



# Application of Reduced Second Order Response Surface Model of Convex Optimization in Paper Producing Industries

Casmir Chidiebere Onyeneke, Effanga Okon Effanga

Department of Mathematics/Statistics and Computer Science, University of Calabar, Calabar, Nigeria

## Email address:

casdyx@yahoo.com (C. C. Onyeneke), coeffanga@yahoo.com (E. O. Effanga)

## To cite this article:

Casmir Chidiebere Onyeneke, Effanga Okon Effanga. Application of Reduced Second Order Response Surface Model of Convex Optimization in Paper Producing Industries. *International Journal of Theoretical and Applied Mathematics*. Vol. 2, No. 1, 2016, pp. 13-23. doi: 10.11648/j.ijtam.20160201.13

**Received:** July 25, 2016; **Accepted:** September 10, 2016; **Published:** October 17, 2016

---

**Abstract:** We are in the middle of an international financial crisis, which intensifies the demands on high quality, reliable and efficient solutions that optimize production processes, making production more efficient while minimizing cost, produce more with high quality, with few raw materials and less energy. It is these circumstances that necessitates the use of response surface methodology to search for the optimal conditions for improving grinding process in case of convex situations in paper producing industries. The uniqueness of this work focused on modeling and adopting the necessary assumptions and conditions to further reduced the formulated second order response surface model to obtain a more adequate model that best optimized the production process. The design was based on the use of central composite rotatable approach known as CCRD with the grinding fineness as the response. The conditions were subjected to experimental method, search method, graphical method and feasible region approach to generate the result which is not significantly different from each other using the reduced model. We could established nine grinding conditions which involves one center points with four factorial and four axial points. The reduced second order response surface model was optimized to obtain the best grinding at machine voltage of two hundred within fifty minutes. It is on this condition that the response variable gave the value of 1399.36 meshes.

**Keywords:** Optimization, Response Surface, Differentiable, Convexity, Models, Optimal Solution, Lack-of-Fit, Design, Experiment, Analysis, Maximization, Quadratic, Contour Plot

---

## 1. Introduction

The words of a famous decision maker made us to understand that “ultimate goal of all decisions is to either minimize the efforts required or maximize the desired benefits” [1]. The conditions for improving grinding process represent a method that can improve the fineness of grinded calcite and barite by optimizing the machine voltage and time in the paper producing industries. The applied process varies in machine voltage and grinding time measured in volts and minutes respectively. It became obvious that the level of grinding acceptability is a function of the grinding time and machine adequate voltage regulation. In other words, the level of fineness and grinding acceptability are significantly influenced by the voltage of grinding machine and the grinding time. Grinding at low voltage may decrease the fine quality and consequently may lead to the production of

inferior paper and undesirable effects such as low quality and dirt. Grinding time affects the productivity of grinding process. Short period of grinding may decrease the productivity whereas long period may increase the production process.

In order to explore for the best possible level of voltage and grinding time, response surface methodology (RSM) was a very practical approach to find, as maximum as possible, the fineness. RSM enables the assessment of varied impact of complex effects on dependent variables as result of many interactions on independent and factored variables. This implies the advantages of using RSM are reported to be the reduction in the number of experimental runs needed to evaluate multiple variables, and the ability of the statistical tool to identify interactions [2]. Hence, it is less laborious and saves time compared to one-variable at a-time [3]. The Central Composite Rotatable Design (CCRD) of RSM was

applied as analytical techniques of improving and optimizing method for the grinding process and maximization of fineness of grinding industries.

Considering the relevance of this statistical tool in design, analysis and interpretation of statistical experiments, it is regarded as vital tool employ for the collection of mathematical and statistical techniques useful for modeling and analyzing problems in which a response of interest is influenced by several variables and the objective is to optimize this response [4]. As an instance, the degree of fineness in the paper producing industries is a function of positive quantity of machine voltage  $x_1$  with respect to grinding time  $x_2$ . In effort to obtain desired grinding quality, treatments  $x_1$  and  $x_2$  must be optimally combined. In this case, machine voltage and grinding time are continuous variables. It is worthy to note that continuous treatments are more appropriate in emergent, developing and approximating the target variable. This research captures the rate of fine process  $y$  as the objective variable and it is determined by machine voltage and grinding time. Its mathematical expression is given as

$$y = f(x_1, x_2) + e \tag{1}$$

The variables  $x_1$  and  $x_2$  are independent variables [5].

$$y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \beta_{11}x_{11}^2 + \beta_{22}x_{22}^2 + \beta_{12}x_1x_2 + \varepsilon \tag{3}$$

In fact, wide-range of RSM objectives considers utilizing the application either the combination of the two models or any one of them. Each and every one of the levels of the models is not dependent on the stages of co-existing factors. However, higher other models can be accepted when it fits and also deemed suitable for approximation and optimization of response surface. It is advisable to use higher other models especially when they are efficient enough to produce an appropriate approximated result suitable for further decision and forecast of other responses. At this point, what facilitates the competent and efficient of the approximated response surface model is the appropriate method of data collection and presentation used. When the right techniques are implemented to the design of experiment, adequate date capable of analyzing and predicting results will be obtained by diligently following the method of Least-Squares for parameter estimates in the polynomials.

The analysis of response surface is carried out by using an integral function of the surface plot. Since the design is planned to execute tasks associated with appropriate fitting of response surface, there is no gainsaying the fact that the object of using Response Surface Methodology cannot be overemphasize. These objectives can easily be accomplished by

- Viewing and taking recognitions of scenery of the response pattern in form of confined utmost, neighboring bare minimum and edge ranks.
- Also ensure the area where the best possible result occurs is identified. In other to achieve the most powerful and quick high or low response, substantial

The response  $y$  is a dependent variable and  $e$  is the experimental miscalculation expression (error) which is assumed to be normally distributed with zero mean as measure of central tendency with variance,  $\sigma^2$  as measure of dispersion.

The true objective function  $f$  is non determinant in most cases of Response Surface Methodology problems. It is this that gave rise to the use of approximation process. In attempt to get a defined and appropriate approximation of  $f$ , there is need to launch a lower order polynomial in minor region. This order is called a simple or linear first order response surface. If it adequate enough to define and provide sufficient information for the target objective and can also be determined by the independent variables, then the linear function is adopted as the first order model and also seen as the optimizing function. The equation algebraically expressed the liner RSM-model as

$$y = \beta_0 + \beta_1x_1 + \beta_2x_2 + \varepsilon \tag{2}$$

When the response surface is wrapped due to curved nature, we proceed to use a higher degree polynomial. Then due to the fact that the first order model cannot account for curvatures that exist in researches; the second order model is often preferred (see, [6]).

effort is required to optimize the process.

## 2. Research Methodology

Most of the previous studies reviewed in cause of this work emphasized much on the relevance of applying response surface methodology in every facet of the society. It is used in industries, government, ministries, militaries, education, planning, business development, agriculture, project management and many more. It enables researchers in collecting, describing, analyzing and interpreting scientific process and products for the purpose of attaining optimum solution. Response-Surface-Methodology is mostly applied in production industries, engineering, biological and physical sciences, environmental resource managements, social sciences, food science and technologies and crop farming. In the view of [7], since RSM has an extensive application in the real-world, it is also important to know how and where Response Surface Method of optimization started in the history. It was during the research of Box and Wilson that RS-methodology was invented for optimization and decision making. It was then that the almighty assertion was made, “it is more appropriate to use first degree polynomials to estimate the response indices” [8]. Here, it is worthy to note that the first order polynomials are nothing but an approximating function. It is never the accurate solution. However, the model is easy to apply during estimation even when no much information is acquired about the generating process. Furthermore, RSM was first used in 1930s when the response surface curves were mostly used to determine levels

of optimum.

It was in the research of [9], the orthogonal design was motivated by Box and Wilson in the case of the first-order model. Since then, many scientist and researchers especially those in the field of engineering have developed a huge knowledge of the use and application of Central Composite Designs and three-factored design methods of analysis and optimization. As the research continued, many more cost-effective and sufficient designs have been discovered. Since then, lots of research works have been reported on design and analysis of experiments which have huge link in optimization and response surface methods. One of the mostly used that has little work and stress during its application is the three-factorial level method of design. Its work is limited and open research fractional design on the objectives. It was stated categorically in the research and analysis done by [10], experiment design for Factor at three levels is a helpful resource conducting this kind of design. Many more models of the three factorial design models and their alias were represented in the chats of their works.

The importance of the development of optimal design theory in the field of experimental design emerged following World War II. This was a result of the need to fully utilize the technological challenges facing the military administration and the world at large during the time. It was through the application of operations research in the form of process and product optimization that many lives and properties were saved from hunger, starvation and danger. This was also revealed in the work of [11], optimum conditions comprises of maximum or minimum function obtainable to satisfy the objective of the research. He was one of the various authors who published their work on optimality. Though [12], established the optimum level in conditions of applying edible coating emission on guava using response surface methodology, little or no application of Response Surface Methodology was observed in manufacturing and production industries. In Nigeria and similar developing countries, there is a need to adopt the optimum use of RSM in all areas of the economy to attain the most favorable decision for the betterment of the countries' economy.

In industrial optimization, it is worthy to identify the saddle, maximum and minimum points of the system. The most interested point is to note the pattern which the system assumes, say, saddle, minimum or a maximum point. As this interest arises in manufacturing and production industries, RSM becomes the most popular tool used to describe and analyze the systems. Increasingly, many research works have been conducted in chemical producing industries to ascertain the best attainable regions in the process based on the analysis of the optimum response. It is now obvious that the application and development of RS-methodology spreads across all areas of life and still remains the best tool for further prediction and decisions making.

Designs for fitting Response Surface Models have various approaches and categories. The range starts from the first order model which is applied whenever the description of a flat surface is needed. One of the major assumptions of this

model is that the plane exterior is probably stylish. The model of first-order response surface may not be applicable in the analysis of the maximum, minimum and ridge outlines. The most appropriate time to use the first-order model is when there is no curvature in the surface region. Also when the region is not too big, the approximating function  $f$  becomes more suitable for analysis and optimization of the linear objective function. In order words, first order response surface model is considered to be more suitable for the approximation and optimization of the factual plane in diminutive area  $x$ 's. Our motivation at this point is to check the suitability of the first-order response surface model in optimizing the grinding process in the selected industries. If it does not, we proceed to second order model. The data gave a set of observation collected during the grinding experiment conducted in the optimization of grinding conditions in Cassa Paper Industries. We studied the effects of grinding time ( $t$ ) and machine voltage ( $V$ ) on the optimization of grinding process through fine quality of grinded calcite and barite powder when grinded with Zenith powder grinding machine.

We studied this real-life experiment to enable us apply response surface methods on two continues treatments of independent response variables. This Case Study also allowed us to demonstrate when first-order model is adequate to the given data versus when it is not. With this respect, it is essential to illustrate a first-order design. The situation of the work together with the viability of this case study enables us to apply and analyze the two cases of response surface models. The first case was to show when the first order response surface model was appropriate to further decision making and when it is not. Also to proffer appropriate assumptions that guides every operations research experimenter on what to do when the first order-model fails to show adequate approximation for a given model. In order to fully understand the serene of this research, it is better to first illustrate the first order experimental design.

Among all the available designs used for fitting and demonstrating response surface, the Central Composite Design remains the most widely used in modeling and analysis of the experiments. It was invented by one of the famous scientist and it is made up of factorial points (Box et al 2005). The Central Composite Design (CCD) consists of the central point, plus or minus axial points and the factorial points which ranges from  $2^n$  designs and  $2^{n-k}$  incomplete designs. The  $2^n$  axial point is represented as follows

$$\begin{pmatrix} x_1 & x_2 & \dots & x_q \\ -a & 0 & \dots & 0 \\ a & 0 & \dots & 0 \\ 0 & -a & \dots & 0 \\ 0 & a & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ \cdot & \cdot & \dots & \cdot \\ 0 & 0 & \dots & -a \\ 0 & 0 & \dots & a \end{pmatrix}$$

Central Composite Design experiment is mostly derived a repeated order of experiment with definite pattern of layout. The number of axial points can easily been adjusted when

there exist a lack of fit in the first order model. The essence of doing this is to form a curvature which results to the second order quadratic model. However, when there subsists proof of lack-of-fit in the first order model, axial points can be added to the quadratic terms with more center point to form a complete Central Composite Design. In this case, the distance between the axial and the design center point is denoted by  $a$  while  $n_c$  represents the number of center points from the origin. Therefore, the two parameters in Central Composite Design are  $a$  and  $n_c$  standing distance of the axial and number of the center points respectively. The information about the curvature of the design in captured and taken care of by the centre runs introduced during the design of the experiment. In case of a significant curvature, the added axial points then enable the experimenter to attain a competent inference of the quadratic expressions.

There are many ways of selecting the number of center points and the axial points ( $n_c$  and  $a$ ). The first approach is to run the central composite design in fractional blocks. A set of contiguous and relatively identical trial situations that enables a researcher to gather data that perform same action as a unit is define as a block. It is through these blocks that an experimenter partitions the observations into groups and carries out the research in the individual groups. This helps to determine the level of dispersion and variability that exist in each axial, factorial and number of center points of the groups. We used Figure 3 to demonstrate a practical view of Central Composite Design where  $q$  represented 2 factors. In order to obtain a reliable and the most efficient result, one may consider the adoption of incomplete block design during the design and analysis of the experiment. When all treatment experiment cannot be run in a single block, there is an alternative way which considers splitting the blocks into groups. The block effect is expected to describe a set of axes all at right angle to each other in order to maintain the contour of the response surface. When this happens, we say that the treatment effect is orthogonal to the block effect. The question now is how to achieve this orthogonality in a Central Composite Design experiment. This can be obtained by chosen the right center point, axial point and factorial point especially in the case of disintegrated and axial layout.

Furthermore, the axial and number of center points that is,  $a$  and  $n_c$ , are selected to ensure there is no blockage on the Central Composite Design pattern. Now, we considered a case where the design is rotatable. When an origin is introduced in form of the center point and the accuracy of the approximated response plane at some points  $x$  is dependent only on the distance from the origin then we called the design a Rotatable Central Composite Design. When the design is repeated about the center on a rotatable way, the variance  $\sigma$  of  $y$  estimate is constant. It is obvious that every operation researcher seeks for the attainment of the unknown optimum point. As a result, it is more reliable to make use of the rotatable design with equal exactitude. When this is achieved, the response surface analysis estimates the points in all directions. The Central Composite Design (CCD) becomes rotatable based on the choice of the axial points. It was stated

by [13], “In order to obtain a full factorial design, the experimenter uses  $a = 2^{q/4}$  while  $a = 2^{(q-k)/4}$  is used for a fractional factorial research”. We used the Rotatable Central Composite Designs know as the RCCD of RS-Methodology and the paper mesh as the response to generate nine grinding conditions. These conditions are

- a) Four factorial points
- b) Four axial points
- c) And one center point.

Each and every one of the axial and factorial points was varied over positive and negative alpha and distance from the one center point. On the basis of this set-up, we could generate 33 runs of experimental conditions.

It is more appropriate and easier to use coded values instead of the actual values. Therefore, we coded the actual values by simplifying and reducing them into (-1.41, 1.41) distance. We recalled the treatment factors into the range of  $\pm 1.41$ . In order to simplify the calculation, it is appropriate to use coded variables for describing independent variable where zero is the center of the design. The direct variables (V and T) are called the natural variable because they assumed the real values of voltage and time correspondingly. Therefore, we followed the following procedure to convert the normal erratic values to coded variables.

$$x_i = \frac{V_i - 220}{20} \quad y_i = \frac{T_i - 30}{20} \quad (4)$$

Even though the ANOVA can be generated through the computer software known as the Design-Expert 8.0.7.1, we still saw reasons to manually analyze and compare our results with that of the computer software. There are several other computer soft-wares such as the SPSS, Minitab, and other, which we used to compare our results for efficiency and sufficiency. It is imperative to illustrate the approach used to calculate and analyze the data. It is worthy to note the in analyzing the response surface, the first stage is to approximate the parameters of the response surface model. The method of least square is very appropriate for the parameter estimate of the regression and to gather information on how fit the estimated model is to be used in forecasting and further decision making. One of the major characteristics of the predictable response surface is that it is usually curved with a hill also called the crest. This is seen at the unique estimated of highest response. The estimated surface also contains a unique optimum point called saddle surface or a trough with no unique upper or lower limit.

We further decided to use the Analysis of Variance Table to examine the effect and role of each and every one of the linear, quadratic and cross models. The essence was to check and ascertain the model that is most statistically fit for the approximation of the response pattern. It also helped us to analyze the outstanding error that is responsible for Lack-of-Fit (LOF) and to check for the model that sufficiently represented the true response surface. The effect of each factor is inconsistent when compared to the level of statistical accuracy measure. It is also by the aid of Analysis of Variance that the experiment can be adequately predictable

when the varied factor is removed where  $F_{cal}$  = calculated value,  $Q$  = observations and  $b$  = number of independent variables ANOVA table enables the goodness of fit of a test to be checked. The quotient of the Average (Mean) Sum of Squares of regression model (MSM) and Average (Mean) Sum of Squares of Error (MSE) give the value of F calculated.

$$SSE = SST - SSM$$

$$SSO = y'y - \frac{1}{n} \left( \sum_{i=0}^n y_i \right)^2$$

and

$$SSM = \hat{B}'X'y - \frac{1}{n} \left( \sum_{i=0}^n y_i \right)^2$$

$$SSE = y'y - \frac{1}{n} \left( \sum_{i=0}^n y_i \right)^2 - \left[ \hat{B}'X'y - \frac{1}{n} \left( \sum_{i=0}^n y_i \right)^2 \right]$$

implies  $SSE = y'y - \hat{B}'X'y$ . Where the  $SSE$  is the difference between the deviation from the total sum of squared error and the deviation from the between error sum of squares.

Using the significant approach, we could reexamine and compared how suitable the regression model could fit the analysis, interpretation and optimization of the entire model. The replicated observations were used to compare the variability that exists among within the regression model with pure variation within replicated experimental conditions. We used this to measure the reliability of the quadratic model when linear models do not satisfy the necessary assumptions needed for adequate approximation of the objective function. Let us consider the case of replicated observations that ranges in  $n_i$  or  $n_j$  with the respective response values of  $Y_{i1}, \dots, Y_{inj}$  each occurring at the same value  $x_i$  factors, it makes the further forecasting of  $x_i$  possible. This is done in two ways: the first approach is by estimating the significant of  $\hat{Y}_i$  using the model parameter of the formulated response model while the second method is by applying the mean value  $\bar{y}_i$  of the simulated quantities. It is possible to break the lack of fit residual error into components of pure error which is the deviations from the mean. The next component is the variation of the individual replicates around the predicted model called error due to bias. The partition of the various components of the lack of fit error is illustrated below

$$MaxY = 54090.41 - 464.109x_1 - 23.1095x_2 + 1.003125x_1^2 + 0.464453x_2^2$$

$$\frac{dy}{dx_1} = -464.109 + 2.006x_1$$

$$\frac{dy}{dx_2} = -23.109 + 0.928x_2$$

$$\sum_i \sum_j^{n_i} (Y_{ij} - \hat{Y}_i)^2 = \sum_i \sum_j^{n_i} (Y_{ij} - \bar{Y}_i)^2 + \sum_i n_i (Y_{ij} - \hat{Y}_i)^2$$

where the first part of the equation represents sum of squared overall (SSO) which gives the summation of squared regression (SSR) and sum of squared errors (SSE) respectively.

Significant lack of fit occurs when pure error is less than the bias error. A model is considered to be adequate in a situation where both components estimate the supposed level of error.

When the second derivative of a function is not negative, the function is considered to be convex only on the condition that it can be differentiated up to two times. This is typically used to check if a function is convex. Diagrammatically, a convex function has upward curvature (curves up) with no crook if and if only it is twice differentiable. This gives a practical test for convexity. The infection continuous to climb until it reaches the apex (maximum point) before it starts to climb down the hill. When it is differentiated up twice, the value obtained must be positive at every single peak before it is considered to be a strict convex function.

In wide situation, a set of continuous function is considered a convex set when the result of the function gives a Hessian matrix which is non negative. This is practically applied in the case of multiple variables that satisfy the condition of being positive semi-definite or entirely positive at all the non exterior points is considered to be a convex set. In this section, test for convexity of the objective function was carried out before the actual optimization process to know if the approximated function is convex.

Since we were walking on a quadratic model, we decided to test for convexity of the approximated reduced second-order (quadratic) response surface model. The essence was to ensure we did not differentiate and substitute the values in order to get the optimum value. If we do, we would only succeed in get the minimum process, which was not our objective. Visualizing the sketch on figure 1, it was observed to be convex, in such case, any movement from the minimum point  $x^*$  gives a value greater than that of  $x^*$  in the objective function.

If the  $f(x_1, x_2)$  is concave, then the  $maxf(x_1, x_2)$  occurs at the point where

$$\frac{dy}{dx_1} = 0, \frac{dy}{dx_2} = 0$$

From the Hessian-matrix

$$H = \begin{bmatrix} 2.006 & 0 \\ 0 & 0.928 \end{bmatrix}$$

$$|H| = 1.86$$

Since the Hessian-matrix is not negative, there is strong existence of convexity.

Since  $f(x_1, x_2)$  is convex then we examine the following theorem

Theorem 3.1 If  $F(x)$  is convex in the interval

$$a \leq x \leq b$$

then  $\max F(x) = F(a)$  or  $\max F(x) = F(b)$ .

Proof. Let  $x > x^* \Rightarrow F(x) > F(x^*)$

and  $x < x^* \Rightarrow F(x) > F(x^*)$ .

But  $b > x^* \Rightarrow F(b) > F(x^*)$

for  $a < x^* \Rightarrow F(a) > F(x^*)$ .

If  $F(x) \leq F(b) \forall x > x^*$  then  $\max F(x) = F(b)$ .

But if  $F(x) \leq F(a) \forall x < x^*$  then  $\max F(x) = F(a)$ ,

where  $x^*$  = minimum value and  $F(x^*)$  = value of  $x^*$  on the objective function.

Let  $x < x^* \Rightarrow F(x) > F(x^*)$

If  $F(x) \leq F(b) \forall x > x^*$  then  $\max F(x) = F(b)$

But if  $F(x) \leq F(a) \forall x < x^*$  then  $\max F(x) = F(a)$

Now, let  $x^*$  be the minimum point of  $F(x)$ .

That is  $f(x^*) \leq F(x)$ , for  $\forall a \leq x \leq b$

$$F(x) \geq f(x^*) \forall x \geq x^*$$

or  $F(x) \geq f(x^*) \forall x \leq x^*$ .

But if  $F(x) \leq f(b), \forall x \geq x^* \Rightarrow \max F(x) = F(b)$

or if  $F(x) \leq f(a), \forall x \leq x^* \Rightarrow \max F(x) = F(a)$ .

From the above analysis, we could come up with the following three conclusions

(i) If  $F(a) = F(b) = Z$ , then  $\max F(x) = Z$  where  $Z$  = the optimum level

(ii) If  $F(a) > F(b)$ , then  $\max F(x) = F(a)$

(iii) If  $F(a) < F(b)$ , then  $\max F(x) = F(b)$ .

From the forgoing, we can state the following Corollary by generalizing the above conditions. If  $F(x)$  is convex in the domain  $D(x) = \{x: Ax \leq b\}$ ,  $\max F(x)$  occurs at the boundaries or corner points.

### 3. Results and Discussion

Model adequacy was very necessary in the approximation and optimization of the objective function. "It is used to measure the strength of association among variables; the impacts of independent variables are studied simultaneously to check the variation effect" [14]. It is not good to use insufficient experimental design to represent a response surface model. We tested if the model and experiment followed a normal distribution. Also, we examined the distribution of the error terms because when the errors are normally distributed, the design is said to be unbiased [14]. We did some variance and regression to ascertain if there exists any lack of fit in the model. When the model is not fit, the analysis will show non-significant lack of fit and that means the model is not adequate enough for further decisions [15]. A popular statistical software used for design and analysis of experiment (Design-Expert version 8.0.7.1) was employed to carry out both regression and variance analysis of the models. When we critically examined the model and obtained the f-value of 83.29 which indicated significant model parameters. The model error of zero percent is the

probability of the model F-value which occurred as a result of obstructions in the experiment. The amount of "Prob > F < 0.05" indicates significant model terms. The linear analysis done showed that model parameters for machine voltage and grinding time are significant in the approximated model. When the (Prob>F) is greater than 0.1, there is an evidence that the model terms are not present. These model terms that are not significant add to reduce the power of the design. However, there are cases where model reduction helps to improve the estimated model if there are many non significant models that are not used to maintain the pecking order. The analysis revealed "Lack of Fit F-value" and entails lack of efficiency. It is the aim of every researcher to formulate a model that fits and represent the true pattern of the situation. As a result any model that shows presence of lack of fit is considered to be a inadequate for further optimizations purpose.

The predicted residual squared has a value of 0.8206 which is in realistic conformity with the adjusted residual squared of value 0.8372. Adequate precision determines the indicator to interaction relative amount. The relative yearning ratio is expected to be greater than four where V stands for variation. Our relative amount of 20.95 designates a satisfactory indicator of high exactitude. The linear model may considered to be relevant when a researcher wished to take the helm of the experimental space but it is not adequate to adopt in optimization processes such as the grinding excellence in paper producing industries. We described the low and high confidence interval, Variance of Indecent Factors (VIF) and coefficient estimate. The VIF of one showed a desirable effect. It indicated the presence of one independent effect where  $\alpha$  represents the intercept. In the first model, there was only one independent variable which was represented as the grinding process in terms of the mesh value of the grinded calcite and barite of the paper producing industries. The coefficients of the respective model parameters were further subjected into lack of fit test. This explains how sufficient, efficient and unbiased the estimated model could explain and truly represent the design and true nature of the experimental region. Suitable variance of independent factor is expected to be one. The analysis could give a true and desirable variance of independent factor of one. The following equations are generated for the model. They are for the coded and actual factors respectively. It is a point of notice that both equations give true representation of the approximated linear model for the design of the experiment.

Coded Equation:

$$Y = 831.74 - 454.67x_1 + 95.15x_2 \quad (5)$$

Actual Equation:

$$Y = 5690.39 - 22.73x_1 + 4.76x_2 \quad (6)$$

Generally, it is expected that a fit model should be able to adequately approximate the true situation in the model form. Any regression model that explains the standard errors of the

dependent and independent variables in a model is considered to be normal. There are assumptions which every researcher should have in mind while formulating, analyzing and interpreting any estimated regression model. The most common but highly effective one is that the standard error terms  $e_i$ 's should be independently, identically and normally distributed. The next is the mean and variance of the  $e_i$ 's must be zero and  $s^2$  respectively.

Since the model shows presence of unfit model, the linear model is considered inappropriate for further decisions. When there is indication of explained lack-of fit it shows that the model is not adequate for further analysis of optimal condition of grinding process in the industries. Therefore we decided to further the analysis with second order response surface also known as quadratic model.

#### *Second-Order Response Surface Analysis*

We stated earlier that the first order response surface model does not adequately approximate the true model in cases of curved or warped situations. On the basis of the analysis done earlier, it was obvious that the linear model is insufficient enough for the optimization process. We therefore decided to divert to the use of the quadratic model since there is significant lack of fit in the linear model. The parabolic curve of the second order response surface helped us to detect the central unite of the experiment. When we examined analytically the model  $F_{value}$  and discovered that the value of 2,265.87 indicated a significant level which is a huge evidence that there is presence of quadratic model which suitable for the design. Using the  $P_{value}$ , it shows that there exist a probability value of 0.01 percent for "Model F-Value" occurring as a result of obstructions in terms of error. In the analysis, any value greater than 0.05 implies a significant model term.

The parameters for machine voltage, grinding time and their respective parabolic terms were significant. The interaction effect between the machine voltage and grinding time is 0.4046 which is greater than the generally accepted region of for significant models. Therefore, we considered interactive effect as a non significant term which implies that it has no effect in the optimization process. The grinding voltage and time are independently distributed. This incident reflected our mind to the assumption that the instructions of non significant model terms in the approximation process adversely effect the objective function. So, discarding the interactive effect may help to improve our model as far as it is not present for hierarchy measures. Considering the "Lack of Fit F-value" of 1.43, a clear conclusion was drawn which holds that Lack of Fit is not significant compared to the unadulterated variation. The 25.88 percent probability is the

rate by which the lack of fit occurred due to random distribution of errors. This desired status of lack of fit was desirable, we wanted the model to fit where  $M_{value}$  represented the mean value and NA is used to represent the terms that are not applicable.

We noted that situations with much influence of one implies the predicted residual squared and predicted residual error sum of square values are not defined. Still on the cross examination of the suitability of the model, we observed that the adequate precision value of 43.737 measures the signals to noise proportion. As stated earlier, we cherish a ratio not smaller than four. It is clear that our ratio 43.737 is greater than four and it is acceptable because it shows adequate signal. Therefore, this model can be adopted to plot a course for the experimental design and analysis of situations in the case study and also to obtain the optimal conditions for grinding process in paper producing establishments.

The Lack of Fit (LOF) test measured the dispersion range of the data from the fitted model. if the fitted model is does not adequately represent the data used, the lack of fit value for such model shows up. When a model fails fit the data well, the Lack of Fit (LOF) test indicates a significant value. According to the words of famous statistical analysts [16], "A model is rejected if the result shows evidence of LOF when tested". The analysis of variance to the models source revealed that the LOF test for the quadratic model indicated no significant value proved by a high of  $P_{value}$  (Probability >F) of 0.2588. According to [17], the analysis suggested that the quadratic polynomial model was statistically adequate and could be used to predict the new response.

The coefficient-of-determination, Residual squared show a reasonable quadratic model at a high level of 0.9801 which also reflected the degree of fit of the model [18]. This includes that 98 percent of the variations could be explained. Furthermore, we considered reducing the models by dropping the insignificant model provisos; this might improve the values of residual squared and adjusted residual squared in the quadratic polynomial case. In the meantime, the rate by which the model is suitable for each and every one of the design point is determined by the predicted-RESS value.

When the model was analysed, the parameter estimates were summerized and abbreviated in the to formulate the approximated model for the function. We could generate the two equations for the coed factors and the actual factors and the approximated models are presented in their respective terms as follows:

Coded Equation:

$$Y = 262.5 - 454.67x_1 + 95.15x_2 - 16.41x_1x_2 + 401.25x_1^2 + 185.78x_2^2 \quad (7)$$

Actual Equation:

$$Y = 53819.70 - 462.88x_1 - 14.09x_2 - 0.04x_1x_2 + 1.00x_1^2 + 0.46x_2^2 \quad (8)$$

Where  $x_1$  = Machine Voltage and  $x_2$  = Grinding Time

The lesser the quantity of the predicted-RESS, it is the more favorable the model. It is recommended that the model

should be fit; this is why care should be taken in analyzing the model to check for its adequacy. The PRESS value of the second order response surface is lesser than the first order

response surface model. Since the lesser the PRESS, the more adequate the model becomes, we decided to use the quadratic model for further the optimization process.

*Model Reduction Optimization – Search Method*

The outcome of the analysis of variance for the reduced second order response surface model was shown in Table 1. We paused to do some comparisons between the reduced and unreduced model to check if there is any significant difference between the two. The interaction between machine voltage and grinding time represented by AB was not significant. The addition of the term posed huge distractions on the value of F calculated. There is a reduction in F

(1.4294) when the term AB was removed from the model. It was suggested to dropping or fixing the non significant term(s) as one level helps to improve the model except in cases where hierarchy is important. As a result of [19], we decided to remove the term AB in order to improve the quadratic model. Additional study with the LOF test revealed that the higher value of probability>F was observed in the reduced model (0.3130) than that of the initial model (0.2588). This could also indicate the reduced second-order (quadratic) is more suitable for the data than the initial models.

*Table 1. Analysis of Variance for the Reduced Second-Order Model.*

Sources	S-Squares	d. f	M-Square	F <sub>value</sub>	P <sub>value</sub>	
Model	7,981,945.63	4	1,995,486.41	335.556	0.0001	significant
A-Voltage	6,615,220.06	1	6,615,220.06	1,112.40	0.0001	
B-Time	289,733.96	1	289,733.96	48.721	0.0001	
A <sup>2</sup>	588,805.71	1	588,805.71	99.0122	0.0001	
B <sup>2</sup>	126,225.09	1	126,225.09	21.2257	0.0001	
Residual	166,510.43	28	5,946.80			
Lack of Fit	28,894.81	4	7,223.70	1.2598	0.313	not significant
<Pure Error	137,615.63	24	5,733.98			
Cor. Total	8,148,456.06	32				

Where, d. f. = degree of freedom.

Table 2 was used to ascertain the possible level of coefficient for the entire model and the respective responses accrued from them. Whatever happened to the model in terms of coded factors is applicable to the actual values. Therefore, the basis of our analysis here focused on the coded factors since they are easily calculated and makes work less complicated. Decisions are made on the relative

factor using the coefficients and the effect of the model parameters on the entire model. A typical example of this is the coefficient of the coded Factor B (95.15) in the converted equation is a large amount compared to the coefficients of Factor A (-454.67). This is an indication that the grinding time contributes more of positive amount to the paper mesh level compared to the machine voltage.

*Table 2. Confidence Interval of the Reduced model.*

Factor	Coefficient	D. F.	Error	95 CI(Low)	95 CI (High)	V. I. F.
Model	262.5	1	77.1155	104.536	420.4639	
A-Voltage	-454.6708	1	13.6322	-482.5952	-426.7466	1
B-Time	95.15348	1	13.6322	67.2291	123.078	1
A <sup>2</sup>	401.25	1	40.3246	318.6486	483.8513	4.6401
B <sup>2</sup>	185.781	1	40.3246	103.1799	268.3825	4.6401

However, we can still conclude based on the absolute value that the value [454.67] has a very huge effect in the grinding process. Every slide adjustment in the machine voltage affects the system seriously. It is worthy to note the final model parameter which was used to optimize the response in grinding process was the Reduced Second Order

Response Surface Model (RSORSM). The reduced model can also be represented by the actual equation. The equations demonstrates association between the independent variables, the combination of the factors and the corresponding responses that yields from them the conditions to improve grinding process in paper producing industries.

$$Y = 54090.41 - 464.109x_1 - 23.1095x_2 + 1.003125x_1^2 + 0.464453x_2^2 \tag{9}$$

According to [20], the coefficient of variance for the unreduced model should be more that of the reduced model. Our reduced satisfied this assumption with its coefficient of variance value of 9.2 percent. It was 0.05 percent less than the coefficient of variance of the unreduced model of 9.32 percent. It was also suggested that, for a good fit of a model, the square of the residual  $R^2$  should be not be less than 0.80 residual squared is used to calculate the explain variables. It also shows how a formulated model fit the

design and analysis of the experiment. In observing the residual squared of the reduced model, the 97.96 percent value means that level of variability that existed in the model was explained up to 97.96 percent. There is another index which measures the variability around the mean. It centers on what happened to the individual factors with respect to the center point. This is known as the adjusted residual squared and it is used to determine the level of adjustment applied to measure the dispersion on the average point. The higher the

terms of the model increases, the lower the adjusted residual squared if the additional terms do not contribute to the improvement of the model. The evidence is clearer in the two models we examined (standard and reduced), when the interaction effect between machine voltage and grinding time was added, there was a significant decrease in 0.0002 that is from 0.9766 to 0.9764. This happened because the interaction effect did not add notably to the  $Y$  response. We still recalled that adequate precision was used to determine the disturbances that contributed to random errors in a proportional rate. We didn't forget the clue given that adequate precision should be a ratio greater than four before it can be considered as an adequate signal. The value of adequate precision (48.156) of the reduced model was higher than the minimum value of four recommended and therefore the model can be used to plot a route for the experimented region. Statistical summary obtained from the reduced CCRD Model in the Optimization of process to improve the grinding conditions in paper producing industries. The model adequacy can be seen from the values of Residual Squared, Adjusted Residual Squared, Adequate precision and Coefficient of Variation.

Having obtain the feasibility region of the functions which enables us to know the boundaries by which the approximated objective functions cannot exceed, we then further the research by confirming the outcomes of the three dimensional contour plots with the search method. We used the graphical method to ascertain the range by which the optimum values lies but we could not get the exact optimum point. Therefore, the introduction of the search method enables us to apply vividly the ranges, limitations and importance gathered to get the most advantageous process that improves products in paper industries. In the effort to get the best possible solution using the numerical search method, we first set the limitations and coverage of the range of data available. It is through the desirability of the values that we could actually take decisions on presented outcomes. Magnitude is used to check the level of importance attach to individual factors and response. It is an instrument that enables an operation researcher to apportion priorities to the expected goals and objectives for the approximated function. The higher the importance, the more the significance of variables used in the research. Design Expert Software version 8.0.7.1 enables users to apply up to five levels of emphasizes ranging from one to five that is, (+) to (+++++). The magnitudes are low, lower, medium, high, and higher respectively. When optimizing, we set the importance for all the variables at the medium level since none is more significant than others.

In searching for the best and optimum fineness, proper consideration was given to the required outcomes such as the grinding fineness should be as maximum as possible. In view of the above considerations, it was therefore decided to set a maximum goal for fineness with the machine voltage preferably not exceeding the range and the grinding time

within the desired interval. When these conditions were set, the RSM software automatically generated many solutions for the optimized grinding. Desirability represents the individual level of attractiveness and interest which favours the objective of the research. Every operation researcher wishes to obtain a cachet which tends to unity or one. In the search method, the range of desirability for the available response values ranges from 0.033 to 0.766. This level of interest is arranged in a descending order where the most favorable range is written in bold and presented before other attraction. Design expert combines all the respective individual allure starting from the design, formulation, approximation, analysis and maximization of the optimum fineness to one level. According to [21], when an individual or extra response is not within the desired limit, the resulting value of zero emerges. In an ideal solution, the desired value tends to one and it is highly recommended.

The optimum condition for maximizing fineness was identified as voltage of grinding machine 200 volts and grinding time 50 minutes. Using this optimized grinding condition, the predicted response of fineness was about 1,399.36 meshes. It should also be mentioned that the desirability value of all solutions showed satisfactory good values. The value can range from zero to one and it should only be evaluated relative to the upper and lower limits that were chosen for the responses and variables. In this case, upper and lower limits of all variables were set according to the ranges of study while the fineness was set to be at maximum. At these intervals, the predicted grinding volume and the upper and lower values were given in terms of the fineness level. The standard deviation of the two independent variables were given as zero which shows that the error terms were normally and independently distributed among the factor levels. The analysis in this case was done with the actual value and it was observed that each and every one of the factors was adequately represented and they gave the best mesh value of 1,399.36 which was most favorable and suitable at an encouraging voltage of 200 and grinding time of 50 minutes. It became a shock that such volume of grinded calcite, dolomite, crowncassa or barite powder was obtained within the limited time and voltage.

#### *Surface Optimization – Graphical Method*

Fig. 1 is a three dimensional graph which demonstrates the combination of the two factors grinding voltage and time used during the grinding process to give the response values which satisfied the conditions attached to the process. There are significant boundaries in terms of limited time, range of machine voltage and the mesh levels which the experiment should not exceed. The plot accommodates a spherical region where the lowest value of fineness is observed at machine voltage range from 224-240 volts and grinding time of 10-42 minutes. The highest value of fineness is observed at voltage and grinding time ranging from 200-208 volts and 42-50 minutes, respectively.

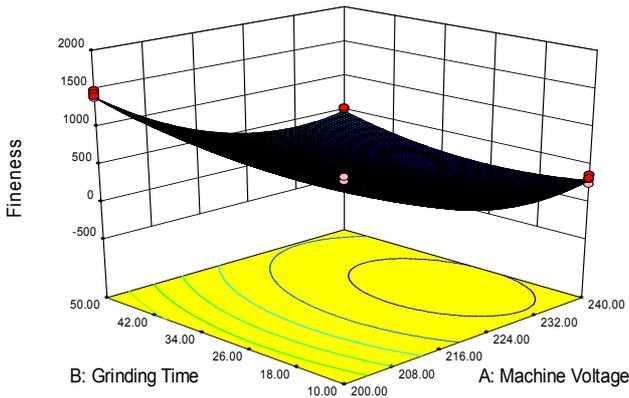


Fig. 1. Three Dimensional Surface Optimization.

Fig. 2 describes the feasible region of the model. The upper and lower bonds for both machine volts and grinding time are represented as  $V_1, V_2$  and  $T_1, T_2$  respectively. The rectangular boundary indicates the border by which we cannot exceed due to limited resources and time.

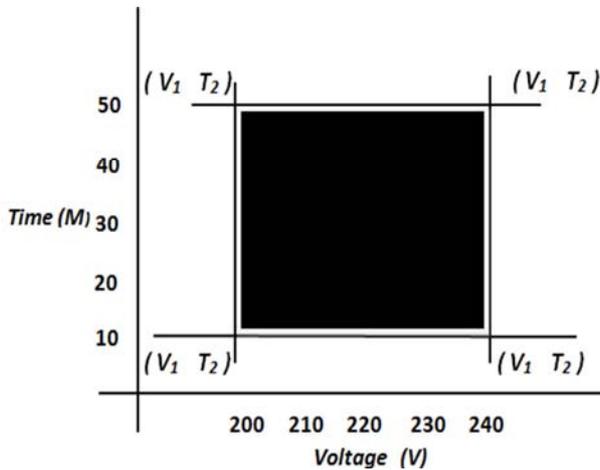


Fig. 2. Sketch of the Feasible Region.

The screen shot in Fig. 3 is a graph displaying graduated contours of the optimum points – from lower desirability to high desirability.

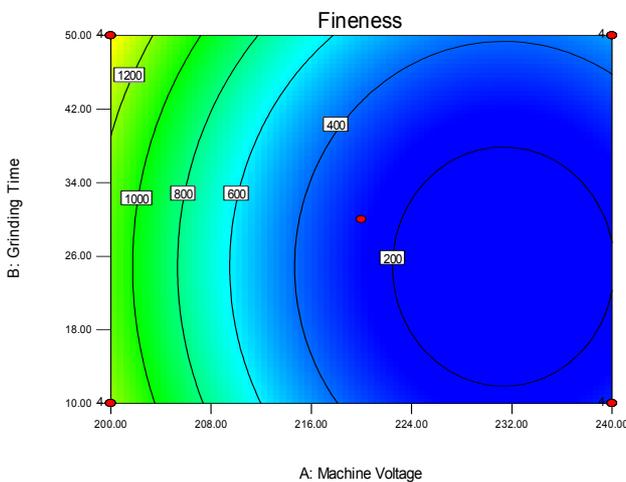


Fig. 3. Contours plot of the solution.

The thick blue colour shows the least desirable regions while the movement towards the pale blue is the highest value. It represented the level of interest and ability to satisfy the objective of the research. The desirability's were graduated from at the levels of the voltage and time. The higher the desirability, the more acceptable the outcome of the combination of the factors that is been produced at the level. The desirability of 0.766 was observed as the best level of the outcome. It is recommended because the closer the desired outcome to one, the better the optimum level of the research.

### 4. Conclusion

In this study, RSM was considered to have succeeded in maximizing the grinding fineness as a function of voltage of the grinding machine and grinding time. The optimum process conditions for application of grinding fineness were best at machine voltage (200V) and grinding time (50mins). Under this optimum condition, the predicted grinding fineness of the paper chemicals (calcite, barite) and the experimental results gave close values of about 1,399.36mesh. The response surface method predicted and experimental value of the grinding process showed appreciably evidence of similarity and reliability among the two. It is then means the predicted and estimated method can be adopted whenever production is taking place in the paper producing industries and any other producing company of the brand.

The paper producing industries uses grinded calcite frequently in the production of many toiletries, coded drugs, pomades, tissues, exercise books and many more. Similarly, grinded and processed barite is progressively used more than before in the manufacturing sectors and even in individual houses. The two are used as protective materials because they are they are more economical than other resources. Their excellence varies between 1,250 and 1,400 mesh. Considering about the fineness and optimum machine voltage (200V) and grinding time (50Mins), Zenith grinding mall offers unique barite and ultimate calcite powder ranging up to 1,399.36 when the process is fully optimized using Rotatable Central Composite design method of Response Surface Optimizations.

In paper making industries, the two most important materials used are ultrafine grinded barite and calcite within the range of 1,250 to 1,400 mesh. One of them is powerful in filler substitution of kaolin or talc and it is also called calcium trioxosulphate or carbonated calcium if it is not called calcite. In the same way, grinded barite is extensively applied as stuffing in paper producing industries. Using the response-surface methodology of rotatable central composite designs (RCCD), we obtained a reduced quadratic model that met our barite and calcite production prerequisites used in favor of improving grinding conditions in paper producing industries.

### References

[1] Dixit, A. K. (1990). Optimization Economic Theory. USA: Oxford University Press.

- [2] Chen, M. J., Chen, K. N. & Lin, C. W. (2004). Sequential Quadratic Programming for Development of a new probiotic dairy with glucono-lactone. *Journal of Food Science*, 69(22), 344-350.
- [3] Lee, J. Y., Landen, W. O. & Eitenmiller, R. R. (2000). Optimization of an Extraction Procedure for the Quantification of Vitamin E in Tomato and Broccoli using Response Surface Methodology. USA: FoodComposition and Analysis publishers.
- [4] Montgomery, D. C. (2005). Design and Analysis of Experiments: Response Surface Method and Designs. New Jersey: John Wiley and Sons Inc.
- [5] Guan, X. (2008). Optimization of Viscozyme L assisted extraction of oat bran protein using response surface methodology. Food Chemistry Publication.
- [6] Nwabueze T. U. (2010). Basic Steps in Adopting Response Surface Methodology as Mathematical Modeling for Bioprocess Optimization in the Food Systems, International. *Journals of Food Science and Technology*. 12(7), 104–124.
- [7] Hill W. J. and Hunter W. G. (1966). Response Surface Methodology, a Review. *Science Books Publishers* 10(5), 345-456.
- [8] Box, G. E., Hunter, J.S. & Hunter, G. W. (2005). Statistics for Experiments. New Jersey: John Wiley and Sons, Inc.
- [9] Myers, R. H., Khuri, A. I. & Carter, W. H. (1989). Response Surface Methodology. *Journal of Technometrics* 31 (2): 137-153.
- [10] Connor, W. S. & Marvin, Z. A. (1959). Fractional Factorial Experiment Designs for Factors at three-levels. Washington: U. S. Government Publishing Company.
- [11] Elfving, G. I. (1952), Optimal Allocation in Linear Regression Theory. *Annals of Mathematical Statistics*, 25(1), 255-263
- [12] Zahid, A. M., Cheow, C. S. & Norizzah, A. R. (2011). Optimization of Process Conditions for the Application of edible coating emulsion on guava (*Psidium guajava*) using response surface methodology. Singapore: IACSIT Press.
- [13] Oehlert, G. W. (2000). Design and Analysis of Experiments Response surface design. New York: W. H. Freeman and Company.
- [14] Onyeneke C. C. (2015). Multivariate Approach to Partial Correlation Analysis. *Science Journal of Applied Mathematics and Statistics*. Vol. 3, No. 3, 2015, pp. 165-170. doi: 10.11648/j.sjams.20150303.20.
- [15] Dave, Rudri, T. V. Ramana Rao, and A. S. Nandane (2015). RSM-Based Optimization of Edible-Coating Formulations for Preserving Post-Harvest Quality and Enhancing Storability of Phalsa (*Grewia asiatica* L.) RSM-Based Optimization of Coatings for Phalsa. *Journal of Food Processing and Preservation*.
- [16] Myers, H. R. & Montgomery, D. C. (1995). Response Surface Methodology: Process and Product Optimization Using Designed Experiments. New York: John Wiley and Sons Inc.
- [17] Devaki, C. S., and K. S. Premavalli. (2012). Development of Fermented Beverage Using RSM and Nutrients Evaluation – I. Fermented Ashgourd Beverage, *Journal of Food Research*.
- [18] Haber, A. B. & Runyon, R. A. (1977). General Statistics. USA: Addison-Wesley Press.
- [19] Jacqueline George, Stephy, Rajan Gandhimathi, Puthiya Veetil Nidheesh, and Sreekrishnapurumal Thanga Ramesh (2016). Optimization of salicylic acid removal by electro Fenton process in a continuous stirred tank reactor using response surface methodology, *Desalination and Water Treatment*.
- [20] Joglekar, A. M. & May, A. T. (1987). Product Excellence through Design of Experiments. *Journal of Cereal Food World*, 32 (12), 857-868.
- [21] Myers, H. R. Montgomery, D. C & Anderson C. C. (2009) Response Surface Methodology. New York: John Wiley and Sons.