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Controller Design for Three-Mass Resonant System Based on Polynomial Method

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Abstract: The mechanical system in the industrial electrical drives can be modelled by a multi-mass system. Torsional vibration suppression and attainment of robustness in motion control systems is a requisite in industry applications. In this paper, the system that is three-mass system is considered. A speed control method based on polynomial method introduced to imple-mentation of torsional vibration is proposed. The effectiveness of the proposed controller is demonstrated by transfer function analysis and simulation results. The simulation results show that the proposed controller improves dynamic performance and suppresses torsional vibration of three-mass resonant system.

Keywords: Three-Mass System, Speed Control, Polynomial Method, Transfer Function

1. Introduction

In industrial motor drive systems, a shaft torsional vibration is often generated in which a motor and load are connected via a flexible shaft [1-3]. A mechanical system composed of some masses connected with flexible shafts such as steel rolling mills, variable-speed wind turbine, industrial robot, ele-vator systems and so on, is called multi-mass resonant sys-tem [4, 5]. This simplified approach has been used in different industrial applications [6-8]. Vibration suppression and high speed response control in motion control system is an important problem in industry applications [9-11]. Control problems of a multi-mass system is especially difficult when not all system variables are measurable, which happens very often in industrial applications [12, 13].

Several references in technical literature can be found on the application of control methods to improvement perf-ormance of the multi-mass [14-16]. A vibration-suppression control method for both two- and three-mass system is prop-osed in [17], which uses controller with three elements: the disturbance observer, the imperfect derivative filter, and the feedback gain. In order to reduce the computational comp-lexity, the model predictive control for high-performance sp-eed control and torsional vibration suppression in the elastic three-mass drive system is described in [18]. An algorithm to design a speed control strategy of a two-mass resonant syste-m by a integral–proportional-derivative (PID) controller bas-ed on the response frequency and the response step is deve-loped in [19]. In order to have sufficient damping, in [20] thr-ee different controller base on PID controller are designed for the speed control of a two-mass system using a normalized model and polynomial method. The PID, PID-P and modify PID controllers in order to control torque of a two-mass reso-nant system is presented in [21], which the pole-placement controller such as coefficient diagram method and integral of time multiplied by absolute error is used to assign closed-loop poles of the system characteristic equation.

Control loop strategies of multi-mass system to reduce the effects of resonance are very different [22, 23]. In this paper the speed control of three-mass system based on the polyno-mial method is proposed. This paper is organized as follows. Section II describes modeling of the three-mass system for simulation. The open-loop system is represented in the mat-rix form in section III. The proposed controller structures and problem formulation are described in Section IV. The simula-tion result is included in Section V to verify good performa-nce obtained the proposed controller, and conclusions are provided in Section VI.

2. Mathematical Model of Three-Mass System

A motor drive system is composed of a motor connected to a load machine through a flexible shaft. Simplified model of the three-inertia torsional system (motor, load and gear) is shown in Figure 1 [24]. Here ω_M is motor speed, ω_L is load speed, T_S is shaft torsional torque, T_G is gear torque and ω_G is gear speed. The control input is motor torque (T_M) and the input disturbance is load torque (T_L). The speed motor is the measurable, but we have no sensor to measure load speed. The system parameters used for study and simulation are listed in Table I [25, 26].

The block diagram of the three-mass system is show in Figure 2. The gear backlash is simplified as a nonlinear element with backlash angle band [-δ, + δ] and elastic coeffic-ient KG [27, 28]. The state equation of the three-mass torsio-nal system is expressed as [29, 30]:

$$\frac{d\omega_L}{dt} = -(\frac{B_L}{J_L}+\frac{B_S}{J_L})\omega_L + \frac{B_S}{J_L}\omega_G + \frac{1}{J_L}T_S' - \frac{1}{J_L}T_L \quad (1)$$

$$\frac{d\omega_M}{dt} = -(\frac{B_M}{J_M}+\frac{B_G}{J_M})\omega_M + \frac{B_G}{J_M}\omega_G - \frac{1}{J_M}T_G' + \frac{1}{J_M}T_M \quad (2)$$

$$\frac{d\omega_G}{dt} = \frac{-1}{J_C}T_S' + \frac{1}{J_C}T_G' - (B_G+B_G)\omega_G + B_S\omega_L + B_G\omega_M \quad (3)$$

$$\frac{dT_S'}{dt} = K_S(-\omega_L + \omega_G) \quad (4)$$

$$\frac{dT_G'}{dt} = K_G(\omega_M - \omega_G) \quad (5)$$

where J_M, J_L, J_G, B_M, B_L, B_G, K_G, K_S are the motor inertia, the load inertia, the gear inertia, the motor viscous damping coefficient, the load viscous damping coefficient, the gear damping coefficient, the gear stiffness, and the shaft stiffness, respectively.
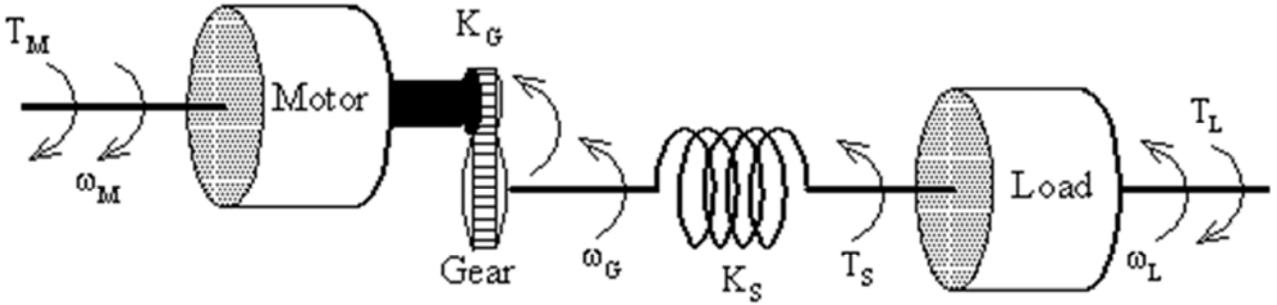
Figure 1. Simplified model of three-mass torsional system

Table 1. Parameters Used in the Study of Three-Mass System.

Parameters	Symbol	Value
motor inertia	J_M	0.0641 Kgm²
motor damping coefficient	B_M	0
load inertia	J_L	0.0523 Kgm²
load damping coefficient	B_L	0
shaft stiffness	K_S	242 Nm/rad
shaft damping coefficient	B_S	0.1 Nm-s/rad
gear stiffness	K_G	2000 Nm/rad
gear damping coefficient	B_G	0.2 Nm-s/rad
gear inertia	J_C	0.0868
resonance frequencies	ω_{r1},ω_{r2}	77.98, 235.75 rad/s
anti-resonance frequencies	ω_{a1},ω_{a2}	63.52, 162.55 rad/s
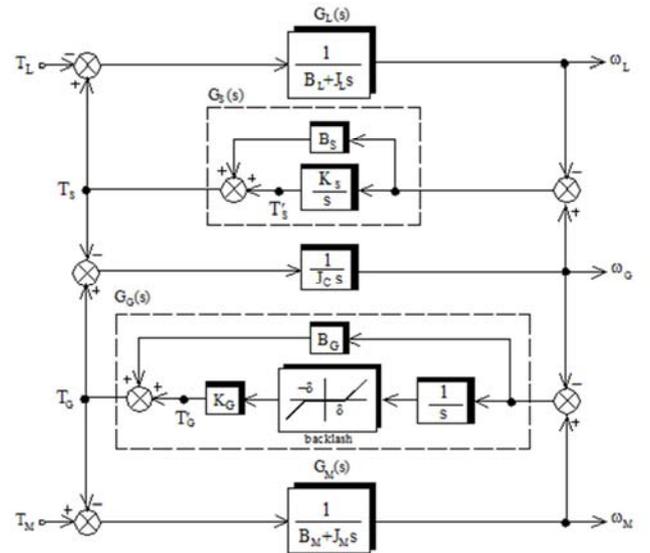
Figure 2. Block diagram of the nonlinear three-mass system.

Also we will have:

$$\begin{cases} T'_S = T_S - B_S(\omega_G - \omega_L) \\ T'_G = T_G - B_G(\omega_M - \omega_G) \end{cases} \quad (6)$$

The characteristic equation of open loop system are given by:

$$\Delta_o(s) = s^5 + p_4 s^4 + p_3 s^3 + p_2 s^2 + p_1 s + p_0 \quad (7)$$

By varying the operating point, the coefficient parameter values p_4 through p_0 also vary. A necessary condition for stability of the system is that all the roots in characteristic equation have a negative real part. By forbear from the value of the damping coefficients, p_0, p_2 and p_4 are zero. The p_1 and p_3 are given by:

$$p_1 = \frac{K_S K_G}{J_L J_M}\left(1 + \frac{J_L + J_M}{J_C}\right) \quad (8)$$

$$p_3 = \frac{K_S}{J_L}\left(1 + \frac{J_L}{J_C}\right) + \frac{K_G}{J_M}\left(1 + \frac{J_M}{J_C}\right) \quad (9)$$

3. Open-Loop Three-Mass System

Let a two-input two-output process be represented by the block diagram shown in Figure 3. The three-mass system can be represented in the following matrix form [31, 32]:

$$\begin{bmatrix} \omega_L \\ \omega_M \end{bmatrix} = \begin{bmatrix} -H_{LL}(s) & H_{LM}(s) \\ -H_{ML}(s) & H_{MM}(s) \end{bmatrix}\begin{bmatrix} T_L \\ T_M \end{bmatrix} \quad (10)$$

The transfer function of the open-loop three-mass system are given by:
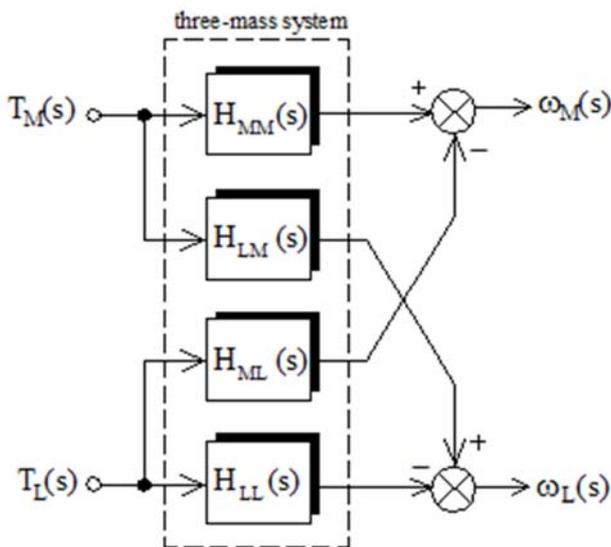
Figure 3. Two-input and two-output of the system.

$$H_{MM}(s) = \frac{a}{b^2 + ac} \quad (11)$$

$$H_{LL}(s) = \frac{a}{b^2 + ac} \quad (12)$$

$$H_{LM}(s) = H_{ML}(s) = \frac{b}{b^2 + ac} \quad (13)$$

where the coefficient a, b and c are given by:

$$\begin{cases} a = \dfrac{-1}{G_L(s)} + \dfrac{G_S(s)[G_G(s) - J_C s]}{J_C s - G_S(s) - G_G(s)} \\[2ex] b = \dfrac{G_G(s)G_S(s)}{J_C s - G_S(s) - G_G(s)} \\[2ex] c = \dfrac{1}{G_M(s)} - \dfrac{G_S(s)[G_G(s) - J_C s]}{J_C s - G_S(s) - G_G(s)} \end{cases} \quad (14)$$

The transfer functions of the motor, load, shaft and gear are $G_M(s)$, $G_L(s)$, $G_S(s)$ and $G_G(s)$.

The plot of the open-loop frequency response from the $H_{MM}(s)$ is for two-mass and three-mass are shown in Figs. 4 and 5, respectively. Two high peaks arise for three-mass system in the gain characteristic plot at resonance frequencies ω_{r1} and ω_{r2} and two down peak arise at anti-resonance frequency ω_{a1} and ω_{a2}. The amount of the attenuation coefficients are deleted, because the values are very small. Open-loop transfer function between the motor torque and the motor speed, which is most important in the closed-loop design can be derived as:

$$H_{MM}(s) = \frac{\omega_M}{T_M} = \frac{1}{\Delta_o(s)}\left[\underbrace{\frac{1}{J_M}}_{b_4} s^4 + \right.$$

$$\underbrace{\frac{\dfrac{K_S}{J_C} + \dfrac{K_G}{J_C} + \dfrac{K_S}{J_L}}{J_M}}_{b_2} s^2 + \underbrace{\frac{K_S K_G}{J_L J_C J_M}}_{b_0}\left.\right] \quad (15)$$

The transfer function can be represented in the form below:

$$H_{MM}(s) = \frac{(s^2 + \omega_{a1}^2)(s^2 + \omega_{a2}^2)}{J_M s(s^2 + \omega_{r1}^2)(s^2 + \omega_{r2}^2)} \quad (16)$$

where ω_{r1} and ω_{r2} are the natural resonance frequencies respectively corresponding to the shaft resonance and the gear-resonance.

4. Applied Speed Control Structure

The direct feedbacks from the load speed and the shaft torque are very difficult in the mechanical system in the industrial application. The block diagram of the controlled plant for the three-mass system is presented in Figure 6. The transfer function of the PID damping controller for the three-mass system in s domain is given by:

$$G_C(s) = (K_P + \frac{K_I}{s} + K_D s)(\frac{T_W s}{T_W s + 1}) \qquad (17)$$

where K_W and T_W are the gain and time constant of the washout term while K_P, K_I and K_D are the proportional gain, integral gain, and derivative gain of the damping controller, respectively. K_F is the forward gain of the controller. The transfer function of the first-order low-pass filter is $K_W/(sT_W+1)$ in the speed controller. The speed command is ω_C. In this I-PD controller, proportional and differential action are worked only on the detected quantity ω_M [33]. The close-loop state space equation can be written as:

$$\begin{cases} \dfrac{d}{dt} X = A X + B U + D W \\ Y = C X \end{cases} \qquad (18)$$

where X is the state vector, Y is the output vector, U is the external or compensated input vector, W is the external disturbance input vector while A, B, C and D are all constant matrices of appropriate dimensions.

With speed controller, the state vector contained seven state variables and the control vector contained command speed as shown below:

$$\begin{cases} X = \begin{bmatrix} \omega_L & \omega_M & \omega_G & T_S^{'} & T_G^{'} & \omega_I & T_X \end{bmatrix}^T \\ U = [\omega_C] \\ W = [T_L] \end{cases} \qquad (19)$$

The state variable T_M is:

$$T_M = -T_X + (K_F + \frac{K_W}{T_W})(\omega_I - K_P \omega_M - K_D \frac{d\omega_M}{dt}) \quad (20)$$

In addition to the five open-loop equation, there are two following equations in closed-loop system:

$$\frac{d\omega_I}{dt} = -K_I \omega_M + K_I \omega_C \qquad (21)$$

$$\frac{dT_X}{dt} = \frac{-1}{T_W} T_X + \frac{K_W}{T_W^2} \omega_I - \frac{K_W K_P}{T_W^2} \omega_M - \frac{K_W K_D}{T_W^2} \frac{d\omega_M}{dt} \quad (22)$$

Closed-loop transfer functions from speed command to load speed is given by:

$$H_s(s) = \frac{\omega_M}{\omega_C} = \frac{K_C T_W H_{MM}(s)}{K_W \left(K_D s^2 + K_P s + K_1\right) H_{MM}(s) + T_W s + 1} \quad (23)$$


motor speed transfer function performance
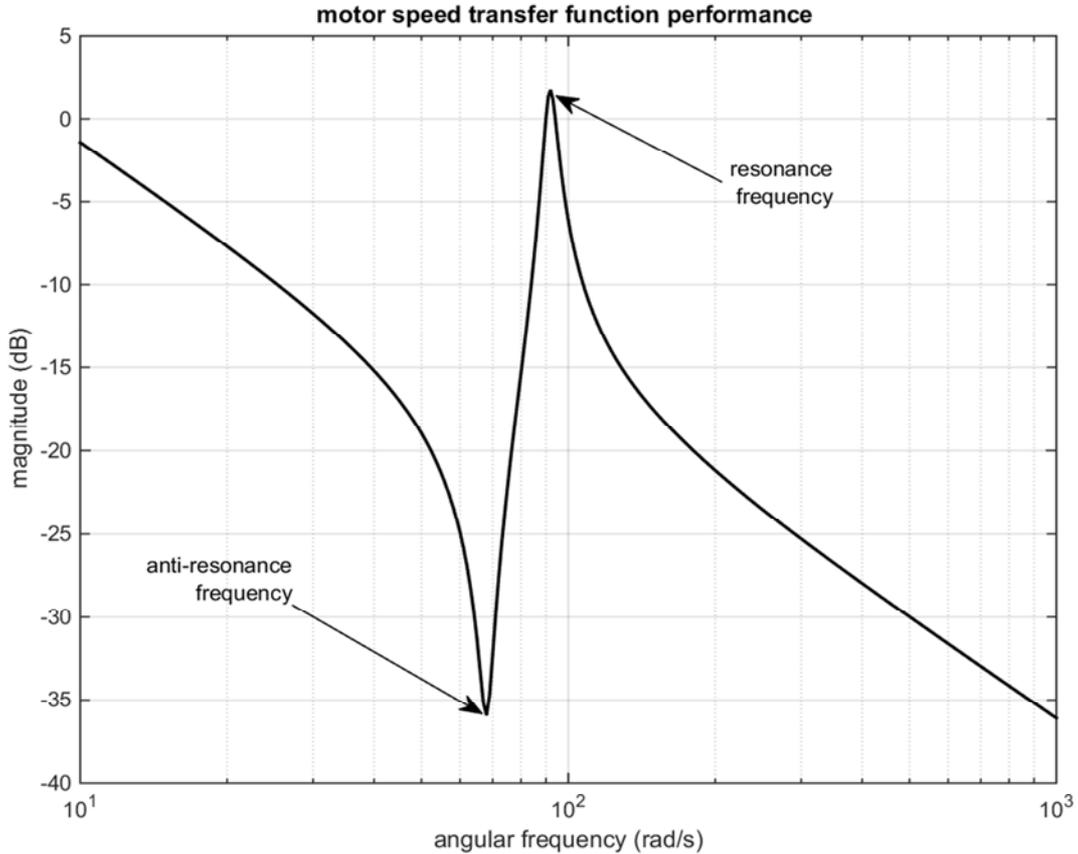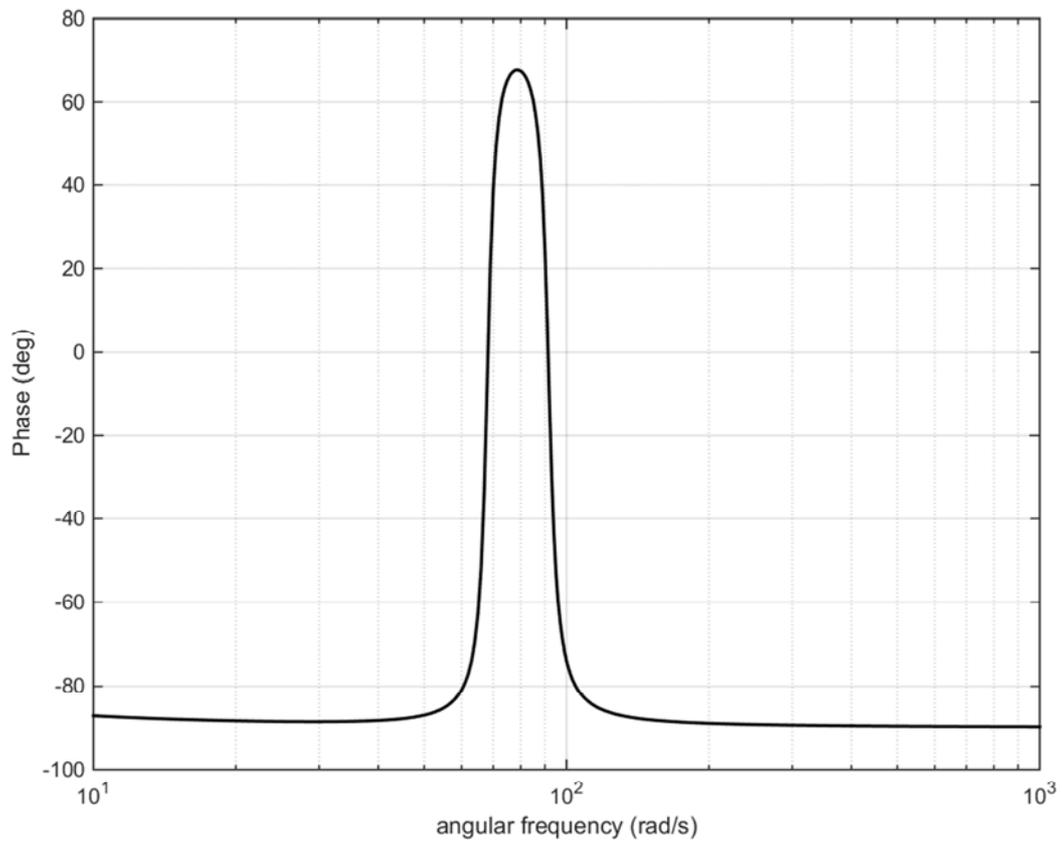
*Figure 4. Bode diagram of magnitude and phase for two-mass model.*
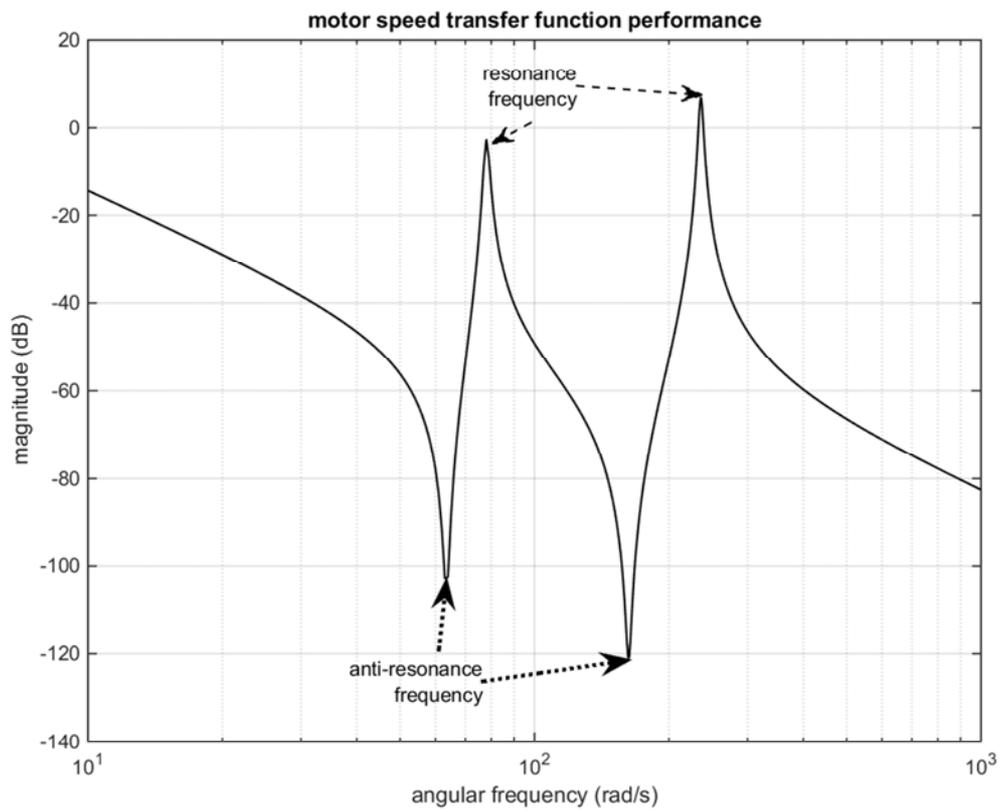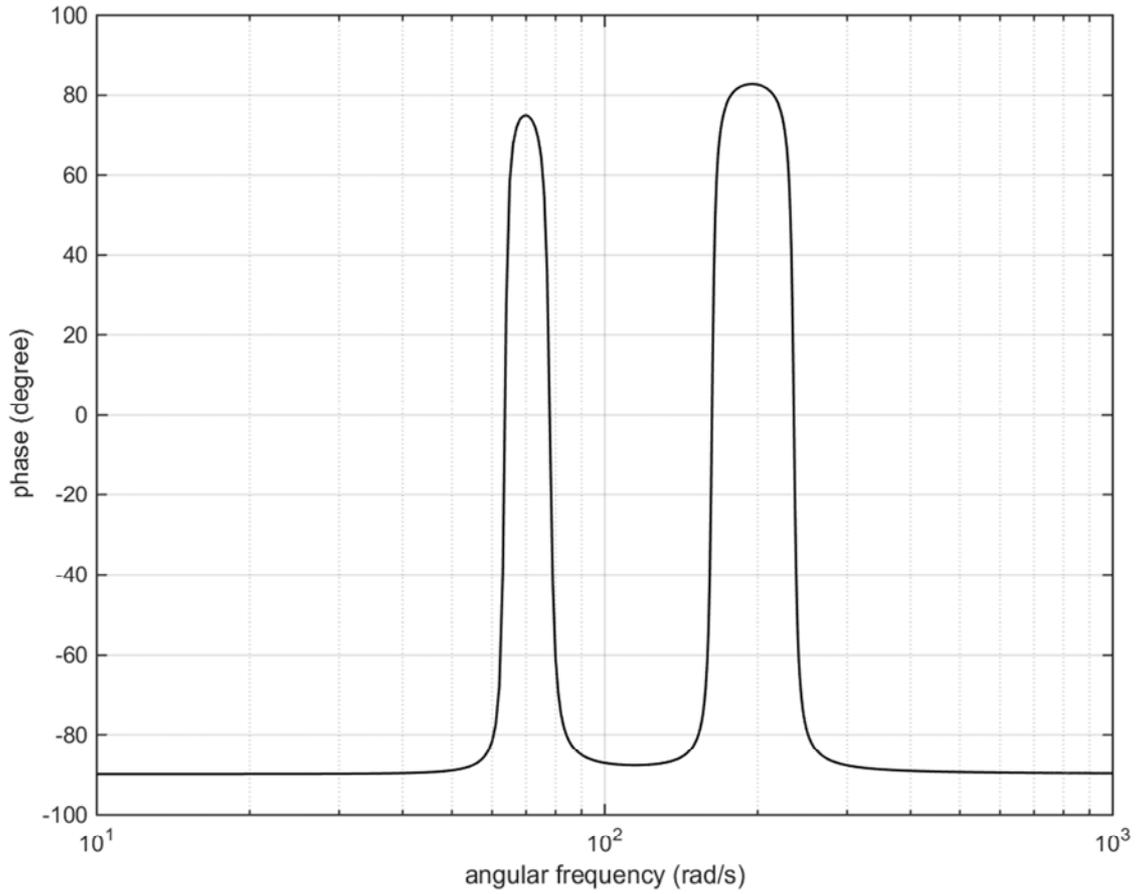
**Figure 5.** *Bode diagram of magnitude and phase for three-mass model.*
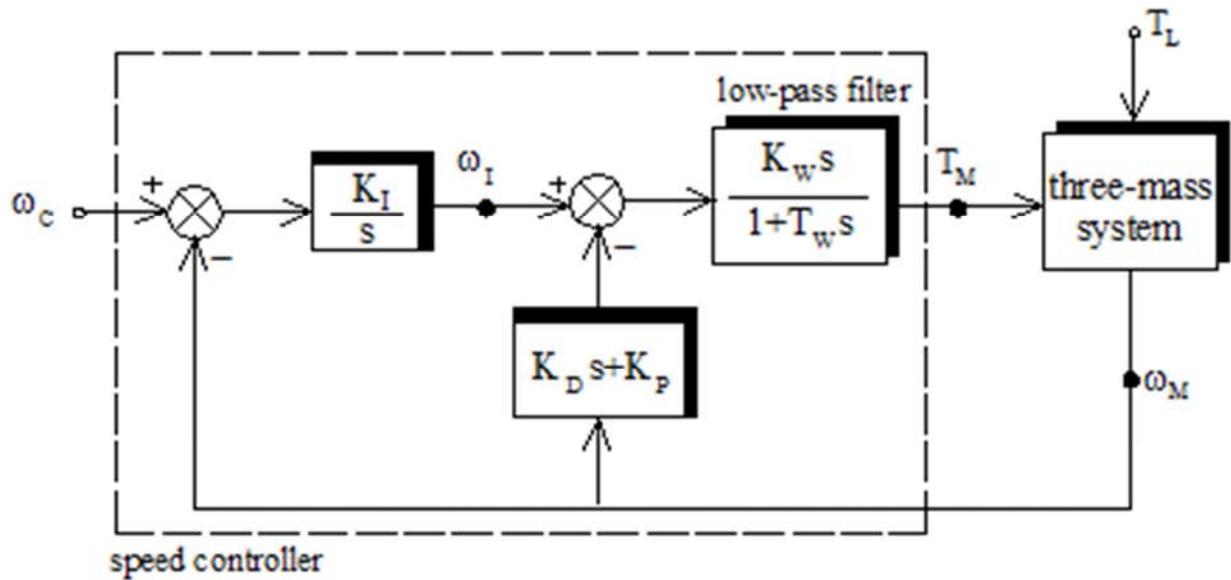
Figure 6. *Controlled plant block diagram.*

The characteristic equation of the close loop three-mass system equipped with controller is given by:

$$\Delta_c(s) = h_7 s^7 + h_6 s^6 + h_5 s^5 + h_4 s^4 + h_3 s^3 + h_2 s^2 + h_1 s + h_0 \quad (24)$$

where the coefficients h_0 through h_7 are given by:

$$h_0 = K_I K_F b_0$$
$$h_1 = K_P K_F b_0 + K_I b_0 (K_W + K_F T_W)$$
$$h_2 = p_1 + K_P b_0 (K_W + K_F T_W) + K_P b_2 K_I + K_F b_0 K_D$$
$$h_3 = T_W p_1 + K_F K_P b_2 + (K_I b_2 + K_D b_0)(K_W + K_F T_W)$$
$$h_4 = p_3 + \frac{K_I K_F}{J_M} + K_P b_2 (K_W + K_F T_W) + K_D K_F b_2$$
$$h_5 = p_3 T_W + (\frac{K_I}{J_M} + K_D b_2)(K_W + K_F T_W) + \frac{K_P K_F}{J_M}$$
$$h_6 = 1 + \frac{K_D K_F}{J_M} + \frac{K_P}{J_M}(K_W + K_F T_W)$$
$$h_7 = T_W + \frac{K_D}{J_M}(K_W + K_F T_W)$$

$$(25)$$

A good set of controller parameters K_F, K_W, T_W, K_P, K_D and K_I will yield a good response and results in the vibration suppression in time domain.

5. Simulation Results

The simulation results of the speed control of the three-mass using the proposed controller will be shown in this section in order to demonstrate the efficiency of the controller. The Simulink model of the system is show in Figure 7. In Figures 8 and 9, the frequency responses considering three-mass system equipped with a speed controller is compared with considering three-mass system without controller. The solid lines show the plots of the closed-loop frequency response from the speed command to motor speed and to shaft torque.

The gain plot of transfer function of motor speed to command speed in three-mass system for two different forward gain is shown in Figure 10. Figures 11 and 12 show the load speed and shaft torsional of the three-mass system with controller. The reference input of 1 rad/s is commanded at t=0, and the 1 Nm disturbance torque is given at t=1.5s. The change of the load speed for three-mass system is show in Figures 13.
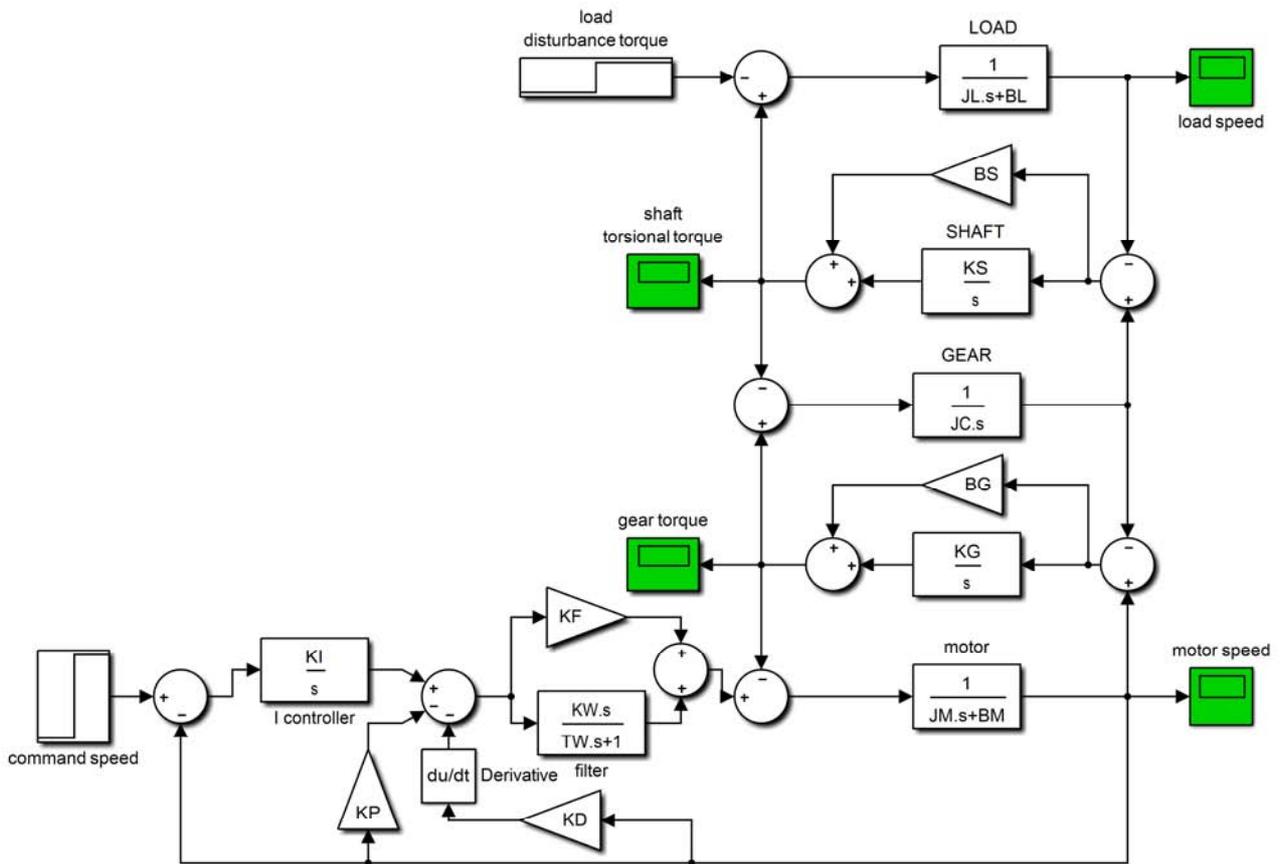

Figure 7. Simulink model of the three-mass system with controller.
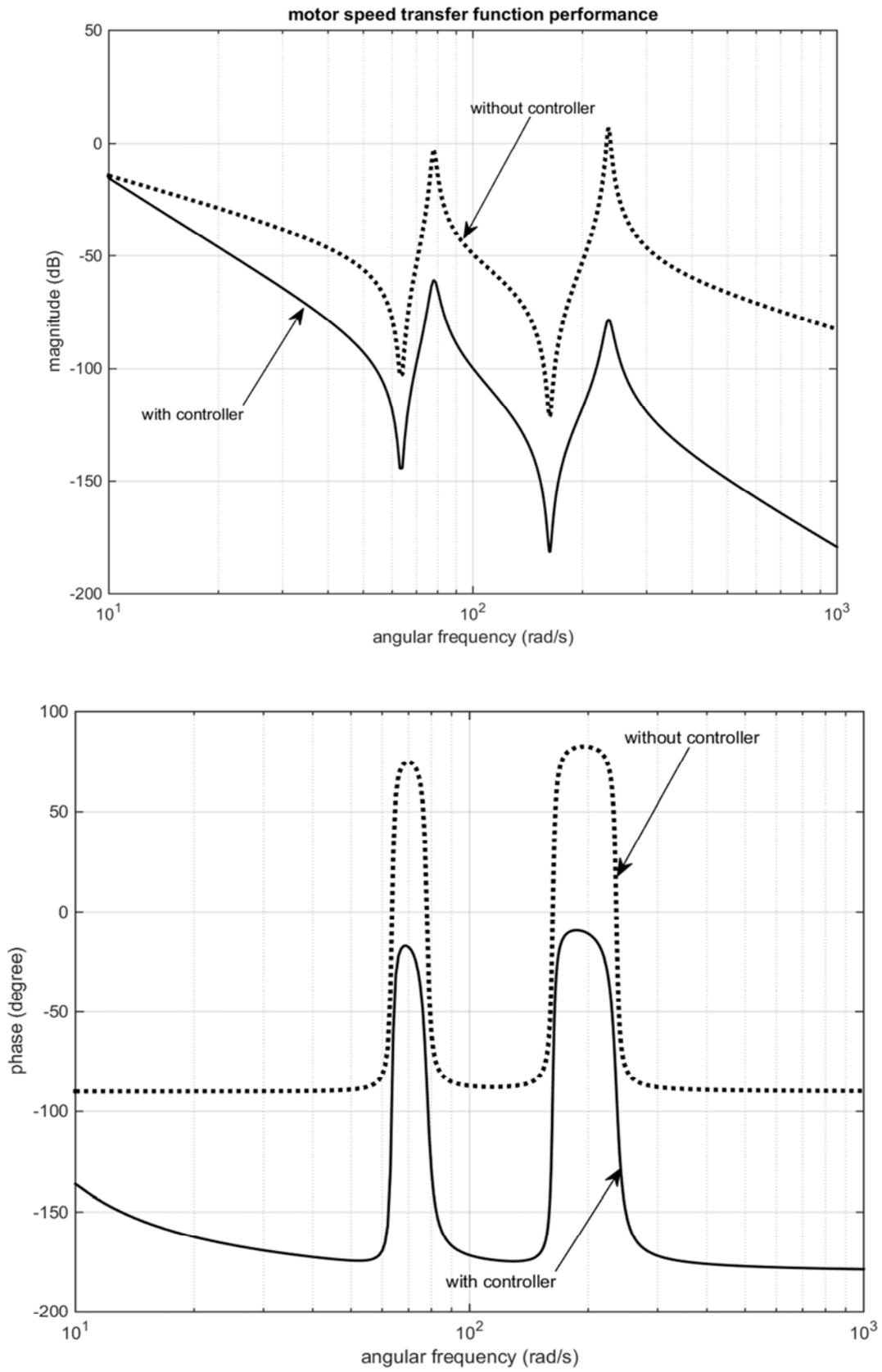
Figures 8. *Comparative frequency response of motor speed transfer function.*
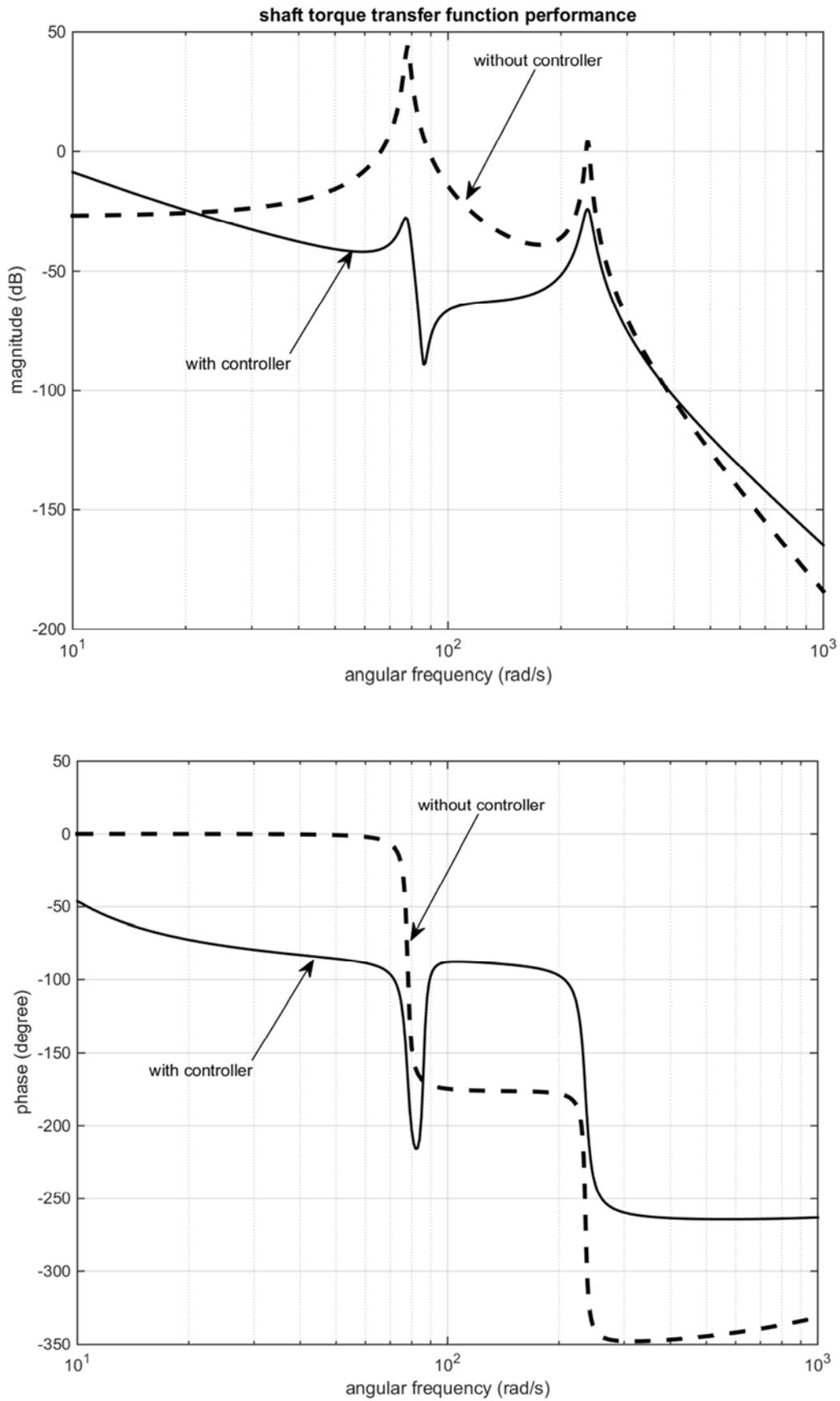
**Figure 9.** *Comparative frequency response of shaft torque transfer function.*
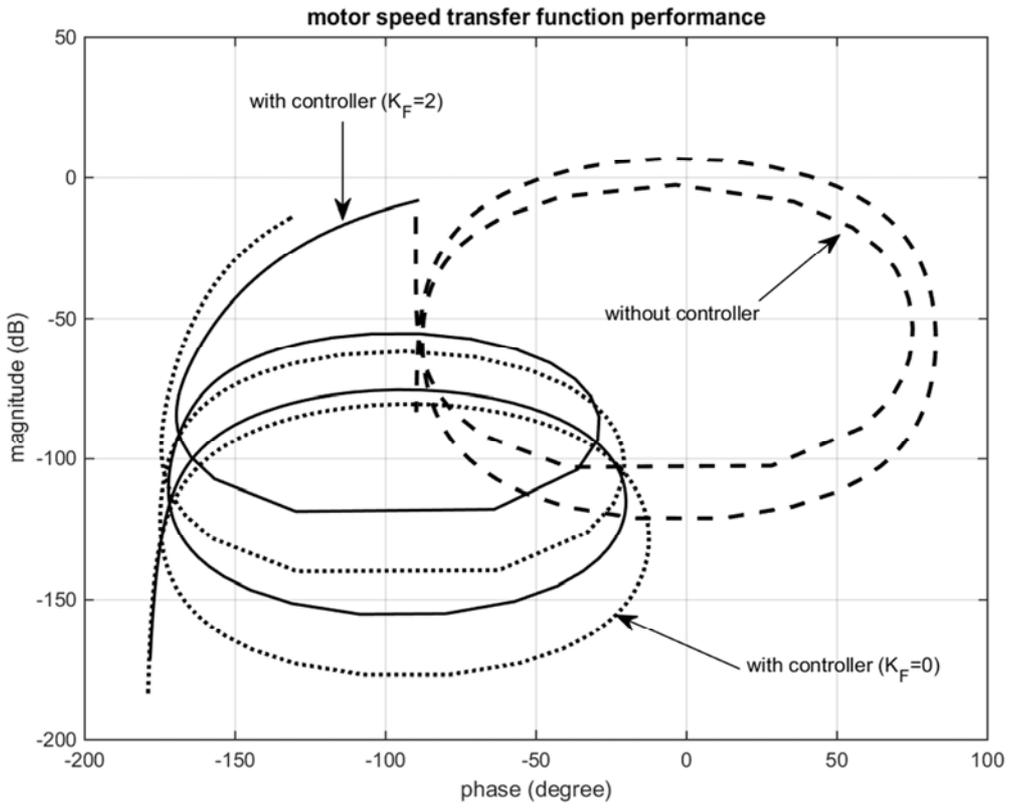
Figure 10. *Gain-phase plot for two different forward gain and without controller.*
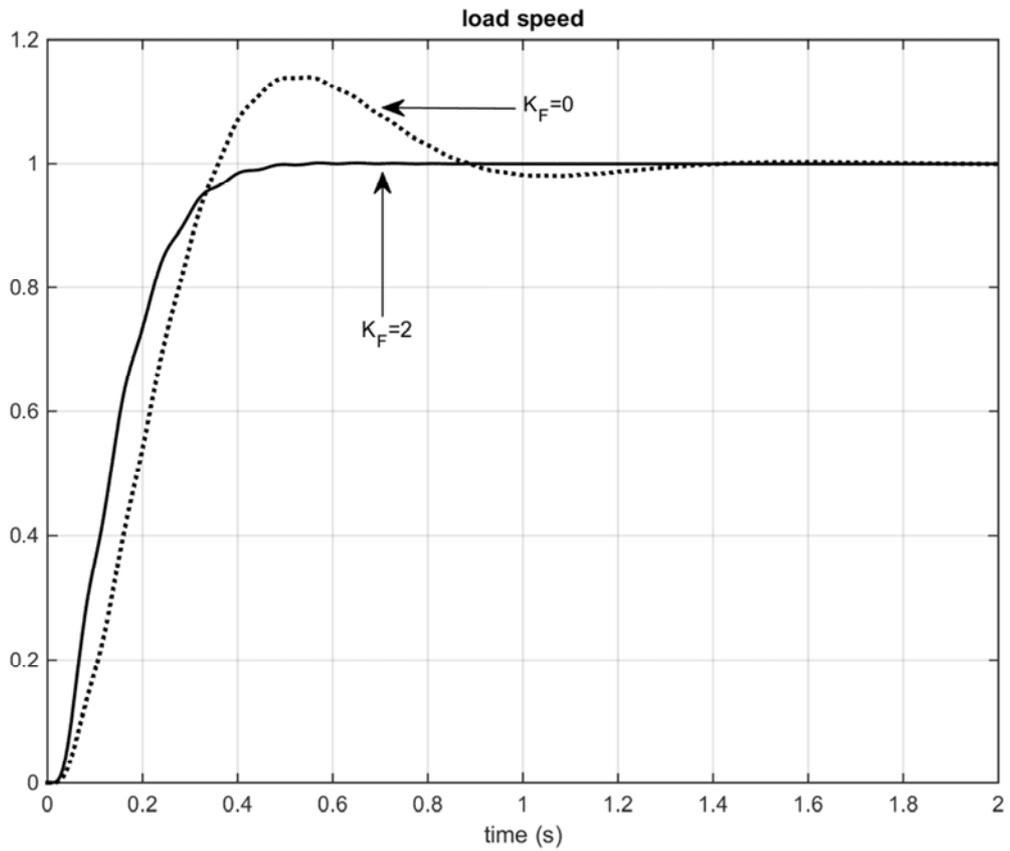

***Figure 11.*** *Load speed with controller for three-mass system.*
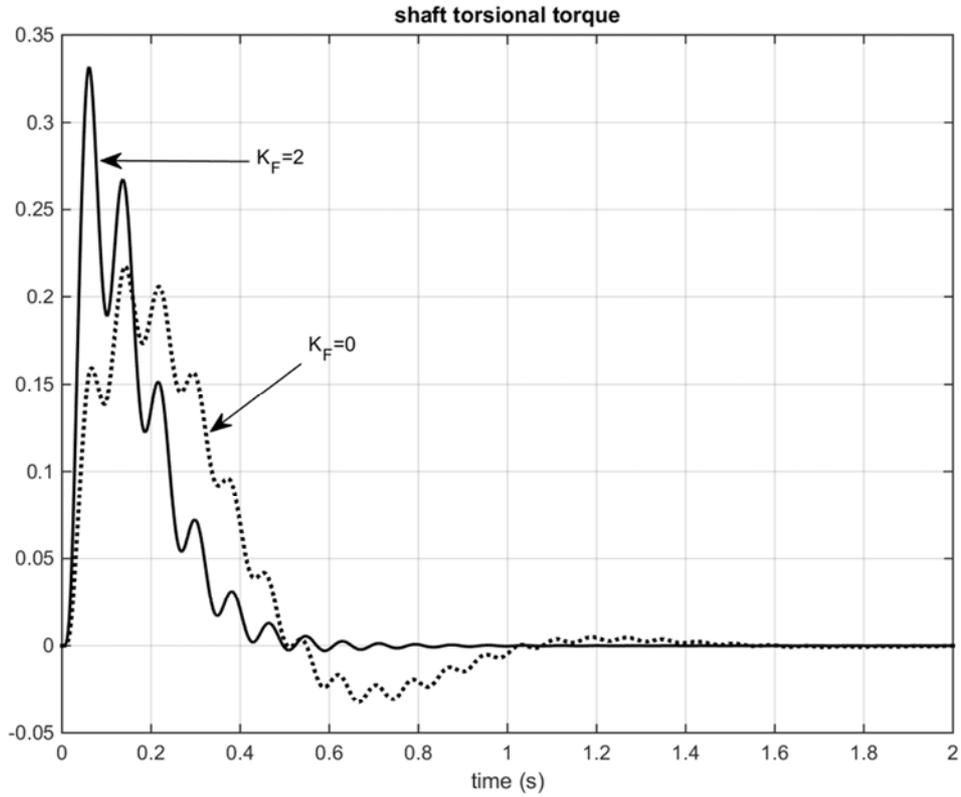
*Figure 12. Shaft torsional with controller for three-mass system.*
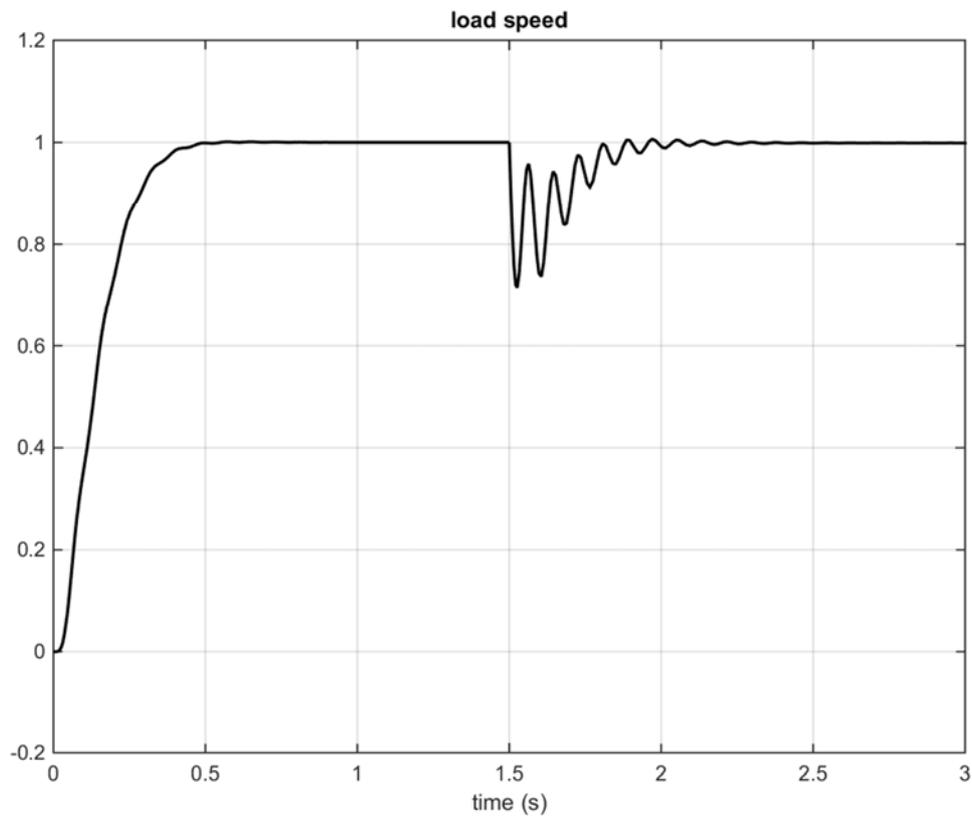
Figure 13. Load speed for change in the input of the three-mass system.

6. Conclusion

Vibration suppression and disturbance rejection control in torsional system is an important problem in the future motion control. In this paper a simple speed control method of three-mass torsional system is proposed. The mathematical model of the open-loop and clos-loop was analyzed. The controller selected is an I-PD controller with the gains of which were select base on polynomial method. With control system, a fast response without oscillation can be achieved. The simulation results show that the proposed controller improves dynamic performance and suppresses torsional vibration of three-mass resonant system.

Acknowledgement

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