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Exponential Entropy Approach for Image Edge Detection

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Abstract: Edge detection is an important pre-processing step in image analysis. Best results of image analysis extremely depend on edge detection. Up to now many edge detection methods have been developed such as Prewitt, Sobel, LoG, Canny, etc. But, they are sensitive to noise. In this paper we propose a novel edge detection algorithm for images corrupted with noise based on Exponential Entropy. The performance of our method is compared against other methods by using various images. It is observed that the proposed algorithm displayed superior noise resilience and decrease the computation time compared with standard approaches. The results indicate the accuracy of the proposed edge-detection method over conventional edge-detection methods.

Keywords: Non-extensive Entropy, Edge Detection, Threshold Value, Gray-Scale Images

1. Introduction

Edge Detection has been very useful low-level image processing tool for image analysis in computer vision and pattern recognition field. In image, edges carries essential information of an object of interest in image as they separate dissimilar regions in an image. Specific linear time-invariant filters is the most common procedure applied to the edge detection problem. In the case of first-order filters, an edge is interpreted as a sudden variation in gray level between two neighbor pixels. In this case the goal is to determine in which points in the image the first derivative of the gray level as a function of position is of high magnitude. Edges in arbitrary directions can be detected, by applying the threshold to the new output image.

Natural images are prone to noise and artifacts. Salt & pepper noise is a form of noise typically seen on images. It is ideally manifested as randomly occurring white and black pixels. Salt & pepper noise creeps into images in situations where quick transients, such as faulty switching. On the other hand, white noise is additive in nature where the each pixel in the image is modified via the addition of a value drawn from a Gaussian distribution. To check the generality of the results, the suggested edge detection algorithm was tested on images containing both these types of noise.

A large number of studies have been published in the field of image edge detection, which attests to its importance within the field of image processing. A lot of edge detection algorithms have been proposed, each of them has its own strengths and weaknesses; for this reason, up to date there does not appear to be a single "best" edge detector. A perfect edge detector should be able to detect the edge for any type of image and should show higher resistance to noise.

Edge detection techniques are classified as follows: the primary order by-product of selection in image process is that the gradient. The second order derivatives of selection in image process are typically computed exploitation Laplacian [1]. For Sobel, Prewitt & Roberts technique performs finding edges by thresholding the gradient for the log. By default edge perform mechanically computes the edge to use. For Sobel & Prewitt strategies, we are able to opt to discover horizontal edges, vertical edges or each. Laplacian of a Gaussian (LOG) [2, 3] finds edges by searching for zero crossing once filtering with a Gaussian filter. Zero crossing finds edges by searching for Zero crossing once filtering with a user-specified filter [4]. The gradient is calculated exploitation the by-product of a Gaussian filter [5]. The strategy used 2 thresholds to discover sturdy & weak edges, and includes the weak edges within the output provided that they're connected to sturdy edges. Therefore; this technique is a lot of doubtless to discover true weak edges. Sobel edge detector technique is somewhat tough than Prewitt edge detector. Prewitt edge detector technique is slightly easier to implement computationally than the Sobel detector. However it tends to supply somewhat noisier results [1]. that yields a double edge image. Zero crossing edge detector supported same thought because the LOG technique however the convolution, is disbursed employing a nominal filter. Canny [6] proposed a method to counter noise problems from gradient operators, where the image is convolved with the first-order derivatives of Gaussian filter for smoothing in the local gradient direction followed by edge detection by thresholding.

In this proposed work we present a new approach to detect edges of Gray-Scale images based on exponential entropy. The proposed method is decrease the computation time and the results were very good compared with the other methods.

The paper is organized as follows: The related work is presented in section 2. A brief introduction of Exponential entropy is given in Section 3. Section 4 presents details of image thresholding. Illustration of the proposed algorithm applied to gray scale images is presented in Section 4. In Section 5, some particular images will be analyzed using proposed method and moreover, a comparison with some existing methods provided for these images. Conclusions is included in Section 6.

2. Related Work

A large number of studies have been published in the field of image edge detection, which attests to its importance within the field of image processing. Many edge detection algorithms have been proposed, each of which has its own strengths and weaknesses; for this reason, hitherto there does not appear to be a single "best" edge detector. A good edge detector should be able to detect the edge for any type of image and should show higher resistance to noise.

Sobel and Prewitt [1] edge detectors are based on the first order derivative of the pixel intensities. The Laplacian-of-Gaussian (LoG) [1, 2, 3] edge detector is another popular technique, using instead the second order differential operators to detect the location of edges. However, all of these algorithms tend to be sensitive to noise, which is an intrinsically high frequency phenomenon. To solve this problem the Canny [6] edge detector was proposed, which combined a smoothing function with zero crossing based edge detection. Although it is more resilient to noise than the previously mentioned algorithms, its performance is still not satisfactory when the noise level is high. Some of the related work in entropy theory is mentioned. Yang et al. [7] proposed GLGM (Gray-Level & Gradient Magnitude) histogram for thresholding. GLGM histogram employs the Fibonacci quantized gradient magnitude to characterize spatial information effectively. B. Singh and P. Singh [8] suggested Shannon Entropy other than the evaluation of derivatives of the image in detecting edges in gray level images, they used a suitable threshold value to segment the image and achieve the binary image. El-Khamy, S. E. et al. [9] used the relation of the probability partition and the fuzzy 2-partition of the image gradient, the best gradient-threshold is automatically and efficiently selected. The selection algorithm is based on the condition for the entropy to reach a minimum value. Elaraby et al. [10] proposed a new edge detection algorithm based minimum cross entropy thresholding for medical images corrupted with noise. El-Owny [11, 12] presented a new approach to detect edges of gray scale noisy images based on Kapur entropy. Elaraby et al [13] proposed a new algorithm for edge detection of noisy medical images based on both Tsallis and Shannon entropies.

3. Exponential Entropy

In this section we present some basic concepts related to probability theory. Entropy is an uncertainty measure first introduced by Shannon [14, 15, 16] into information theory to describe how much information is contained in a source governed by a probability law.

Entropy is defined in terms of the probabilistic behavior of a source of information. Given events $e1, e2, \ldots\ldots, ez$ occurring with probabilities $\mathrm{p}_1, \mathrm{p}_2, \ldots\ldots\ldots\ldots, \mathrm{p}_\mathrm{z}$ being k the total number of states, $\sum_{i=1}^{z} p_i = 1, 0 \leq \mathrm{p_i} \leq 1$.

Shannon entropy is defined as:

$$S(z) = -\sum_{i=1}^{z} p_i \ln p_{i,} \qquad (1)$$

If we consider that a system can be decomposed in two statistical independent subsystems X and Y, the Shannon entropy has the extensive property (additively) $S(X+Y) = S(X) + S(Y)$, this formalism has been shown to be restricted to the Boltzmann-Gibbs-Shannon (BGS) statistics.

It is to be noted from the logarithmic entropic measure (1) that as $p_i \longrightarrow 0$, its corresponding self-information of this event, $I(p_i) = -\log(p_i) \longrightarrow \infty$ but $I(p_i = 1) = -\log(1) = 0$ and $I(p_i = 0) = -\log(0)$ is not defined. Accordingly we see that self-information of an event has conceptual problem. Practically, the self-information of an event, whether highly probable or highly unlikely, is expected to lie between two finite limits.

Pal and Pal [17] proposed another measure called exponential entropy given by

$$eH(p) = \sum_{i=1}^{n} p_i e^{(1-p_i)} \qquad (2)$$

As authors indicate to some advantages for considering exponential entropy over Shannon's entropy, which is widely acclaimed, we find that the measure of self-information of an event with probability p_i is taken as $log\left(\frac{1}{p_i}\right) = -log(p_i)$, a decreasing function of p_i. The same decreasing character alternatively may be maintained by considering it as a function of (1- p_i) rather than of $(1/p_i)$.

The additive property, which is considered crucial in Shannon's approach, of the self-information function for independent events may not have a strong relevance (impact) in practice in some situations. Alternatively, as in the case of probability law, the joint self-information may be product rather than sum of the self-information in two independent cases.

The above considerations suggest the self-information as an exponential function of (1- p_i) and proposed another measure called exponential entropy.

The authors point out that the exponential entropy has an advantage over Shannon's entropy. For example, for the uniform probability distribution P= $(\frac{1}{n},\frac{1}{n},\dots\dots,\frac{1}{n})$ exponential entropy has a fixed upper bound

$$\lim_{n\to\infty} H\left(\frac{1}{n},\frac{1}{n},\dots\dots,\frac{1}{n}\right) = e-1$$

which is not the case for Shannon's entropy.

4. Image Thresholding

The concept of entropy becomes increasingly important in image processing, since an image can be interpreted as an information source with the probability law given by its image histogram [18-22].

For an image with k gray-levels, let $p_1, p_2, \dots, p_t, p_{t+1}, \dots, p_k$ be the probability distribution for an image with k gray-levels, where p_t is the normalized histogram i.e. $p_t = h_t/(M\times N)$ and h_t is the gray level histogram. From this distribution, we can derive two probability distributions, one for the object (class A) and the other for the background (class B), are shown as follows:

$$p_A: \frac{p_1}{P_A}, \frac{p_2}{P_A}, \dots, \frac{p_t}{P_A} \tag{3}$$

$$p_B: \frac{p_{t+1}}{P_B}, \frac{p_{t+2}}{P_B}, \dots, \frac{p_k}{P_B} \tag{4}$$

where

$P_A = \sum_{i=1}^{t} p_i$, $P_B = \sum_{i=t+1}^{k} p_i$,t is the threshold value.

In terms of the definition of expansion entropy, the entropy of Object pixels and the entropy of Background pixels can be defined as follows:

$$eH^O(p) = \sum_{i=1}^{t} p_i e^{(1-p_i)} - 1 \tag{5}$$

$$eH^B(p) = \sum_{i=t+1}^{k} p_i e^{(1-p_i)} - 1 \tag{6}$$

The Expansion entropy $eH(t)$ is parametrically dependent upon the threshold value (t) for the object and background. It is formulated as the sum each entropy, allowing the pseudo-additive property for statistically independent systems. We try to maximize the information measure between the two classes (object and background). When $eH(t)$ is maximized, the luminance level t that maximizes the function is considered to be the optimum threshold value. This can be achieved with a cheap computational effort.

$$t^{opt} = \text{Arg}\max[eH^O(t) + eH^B(t)]. \tag{7}$$

5. Proposed Algorithm

A spatial filter mask may be defined as a matrix w of size $m\times n$. So, we will use the usual masks for detecting the edges. The process of spatial filtering consists simply of moving a filter mask w of order $m\times n$ from point to point in an image. At each point (x,y), the response of the filter at that point is calculated a predefined relationship. Assume that $m = 2a+1$ and $n = 2b+1$, where a, b are nonnegative integers. For this purpose, smallest meaningful size of the mask is 3×3, as shown in figure 1. Image region under the above mask is shown in figure 2.

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(-0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

Figure 1. *Mask coefficients showing coordinate arrangement.*

$f(x-1,y-1)$	$f(x-1,y)$	$f(x-1,y+1)$
$f(x,y-1)$	$f(x,y)$	$f(x,y+1)$
$f(x+1,y-1)$	$f(x+1,y)$	$f(x+1,y+1)$

Figure 2. *Image region under the mask.*

In order to edge detection, firstly classification of all pixels that satisfy the criterion of homogeneousness, and detection of all pixels on the borders between different homogeneous areas. In the proposed scheme, first create a binary image by choosing a suitable threshold value using Tsallis entropy. Window is applied on the binary image. Set all window coefficients equal to 1 except center, center equal to × as shown in Figure 3.

1	1	1
1	×	1
1	1	1

Figure 3. *Window that applied on the binary image.*

Move the window on the whole binary image and find the probability of each central pixel of image under the window. Then, the entropy of each Central Pixel of image under the window is calculated as:

$$H(\text{CP}) = -p_c \ln(p_c).$$

where, p_c is the probability of central pixel CP of binary image under the window. When the probability of central pixel $p_c = 1$ then the entropy of this pixel is zero. Thus, if the gray level of all pixels under the window homogeneous, then $p_c = 1$ and $H = 0$. In this case, the central pixel is not an edge pixel. Other possibilities of entropy of central pixel under window are shown in Table 1.

Table 1. *Entropy of central pixel under window.*

p	***H***
1/9	0.2441
2/9	0.3342
3/9	0.3662
4/9	0.3604
5/9	0.3265
6/9	0.2703
7/9	0.1955
8/9	0.1047

In cases $p_c = 8/9$, and $p_c = 7/9$, the diversity for gray level of pixels under the window is low. So, in these cases, central pixel is not an edge pixel. In remaining cases, $p_c \leq 6/9$, the diversity for gray level of pixels under the window is high. So, for these cases, central pixel is an edge pixel. Thus, the central pixel with entropy greater than and equal to 0.2441 is an edge pixel, otherwise not.

The following Algorithm summarize the proposed technique.

Algorithm: Edge Detection

1. Input: A gray scale image I of size $M \times N$ and t^{opt}, that has been calculated from equation (7).
2. Create a binary image: For all x, y,
   $If\, I(x,y) \leq t^{opt} then\; f(x,y) = 0\; else f(x,y) = 1$.
3. Create a mask w of order $m \times n$, in our case ($m = 3, n = 3$)
4. Create an $M \times N$ output image g: For all x and y, Set $g(x,y) = f(x,y)$.
5. Checking for edge pixels:
   Calculate: $a = (m-1)/2$ and $b = (n-1)/2$.
   For all $y \in \{b+1, \ldots, N-b\}$, and $x \in \{a+1, \ldots, M-a\}$, $sum = 0$;
   For all $l \in \{-b, \ldots, b\}$,and $j \in \{-a, \ldots, a\}$,
   $if\,(f(x,y) = f(x+j, y+l))\; then\; sum = sum + 1$.
   $if\,(sum > 6)\; then\; g(x,y) = 0\; else\; g(x,y) = 1$
6. Output: The edge detection image g of I.

The steps of our proposed technique are as follows:

Step 1: Find global threshold value (t_1) using Expansion Entropy, the image is segmented by t_1 into two parts, the object (Part1) and the background (Part2).

Step 2:Select the locals threshold values (t_2) and (t_3) for Part1 and Part2, respectively.

Step 3: Applying edge detection procedure with threshold values t_1,t_2 and t_3.

Step 4: Merge the resultant images of step 3 in final output edge image.

In order to reduce the run time of the proposed algorithm, we make the following steps:

1) We are use the linear array p (probability distribution) rather than I, for segmentation operation, and threshold values computationt_1,t_2 and t_3, because the run time of arithmetic operations is very much on the $M \times N$ big digital image I, and its two separated regions, Part1 and Part2.
2) We are create one binary matrix f according to threshold values t_1 , t_2 and t_3 together, then apply the edge detector procedure one time then merge the resultant images into one, rather than create many binary matrices f and apply the edge detector procedure for each region individually.

This modifications will reduce the run time of computations.

6. Experimental Results

The proposed approach is tested over a number of different gray scale images and compared with traditional operators to demonstrate the efficiency of our algorithm. The images detected by Canny, LOG, Sobel, and the proposed method, respectively. All the concerned experiments were implemented on Intel(R) Core(TM) 2 Duo 2.20GHz with 3 GB RAM using MATLAB R2012b without pre-processing. As the algorithm has two main phases global and local enhancement phase of the threshold values and detection phase, we present the final results of implementation on these images.

The proposed scheme used the good characters of exponential entropy, to calculate the global and local threshold values. Hence, we ensure that the proposed scheme done better than the traditional methods.

In order to validate the results, we run the Canny, LOG, Sobel and the proposed algorithm 10 times for each image with different sizes. As shown in Figure. 4. It has been observed that the proposed edge detector works effectively for different gray scale digital images as compare to the run time of other methods.

Some selected results of edge detections for these test images using the classical methods and proposed scheme are shown in Figures 5 and 6. From the results; it has again been observed that the performance of the proposed method works well as compare to the performance of the previous methods (with default parameters in MATLAB).

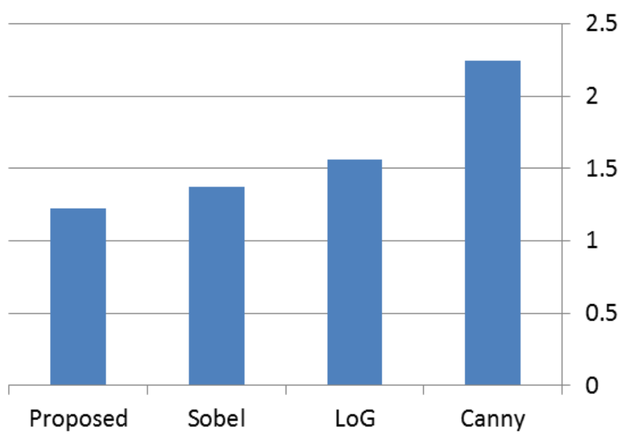

Figure 4. *Chart time for proposed method and classical methods.*

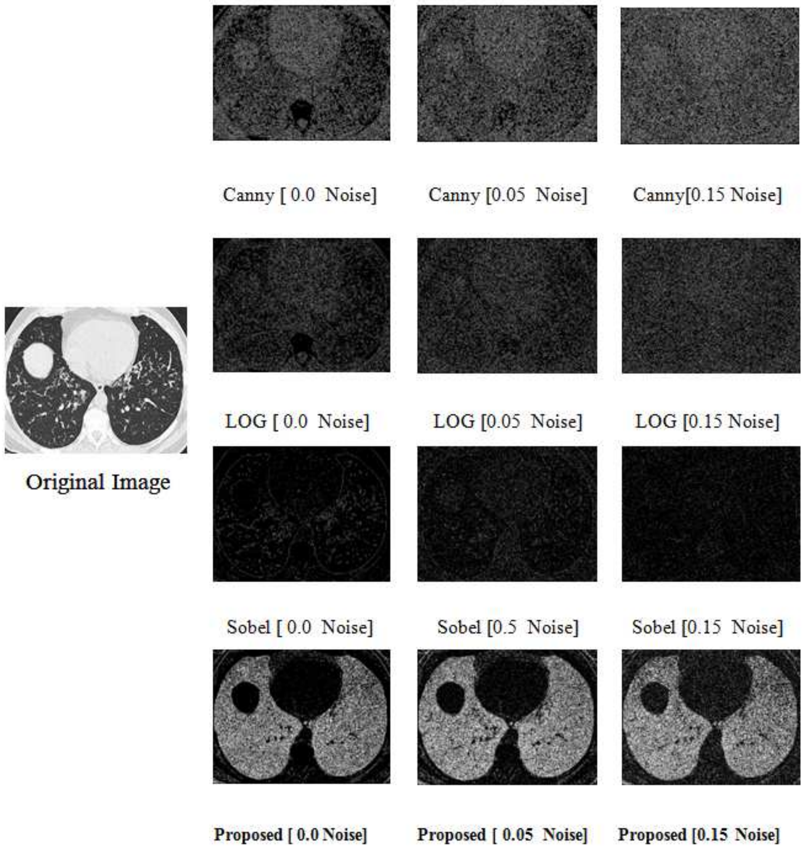

Figure 5. *Performance of Proposed Edge Detector for Lung image.*

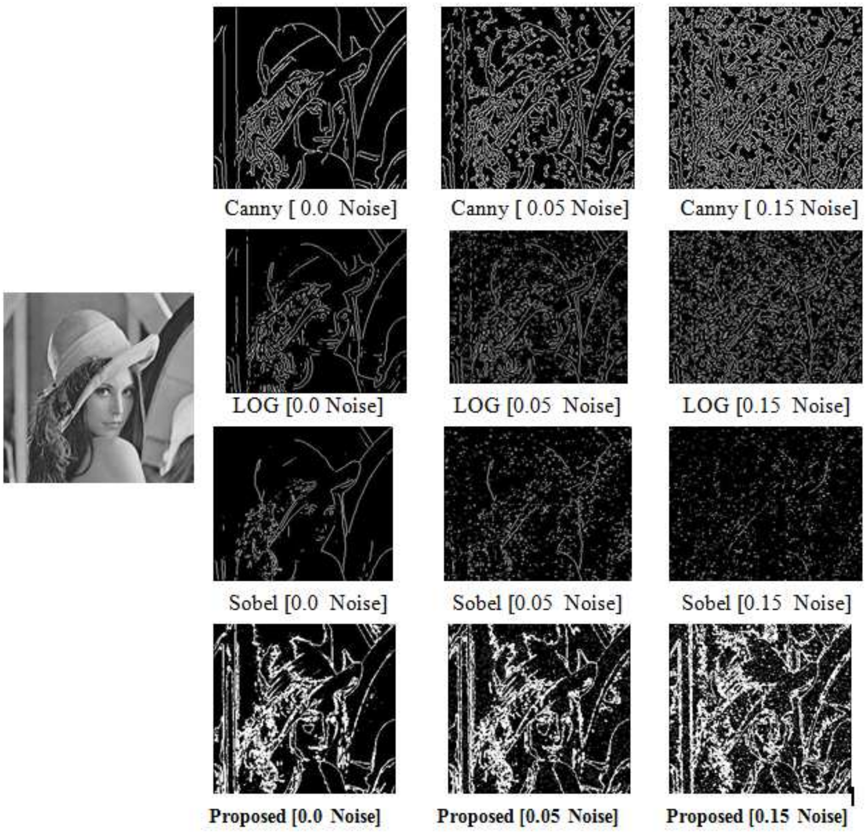

Figure 6. *Performance of Proposed Edge Detector for Lena image.*

7. Conclusion

An efficient approach using Exponential entropy for detection of edges in gray scale images is presented in this paper. The proposed method is compared with traditional edge detectors. On the basis of visual perception and edge counts of edge maps of various gray scale images it is proved that our algorithm is able to detect highest edge pixels in images. The proposed method decrease the computation time with generate high quality of edge detection. Also it gives smooth and thin edges without distorting the shape of images. Another benefit comes from easy implementation of this method.

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