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Using Lagrange Interpolation for Solving Nonlinear Algebraic Equations

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Abstract: Finding the roots of nonlinear algebraic equations is an important problem in science and engineering, later many methods developed for solving nonlinear equations. These methods are given [1-28], in this paper, a new Algorithm for solving nonlinear algebraic equations is obtained by using Lagrange Interpolation method by fitting a polynomial form of degree two. This paper compare the present method with the Famous methods of Regula Falsi (RF), Besection (BS), Modified Regula Falsi (MRF), Nonlinear Regression Method (NR) given by Jutaporn N, Bumrungsak P and Apichat N, 2016 [1] and Least Square Method (LS) given by N. IDE, 2016 [2]. We verified on a number of examples and numerical results obtained show that the present method is faster than the other methods.

Keywords: Nonlinear Algebraic Equations, Least Square Method, Lagrange Interpolation Method, Nonlinear Regression Method

1. Introduction

There are several well-known methods for solving nonlinear algebraic equations of the form

$$F(x)=0 \tag{1}$$

Where f denote a continuously differentiable on $[a,b]\subset\mathcal{R}$ and has at least one root α, in $[a,b]$

Such as Bisection method, Regula Falsi method, Nonlinear Regresion Method and several another methods see for example [2-28].

Here we describe a new method by using Lagrange Interpolation method as a polynomial form of degree two:

$$Ax^2+Bx+C=0 \tag{2}$$

Where A, B and C are a known constants.

We used three points (a, f (a)), (b, (f (b)) and (c, f (c)) where $c=\frac{a+b}{2}$, then we find that, this procedure lead us to the root α of equation (1).

2. The Present Method

In beginning, we define three initial points (a, f (a)), (b, (f (b)) and (c, f (c)), now by using Lagrange Interpolation method for these three points we find the polynomial (2) as suit,

$$P(x)=\frac{(x-c)(x-b)}{(a-c)(a-b)}.f(a)+\frac{(x-a)(x-c)}{(b-a)(b-c)}.f(b)+\frac{(x-a)(x-b)}{(c-a)(c-b)}.f(c) \tag{3}$$

Or the form,

$$P(x)=\frac{(x^2-cx-bx+cb)}{(a-c)(a-b)}f(a)+\frac{(x^2-cx-ax+ac)}{(b-a)(b-c)}f(b)+\frac{(x^2-ax-bx+ab)}{(c-a)(c-b)}f(c) \tag{4}$$

Now by solving the equation of second degree (4), p (x)= $Ax^2+Bx+C=0$ we find the two roots x_1 and x_2 of (2), we choose x_1 or x_2 which verify $x_1\in[a,b]$ or $x_2\in[a,b]$.

3. Algorithm. 1

The present method has 6 steps:

"a" Take $[a,b]$ is an initial interval, which has at least a root in this interval.

"b" Compute $c=\frac{a+b}{2}$.

"c" Determine the Lagrange Interpolation method as a

polynomial form of degree two$Ax^2 + Bx + C = 0$.
"d" Solve the equation of second degree $Ax^2 + Bx + C = 0$ for determine the root of (1), $x = x_1$ or $x = x_2$ which verify $x_1 \in [a, b]$ or $x_2 \in [a, b]$.
"e" Replace the interval $[a, b]$ with $[a, x\ ]\ or\ [x, b]$ which Contains the root.
"f" Return step (2) until the absolute error $|f(x)| < \varepsilon$.

4. Examples

Example 1. Consider the equation: f (x) = $x^2 - (1-x)^5$= 0. Applying present method, by using Maple program, for the initial interval [0.1, 1], we find the approximate equation of second degree (Lagrange Interpolation):

$$-1.8230481482680529243x^2 + 2.8682191491460827 4133x - 774.08319341213797212355 \times 10^{-3} = 0$$

Table 1 presents a comparison of iteration numbers and the error ε between Bisection (BS), Regula Falsi (RF), Modified Regula Falsi (MRF), Nonlinear Regresion Method (NR) given by Jutaporn N, Bumrungsak P and Apichat N, 2016 [1], Least Squart Method (LS) given by N. IDE, 2016 [2] and NEW method.

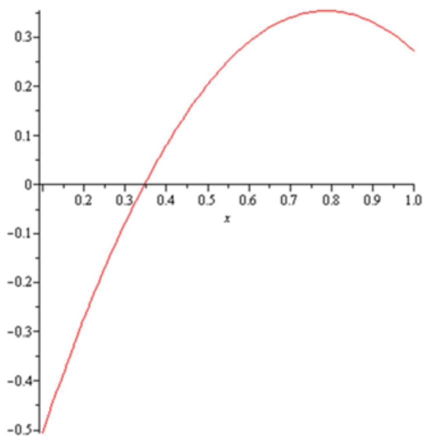
Table 1. *Presents comparison of iteration numbers and the errorε.*

Method	No. Iteration	Approximation Root	$\varepsilon = \|f(x)\|$
BS	33	0.345954815842023	1.10^{-10}
RF	20	0.345954815842023	1.10^{-10}
MRF	5	0.345954815842023	1.10^{-10}
NR	6	0.345954815842023	1.10^{-10}
LS	4	0.345954815671666	1.10^{-10}
NEW	4	0.3459548156373397	$2.840546430459496 \times 10^{-10}$

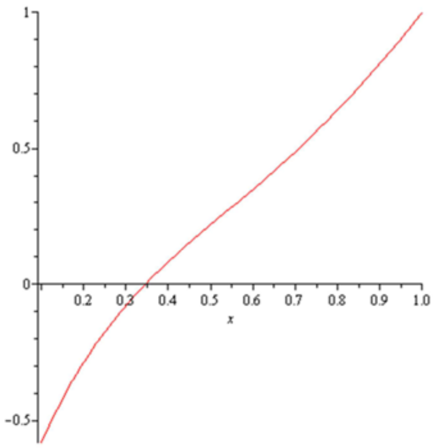
Figure 1 illustrate the plots of approximate solution of the equation

$$-1.8230481482680529243x^2 + 2.86821914914608274133x - 774.08319341213797212355 \times 10^{-3} = 0$$

Moreover, exact solution of the equation for example 1, in the interval [0.1, 1].



$$\text{plot}(-1.8230481482680529243x^2 + 2.86821914914608274133x - 774.08319341213797212355 \times 10^{-3}, \text{x} = 0.1..1)$$



$plot(x^2 - (1-x)^5, x = 0.1..1)$

Figure 1. *The plot of approximate solution. Exact solution for example 1.*

Figure 1 illustrate that, in the interval [0.1, 1] we have the same graph of the exact solution and the solution given by present method.

Example 2. Consider the equation: f (x) = $e^{-e^{-x}} - x$= 0. Applying present method, by using Maple program, for the initial interval [0.1, 1], we find the approximate equation of second degree (Lagrange Interpolation):

$$-69.611131036521966483 08 \times 10^{-3}x^2 - 599.38951620870976209087 \times 10^{-3}x + 362.33026799969775876308 \times 10^{-3} = 0$$

Table 2 presents a comparison of iteration numbers and the error ε between Bisection (BS), Regula Falsi (RF), Modified Regula Falsi (MRF), Nonlinear Regresion Method (NR) given by Jutaporn N, Bumrungsak P and Apichat N, 2016 [1], Least Squart Method (LS) given by N. IDE, 2016 [2] and NEW method.

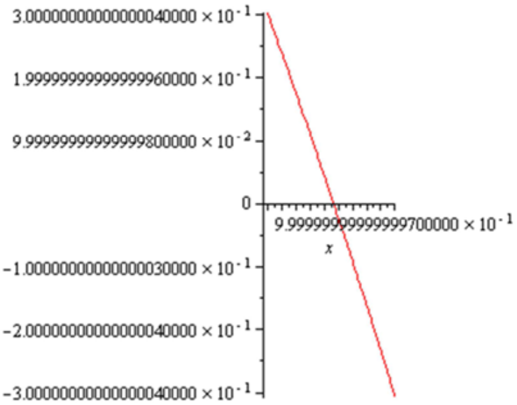
Table 2. *Presents comparison of iteration numbers and the errorε.*

Method	No. Iteration	Approximation Root	$\varepsilon = \|f(x)\|$
BS	29	0.5671432904012503	1.10^{-10}
RF	8	0.5671432904012503	1.10^{-10}
MRF	6	0.5671432904012503	1.10^{-10}
NR	7	0.5671432904012503	1.10^{-10}
LS	4	0.5671432901053765	1.10^{-10}
NEW	3	0.5671432904097838	$4.9519418376550871 \times 10^{-17}$

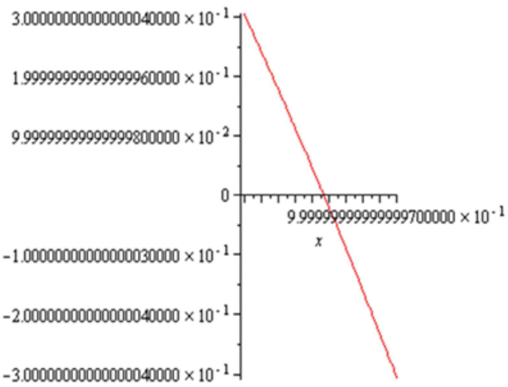
Figure 2 illustrate the plots of approximate solution of the equation

$$-69.61113103652196648308 \times 10^{-3}x^2 - 599.38951620870976209087 \times 10^{-3}x + 362.33026799969775876308 \times 10^{-3} = 0$$

Moreover, exact solution of the equation $e^{-e^{-x}} - x$= 0 for example 2, in the interval [0.1, 1].



$$\text{plot}(-69.61113103652196648308 \times 10^{-3}x^2 - 599.38951620870976209087 \times 10^{-3}x + 362.33026799969775876308 \times 10^{-3}, x = 0.1..1)$$



$$plot(\exp(-\exp(-x)) - x, x = 0.1 .. 1)$$

Figure 2. *The plot of approximate solution. Exact solution for example 2.*

Figure 2 illustrate that, in the interval [0.1, 1] we have the same graph of the exact solution and the solution given by present method.

Example 3. Consider the equation: f (x) $= x.e^{x} - 1 = 0$. Applying present method, by using Maple program, for the initial interval [0.1, 1], we find the approximate equation of second degree (Lagrange Interpolation):

$$2.26139081035670657426x^2 + 198.15694398841245846300 \times 10^{-3}x - 839.76315423983478638222 \times 10^{-3} = 0$$

Table 3 presents a comparison of iteration numbers and the error ε between Bisection (BS), Regula Falsi (RF), Modified Regula Falsi (MRF), Nonlinear Regresion Method (NR) given by Jutaporn N, Bumrungsak P and Apichat N, 2016 [1], Least Squart Method (LS) given by N. IDE, 2016 [2] and NEW method.

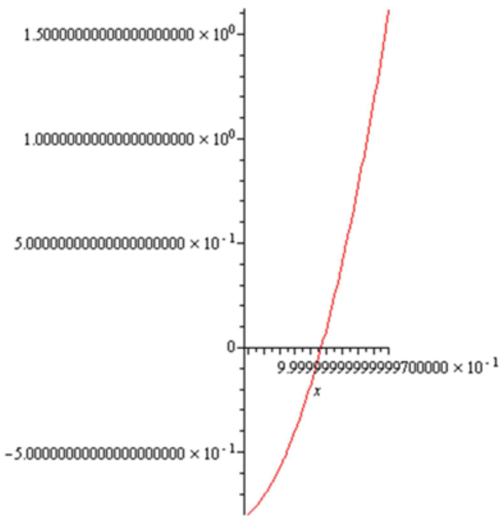
Table 3. *Presents comparison of iteration numbers and the errorε.*

Method	No. Iteration	Approximation Root	$\varepsilon = \lvert f(x) \rvert$
BS	33	0.5671432904097837	1.10^{-10}
RF	20	0.5671432904097837	1.10^{-10}
MRF	5	0.5671432904097837	1.10^{-10}
NR	6	0.5671432904097837	1.10^{-10}
LS	4	0. 5671432904097837	1.10^{-10}
NEW	3	0.5671432903946957	$-4.169198416164994 \times 10^{-11}$

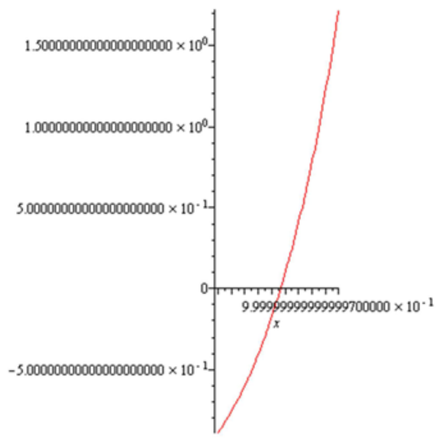
Figure 3 illustrate the plots of approximate solution of the equation

$$2.26139081035670657426x^2 + 198.15694398841245846300 \times 10^{-3}x - 839.76315423983478638222 \times 10^{-3} = 0$$

Moreover, exact solution of the equation $x.e^{x} - 1 = 0$ example 1, in the interval [0.1, 1].



$$\text{plot}(2.26139081035670657426x^2 + 198.15694398841245846300 \times 10^{-3}x - 839.76315423983478638222 \times 10^{-3}, x = 0.1..1$$



$$plot(x \cdot \exp(x) - 1, x = 0.1 .. 1)$$

Figure 3. *The plot of approximate solution. Exact solution for example 3.*

Example 4. Consider the equation: $f := x \rightarrow \cos(x) - x^3 = 0$. Applying present method, by using Maple program, for the initial interval [0.1, 1], we find the approximate equation of second degree (Lagrange Interpolation):

$$-2.92059378515695059141x^2 + 2.04685760393075285711x + 416.15494808520891698808 \times 10^{-3} = 0$$

Table 4 presents a comparison of iteration numbers and the error ε between Bisection (BS), Regula Falsi (RF), Modified Regula Falsi (MRF), Nonlinear Regresion Method (NR) given by Jutaporn N, Bumrungsak P and Apichat N, 2016 [1], Least Squart Method (LS) given by N. IDE, 2016 [2] and NEW method.

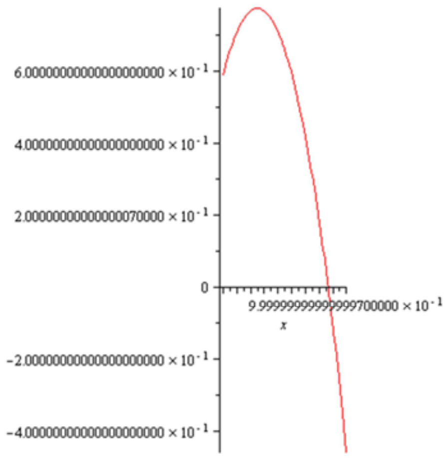
Table 4. *Presents comparison of iteration numbers and the error ε.*

Method	No. Iteration	Approximation Root	$\varepsilon = \lvert f(x) \rvert$
BS	34	0.865470331016205	1.10^{-10}
RF	12	0.865470331016205	1.10^{-10}
MRF	5	0.865470331016205	1.10^{-10}
NR	6	0.865470331016205	1.10^{-10}
LS	4	0.865470331016205	1.10^{-10}
NEW	3	0.8654740331016144	$1.402601312699497 \times 10^{-16}$

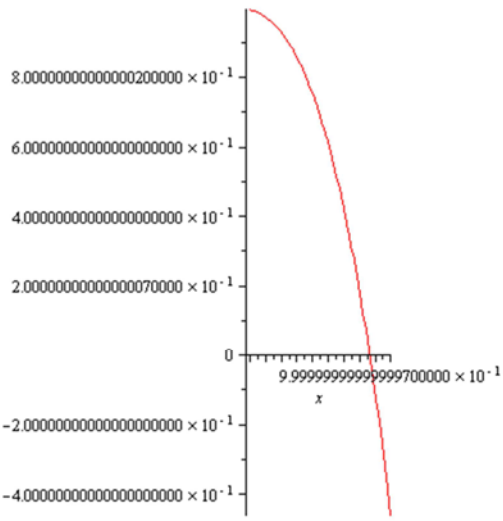
Figure 4 illustrate the plots of approximate solution of the equation

$$-2.92059378515695059141x^2 + 2.04685760393075285711x + 416.15494808520891698808 \times 10^{-3} = 0$$

Moreover, exact solution of the equation $x^2 - (1-x)^5 = 0$ for example 1, in the interval [0.1, 1].



$$\text{plot}(-2.92059378515695059141x^2 + 2.04685760393075285711x + 416.15494808520891698808 \times 10^{-3}, \text{x} = 0.1..1)$$



$$plot(\cos(x) - x^3, x = 0.1 .. 1)$$

Figure 4. *The plot of approximate solution. Exact solution for example 1.*

Figure 4 illustrate that, in the interval [0.1, 1] we have the same graph of the exact solution and the solution given by present method.

5. Conclusion

The present paper suggests a new algorithm for solving nonlinear algebraic equations; this method presented by the algorithm 1. Numerical examples show that this method is remarkably effective for solving nonlinear algebraic equations and it is much faster than the method given by the famous methods of Regula Falsi (RF), Besection (BS), Modified Regula Falsi (MRF), Nonlinear Regresion Method (NR) given by Jutaporn N, Bumrungsak P and Apichat N, 2016 [1] and Least Squart Method (LS) given by N. IDE, 2016 [2]. We verified on a number of examples and numerical results obtained show that the present method is faster than the other methods.

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