**International Journal of Statistical Distributions and Applications**
2023; 9(2): 62-67
http://www.sciencepublishinggroup.com/j/ijsda
doi: 10.11648/j.ijsd.20230902.12
ISSN: 2472-3487 (Print); ISSN: 2472-3509 (Online)

# Bayes Estimator Parameters Exponential Distribution of Type I Sensor Data Using Linear Exponential Loss Function Method Based on Prior Jeffrey

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## To cite this article:

Afriani, Ardi Kurniawan, Elly Ana. Bayes Estimator Parameters Exponential Distribution of Type I Sensor Data Using Linear Exponential Loss Function Method Based on Prior Jeffrey. *International Journal of Statistical Distributions and Applications*. Vol. 9, No. 2, 2023, pp. 62-67. doi: 10.11648/j.ijsd.20230902.12

**Received**: June 11, 2023; **Accepted**: July 4, 2023; **Published**: July 11, 2023

**Abstract:** Lifetime data analysis or survival analysis is a technique in statistics that can be used to test the durability and reliability of a component. The life time data obtained from the life test experiment is in the form of type I censored data if the life test data is generated after the experiment lasts for a predetermined time. This study aims to obtain a parameter estimator from the exponential distribution of type I censored data using the Linear Exponential Loss Function (LINEX) method. The prior distribution used in this study is a non-informative prior with the technique of determining it using the Jeffrey method. So that the research results were obtained in the form of a parameter estimator $\theta$ is $\hat{\theta}$. Furthermore, applied to secondary data on the survival time of patients with chronic kidney failure at one of the Bojonegoro Hospitals was carried out in 2014. The data is divided into two, namely data on patients with initial causes of diabetic (data 1) and non-diabetic (data 2) diseases. Based on the estimation results for case studies of patients with chronic kidney failure, the value $\hat{\theta}_1 = 0.0102608$ for patients with initial causes of diabetic disease and $\hat{\theta}_2 = 0.00712166$ for patients with initial causes of non-diabetic disease. This shows that the possibility of patients with initial causes of diabetic disease to fail (die) is higher than patients with non-diabetic causes of disease.

**Keywords:** Exponential Distribution, Prior Jeffrey, Bayesian Method, LINEX Loss Function

## 1. Introduction

Survival analysis is one of the statistical analyzes used to analyze data on the survival time of an object or individual. Survival analysis includes a variety of statistical techniques that are useful for analyzing various types of random variables. The random variable in survival analysis is survival time or failure tine. Survival analysis is a special statistical analysis that helps analyze a case that cannot be solved by general statistical analysis. Survival analysis is used when the case relates to the time until a certain event occurs. Events in survival analysis can be in the form of disease onset, disease recurrence, recovery, death, or something interesting to observe on a particular object [1].

In the analysis of survival or survival requires survival data which includes survival time and survival time status of the unit under study. Samples obtained from survival trials or experiments can be complete samples or censored samples. Survival tests carried out using a complete sample are called uncensored data, namely data taken if all the individuals or units studied die or fail. The data is said to be censored if the data obtained before all the data is observed for its lifetime, while the observation time has ended or for other reasons [2]. There are 3 types of data censorship, which are type I censored data, type II censored data, and type III censored data. Type I censored data is live test data that is produced after the research has been running for a predetermined time [3]. In addition to the concept of data censorship, several distribution models are used to observe survival. From the survival test data obtained it is expected to follow a certain form of probability distribution which is useful for analysis purposes including the exponential, Weibull, log-normal, log-logistic distribution, and others [4]. One of the distribution functions that is frequently used and has an important role in the analysis of life test data is the exponential distribution.

The exponential distribution is a type of continuous distribution with a population parameter θ which is based on observational data and is also one of the special cases of the Gamma distribution with α=1 [5]. The exponential distribution is graphically similar to the life expectancy of an object or individual, that is, the longer the time elapses, the lower the life expectancy. A continuous random variable X which has an exponential distribution with parameter θ if the probability density function is as follows [6]:

$$f(x) = \theta e^{-\theta x}, 0 \leq x < \infty \text{ dan } \theta > 0 \quad (1)$$

After knowing the data distribution, the next step is to estimate the parameters of the data distribution. In estimation theory, there are 2 approaches for estimating population parameters from data distribution, namely the classical approach and the Bayesian approach [7]. The classical approach bases the parameter estimation entirely on information obtained from a random sample. Some of the classic methods that can be used include the moment method, Least Square, and Maximum Likelihood. While the Bayesian approach is a method that bases inference on the basis of information obtained from samples and other information that was previously available (prior distribution) [8]. In the bayesian approach there are several methods, namely Linear Exponential Loss Function (LINEX), Lindley Approximation, General Entropy Loss Function (GELF), and Squared Error Loss Function (SELF). The LINEX bayesian method is the most common loss function method used for bayesian estimation [7].

The LINEX loss function (Linear Exponential) is a Bayesian estimation method introduced by Varian (1975) [9]. This method has been applied by many researchers, including Ramadhan, et al (2022) [10], Puspitawati, et al (2019) [11], and Sagita et al (2018) [12]. In the Bayesian LINEX method, the likelihood function, prior distribution, and posterior distribution are required. Parameters in the Bayesian method are viewed as a variable and their values are expressed in a probability distribution [13]. The probability distribution of these unknown parameters is chosen subjectively or based on the results of previous studies which are called the prior distributions. There are 2 types of prior distribution, namely prior informative and non-informative prior distribution. Informative prior distributions are prior distributions that are based on known information about the parameter θ, while non-informative prior distributions are prior distributions where information about the parameter θ is not known, including Jeffrey's priors [14]. Both informative priors and non-informative priors will result in a posterior distribution that refers to the Bayesian theorem.

The resulting posterior distribution will have a more significant effect if the prior information used is prior informative, but if the non-informative prior is used as the prior information it will still have an influence on the results of the posterior distribution but not so significantly. However, non-informative priors are preferred because if prior information about the parameters is not available, then the initial uncertainty about the parameters can be measured by the non-informative prior distribution. One of the non-informative reference priors that can be used is Jeffreys' priors. The results of the resulting posterior distribution will then be further processed with the rules of Bayesian theory so that the parameter estimator results from the distribution are obtained.

Several studies have been conducted regarding the estimation of the parameters of the continuous distribution using the Bayesian approach to the survival model, namely the research conducted by Ramadhan, et al (2022). His research explains how to estimate the parameters of the exponential distribution of censored data using the Bayesian Squared Error Loss Function (SELF) method for priors that are informative, namely priors Gamma [10]. Subsequent research was carried out by Puspitawati, et al (2019) which explained how to estimate the parameters of the complete sample survival model (uncensored) exponential distribution using the Squared Error Loss Function (SELF) and General Entropy Loss Function (GELF) methods based on Jeffrey's priors [11].

Based on this background, the authors wanted to know how to estimate the parameters of the exponential distribution of type I censored data using the Bayesian Linear Exponential Loss Function (LINEX) method. Prior distribution used in this research is non-informative prior with the technique of determining it using the Jeffrey method.

# 2. Methods

To obtain parameter estimates for the exponential distribution of type I censored data using the Linear Exponential Loss Function (LINEX) method, the following steps can be taken:

a. Assumes that $x_1, x_2, \ldots, x_n$, are live test data from a random variable of size n that comes from an exponential distribution with parameter $\theta$
b. Perform data censorship by determining the size of the time limit for observation or sensor time to obtain type I censored data $(x_i, \delta_i)$
c. Determine the shape of the opportunity density function (pdf) of the exponential distribution
d. Determine the exponential distribution survival function
e. Determine the shape of the Likelihood function ($L(\theta)$) from the exponential distribution for type I censored data
f. Forming a prior distribution based on non-informative priors using Jeffrey's method, with the following steps:
   1) Determines the value of $ln(f(x;\theta))$
   2) Determines the first derivative of $ln(f(x;\theta))$
   3) Determines the second derivative of $ln(f(x;\theta))$
   4) Determine $I(\theta)$ or Fisher information from $\theta$
   5) Determine the shape of the prior distribution $(P(\theta))$
g. Forming the posterior distribution $P(\theta|X_i)$
h. Parameter estimation uses the Linear Exponential Loss Function (LINEX) method based on the posterior

distribution that has been obtained. With the following steps:

1) Determine the posterior expected value of the LINEX loss function $L(\hat{\theta},\theta)$ which can be written as: $E_\theta\left[L(\hat{\theta},\theta)\right]$.
2) Minimizing the posterior expected value of the LINEX loss function $E_\theta\left[L(\hat{\theta},\theta)\right]$ against $\theta$, so that the bayes estimator parameter $\theta$ is obtained which is denoted by $\hat{\theta}_L$.
3) Specifies the value of $E_\theta[exp(-a\theta)]$.
4) Substitute the result from step 3 into the equation $\hat{\theta}_L$.

The application of the estimation results to real data can be carried out in the following steps:

a. Determine the real data to be applied.
b. Testing the exponential distribution of real data based on the Kolmogorov-Smirnov test.
c. Import data in Mathematica software.
d. Calculating the estimated parameter θˆ based on the LINEX method using Mathematica software.
e. Make an interpretation of the results of step d.

# 3. Discussion

It is assumed that $X$ is a random variable with an exponential distribution. We obtain observational data from variable $X$ as many as $n$ $(x_1, x_2, \ldots, x_n)$. Then the probability density function (PDF) of variable X is as follows:

$$f(x_i;\theta) = \begin{cases} \theta e^{-\theta x_i} & x \geq 0, \theta > 0 \\ 0 & x < 0 \end{cases} \tag{2}$$

The cumulative distribution function (CDF) of variable X is stated as follows:

$$F(x) = 1 - e^{-\theta x} \tag{3}$$

The data used is censored data, so to form the Likelihood function a survival function of the exponential distribution is needed which can be obtained as follows

$$S(t) = 1 - F(t)$$

$$S(t) = 1 - \left(1 - e^{-\theta x}\right)$$

$$S(t) = e^{-\theta x} \tag{4}$$

After obtaining the survival function, the Likelihood function of the random variable censored data can be formed with the following description:

$$L(\theta) = \prod_{i=1}^{n} f(x_i)^{\delta_i} S(x_i)^{1-\delta_i}$$

$$L(\theta) = \prod_{i=1}^{n}\left(\theta e^{-\theta x_i}\right)^{\delta_i}\left(e^{-\theta x_i}\right)^{1-\delta_i}$$

$$L(\theta) = \prod_{i=1}^{n} \theta^{\delta_i}.e^{-\theta x_i\delta_i - \theta x_i + \theta x_i \delta_i}$$

$$L(\theta) = \prod_{i=1}^{n} \theta^{\delta_i}.e^{-\theta x_i}$$

$$L(\theta) = \theta^{\sum_{i=1}^{n}\delta_i}.e^{-\theta\sum_{i=1}^{n}x_i} \tag{5}$$

The shape of the prior P(θ) distribution will be determined using Jeffrey's method. In Jeffrey's method it is known that P(θ) is proportional to the square root of Fisher's information. The information value of Fisher I(θ) is proportional to the negative of the expected second derivative of the natural logarithm of the probability density function of the exponential distribution with respect to θ. The natural logarithm of the probability density function of the exponential distribution can be described as follows:

$$ln\left(f(x;\theta)\right) = ln\left(\theta e^{-\theta x}\right)$$

$$= ln(\theta) + ln\left(e^{-\theta x}\right)$$

$$= ln(\theta) + ln\left(e^{-\theta x}\right)$$

$$= ln(\theta) - \theta x.ln(e)$$

$$= ln(\theta) - \theta x \tag{6}$$

The first derivative of $ln\left(f(x;\theta)\right)$:

$$\frac{\partial\left(ln\left(f(x;\theta)\right)\right)}{\partial\theta} = \frac{d(ln(\theta) - \theta x)}{d\theta}$$

$$= \frac{1}{\theta} - x \tag{7}$$

The second derivative of $ln\left(f(x;\theta)\right)$:

$$\frac{\partial^2\left(ln\left(f(x;\theta)\right)\right)}{\partial\theta^2} = \frac{\partial\left(\frac{1}{\theta} - x\right)}{\partial\theta}$$

$$= -\frac{1}{\theta^2} \tag{8}$$

Based on the results obtained previously, Fisher information is obtained which can be described as follows:

$$I(\theta) = -E\left[\frac{\partial^2 \ln(L(\theta))}{\partial\theta^2}\right]$$

$$= -E\left[-\frac{1}{\theta^2}\right]$$

$$= -E\left[-\frac{1}{\theta^2}\right]$$

$$= \frac{1}{\theta^2} \tag{9}$$

Fisher information above is used to find the shape of the prior distribution. The prior distribution is obtained by calculating the square root of the Fisher Information value. So the form of the prior distribution is obtained as follows:

$$P(\theta) = \sqrt{I(\theta)}$$

$$= \sqrt{\frac{1}{\theta^2}}$$

$$= \frac{1}{\theta} \tag{10}$$

Having obtained the likelihood function and the prior distribution, then the two will be combined to obtain the posterior distribution as follows:

$$P(\theta|X_i) = \frac{L(\theta).P(\theta)}{\int_0^\infty L(\theta).P(\theta)\,d\theta}$$

$$= \frac{\theta^{\sum_{i=1}^n \delta_i}.e^{-\theta\sum_{i=1}^n x_i}.\left(\frac{1}{\theta}\right)}{\int_0^\infty \theta^{\sum_{i=1}^n \delta_i}.e^{-\theta\sum_{i=1}^n x_i}.\left(\frac{1}{\theta}\right)d\theta}$$

$$= \frac{\theta^{\sum_{i=1}^n \delta_i - 1}.e^{-\theta\sum_{i=1}^n x_i}.}{\int_0^\infty \theta^{\sum_{i=1}^n \delta_i - 1}.e^{-\theta\sum_{i=1}^n x_i}d\theta} \quad (11)$$

After obtaining the posterior distribution, it is possible to estimate the parameter θ which was not known before. In this study, to estimate these parameters, the Bayesian Linear Exponential Loss Function method will be used, which is defined as follows:

$$L(\hat{\theta},\theta) = exp\left(a(\hat{\theta} - \theta)\right) - a(\hat{\theta} - \theta) - 1 \text{ with } a \neq 0 \quad (12)$$

Bayesian LINEX estimation of θ in the exponential distribution is obtained by minimizing the posterior expected value of the loss function above which can be described as follows:

Calculating the posterior expected value of the LINEX loss function:

$$E_\theta\left(L(\hat{\theta},\theta)\right) = E_\theta\left[e^{a(\hat{\theta}-\theta)} - a(\hat{\theta} - \theta) - 1\right]$$

$$= E_\theta\left(e^{a(\hat{\theta}-\theta)}\right) - E_\theta\left[a(\hat{\theta} - \theta)\right] - E(1)$$

$$= E_\theta\left(e^{a\hat{\theta}-a\theta}\right) - E_\theta\left(a\hat{\theta} - a\theta\right) - 1$$

$$= E_\theta\left(e^{a\hat{\theta}}.e^{-a\theta}\right) - E_\theta\left(a\hat{\theta} - a\theta\right) - 1$$

$$= E_\theta\left(e^{a\hat{\theta}}\right).E\left(e^{-a\theta}\right) - E_\theta\left(a\hat{\theta}\right) + E_\theta(a\theta) - 1$$

$$= e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right) - a\hat{\theta} + aE_\theta(\theta) - 1$$

$$= e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right) - a\left(\hat{\theta} - E_\theta(\theta)\right) - 1 \quad (13)$$

Then minimize this equation by finding its first derivative with respect to θ and then equating it to zero to obtain the Bayesian LINEX estimator of the parameter θ as follows:

$$\frac{\partial\left(E_\theta\left(L(\hat{\theta},\theta)\right)\right)}{\partial\theta} = 0$$

$$\frac{\partial\left(e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right) - a\left(\hat{\theta} - E_\theta(\theta)\right) - 1\right)}{\partial\theta} = 0$$

$$e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right)(-a) - a = 0$$

$$-a\left(e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right) - 1\right) = 0$$

$$e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right) - 1 = 0$$

$$e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right) = 1$$

$$ln\left(e^{a\hat{\theta}}.E_\theta\left(e^{-a\theta}\right)\right) = ln(1)$$

$$ln\left(e^{a\hat{\theta}}\right) + ln\left(E_\theta\left(e^{-a\theta}\right)\right) = 0$$

$$a\hat{\theta} + ln\left(E_\theta\left(e^{-a\theta}\right)\right) = 0$$

$$\hat{\theta} = \frac{-ln\left(E_\theta\left(e^{-a\theta}\right)\right)}{a}$$

$$\hat{\theta} = -\frac{1}{a}\left(ln\left(E_\theta\left(e^{-a\theta}\right)\right)\right) \quad (14)$$

From this estimator, it is necessary to calculate the value of $E_\theta\left(e^{-a\theta}\right)$ which is the posterior expected value of $e^{-a\theta}$ and can be described as follows:

$$E_\theta\left(e^{-a\theta}\right) = \int_0^\infty e^{-a\theta}.\ P(\theta|X_i)\,d\theta$$

$$= \int_0^\infty e^{-a\theta}\left(\frac{\theta^{\sum_{i=1}^n \delta_i - 1}.e^{-\theta\sum_{i=1}^n x_i}.}{\int_0^\infty \theta^{\sum_{i=1}^n \delta_i - 1}.e^{-\theta\sum_{i=1}^n x_i}d\theta}\right)d\theta$$

$$= \frac{\int_0^\infty \theta^{\sum_{i=1}^n \delta_i - 1}.e^{-\theta(a+\sum_{i=1}^n x_i)}d\theta}{\int_0^\infty \theta^{\sum_{i=1}^n \delta_i - 1}.e^{-\theta\sum_{i=1}^n x_i}d\theta} \quad (15)$$

Then the parameter estimator with the Bayesian LINEX method is as follows:

$$\hat{\theta} = -\frac{1}{a}\left(ln\left(\frac{\int_0^\infty \theta^{(\sum_{i=1}^n \delta_i)-1}.e^{-\theta(a+\sum_{i=1}^n x_i)}d\theta}{\int_0^\infty \theta^{(\sum_{i=1}^n \delta_i)-1}.e^{-\theta\sum_{i=1}^n x_i}d\theta}\right)\right) \quad (16)$$

# 4. Simulation

The application was carried out on secondary data about the survival time of patients with chronic kidney failure in one of the Bojonegoro Hospitals in 2014 [15, 16]. The data is divided into two groups, namely data on patients with initial causes of diabetic disease and data on patients with initial causes of non-diabetic disease. In this study, a sample of 20 patients was taken for patients with early causes of diabetic disease and 63 patients for patients with early causes of non-diabetic diseases who were observed together and the observations were stopped until day 100. After censorship of 20 patients with initial causes of diabetic disease who observed were as many as 12 patients. Meanwhile, of the 63 patients with early causes of non-diabetic disease that could be observed were 30 patients with long shelf life as follows:

***Table 1.*** *Survival time data of patients with early causes of diabetes.*

| Patient | Survival Time (Days) |
|---|---|
| 1 | 2 |
| 2 | 7 |
| 3 | 8 |
| 4 | 9 |
| 5 | 10 |
| 6 | 14 |
| 7 | 26 |
| 8 | 33 |
| 9 | 46 |
| 10 | 67 |
| 11 | 72 |
| 12 | 74 |

***Table 2.*** *Survival Time Data for Patients with Non-Diabetic Diseases.*

| Patient | Survival Time (Days) | Patient | Survival Time (Days) |
|---|---|---|---|
| 1 | 1 | 16 | 22 |
| 2 | 2 | 17 | 23 |
| 3 | 3 | 18 | 24 |
| 4 | 4 | 19 | 30 |
| 5 | 5 | 20 | 32 |
| 6 | 7 | 21 | 33 |
| 7 | 8 | 22 | 34 |
| 8 | 10 | 23 | 39 |
| 9 | 11 | 24 | 43 |
| 10 | 14 | 25 | 49 |
| 11 | 15 | 26 | 71 |
| 12 | 18 | 27 | 72 |
| 13 | 19 | 28 | 89 |
| 14 | 20 | 29 | 93 |
| 15 | 21 | 30 | 99 |

The first step before estimating is to first test the data distribution whether the data follows an exponential distribution or not. A p-value of 0.976 was obtained for patient survival data with initial causes of diabetic disease and 0.878 for patient data with non-diabetic causes of disease. So it can be concluded that the data follows an exponential distribution because the p-value > 0.05.

The estimation results of the θ parameter in patient data with initial causes of diabetic and non-diabetic disease are 0.0102608 and 0.00712166, respectively. This shows that the probability of a patient experiencing failure (death) is 0.0102608 or 1.02608% for patients with the initial cause of diabetic disease. Whereas for patients with the initial cause of non-diabetic disease, the probability of experiencing failure (death) is 0.00712166 or 0.712166%. This means that the probability of experiencing failure (death) for patients with chronic kidney failure with initial causes of diabetic disease is higher than patients with non-diabetic disease initial causes. This is proven based on the statement from dr. Stefanus Cahyo Ariwicaksono, SpU that in diabetes, high blood sugar levels for a long time can cause damage to blood vessels all over the body, including the kidneys. Kidney failure caused by diabetes is known as diabetic nephropathy, and it can cause greater damage to kidney function than non-diabetics [17].

# 5. Conclusion

The form of parameter estimation of the exponential distribution using the Bayesian Linear Exponential Loss Function method on type I censored data based on Jeffrey's priors is as follows:

$$\hat{\theta} = -\frac{1}{a}\left( ln\left( \frac{\int_0^\infty \theta^{\left(\sum_{i=1}^n \delta_i\right)-1} . e^{-\theta\left(a+\sum_{i=1}^n x_i\right)} d\theta}{\int_0^\infty \theta^{\left(\sum_{i=1}^n \delta_i\right)-1} . e^{-\theta \sum_{i=1}^n x_i} d\theta} \right) \right)$$

The estimation results were applied to secondary data about the survival time of patients with chronic kidney failure at one of the Bojonegoro Hospitals in 2014. The estimation result of the parameter θ in the data of patients with chronic kidney failure with the initial cause of diabetes is 0.0102608. While the estimation of the θ parameter in patient data with non-diabetic disease is 0.00712166. This shows that the possibility of a patient experiencing failure (death) is 0.0102608 or 1.02608% for patients with the initial cause of diabetes. Whereas for patients with the initial cause of non-diabetic disease, the probability of failure (death) is 0.00712166 or 0.712166%.

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