P. A. Murad, and Eric Davis. "New frontiers in space propulsion sciences." *Energy Conversion and Management* 49.3 (2008): 436-452.
- [19] Musha, Takaaki. "Possibility of the Space Propulsion System Utilizing the ZPF Field." *AIP Conference Proceedings*. Vol. 1103. No. 1. American Institute of Physics, 2009.
- [20] Haisch, Bernhard, and Alfonso Rueda. "The Zero-Point Field and the NASA Challenge to Create the Space Drive." *NASA Breakthrough Propulsion Physics Workshop Proceedings*. 1999.
- [21] Puthoff, Harold E., S. R. Little, and M. Ibison. "Engineering the zero-point field and polarizable vacuum for interstellar flight." *Journal of the British Interplanetary Society* 55 (2002): 137-144.
- [22] Musha, Takaaki. *Physics of the zero-point field and its applications to advanced technology*. Nova Publishers, 2013.
- [23] Woodward, James F. "Gravity, inertia, and quantum vacuum zero-point fields." *Foundations of Physics* 31.5 (2001): 819-835.
- [24] Haisch, Bernhard, Alfonso Rueda, and Harold E. Puthoff. "Inertia as a zero-point-field Lorentz force." *Physical Review A* 49.2 (1994): 678.