International Journal of Management and Fuzzy Systems
2016; 2(1): 1-5
http://www.sciencepublishinggroup.com/j/ijmfs
doi: 10.11648/j.ijmfs.20160201.11

Attitudinal Character Involved Educational Evaluation Models Under Different OWA Aggregation Operators

Cheng Zhu

School of Mathematical Science, Nanjing Normal University, Nanjing, China

Email address:
zhucheng@njnu.edu.cn, 2536565677@qq.com

To cite this article:
Cheng Zhu. Attitudinal Character Involved Educational Evaluation Models Under Different OWA Aggregation Operators. *International Journal of Management and Fuzzy Systems*. Vol. 2, No. 1, 2016, pp. 1-5. doi: 10.11648/j.ijmfs.20160201.11

Received: April 4, 2016; **Accepted**: May 10, 2016; **Published**: June 3, 2016

Abstract: OWA operators, introduced by Yager, are very important non linear aggregation functions in both academic studies and a myriad of applications. In this study, we use two dimensional OWA aggregation function into pedagogical evaluation practice, which will involve the preferences and experiences of decision makers and teachers. In addition, we also introduce a long time educational evaluation model based on Stancu OWA operators with two same parameters. The model involves time orness degree given by teachers and is useful for monitoring long time teaching and learning process in schools.

Keywords: Aggregation Functions, Orness, OWA Operators, Pedagogical Evaluation

1. Introduction

Pedagogical evaluation methods in educational application are very crucial for the daily works of schools and educational organizations. There are a large variety of models to help educators, teachers and school practitioners judge the current situations of students' learning status. Effective, fair and timely evaluation results can also help students both find their shortcomings in study and incentive them to get more studying motivations. In detail, many evaluation models are based on different aggregation functions [2, 5–6, 11, 13, 20] in certain backgrounds. In practice, the evaluation process involves a large amount of subjective preferences of teachers, and thereby we always need some suitable models that can express the preferences and teaching experiences of first-line teachers. A well-developed model can not only present the true performance of students, but also indicate or predict the possible performance of them in the future. However, simple and single evaluation functions have their inherent limitations. In practice we often need some much comprehensive models to improve those single models. In addition, the preferences of teachers (often showing their precious teaching experiences) should also be remembered to add in.

The subjective preferences of decision maker generally are between two extreme cases, i.e., between absolutely optimistic and absolutely pessimistic, in which fuzzy models are often more suitable than traditional dichotomy. Therefore, we can use a variable over real compact unit interval [0, 1] to model the variation of those preferences. However, many preferences are given by teachers with some Ordered Weighted Averaging (OWA) weights [16, 18]. For example, the preference of a teacher to weight the 4 performances of a student is often represented by an OWA weight based evaluation function: let $\mathbf{w}=(w_i)=(0.1,0.2,0.5,0.2)$ be the weights provided by the teacher, and let $\mathbf{a}=(a_i)=(1,0.9,0.6,0.5)$ be the performance vector of a student over 4 tests and they are ordered from the highest to the lowest, and the weighted arithmetic mean $f_{\mathbf{w}}(\mathbf{a})=\mathbf{w}\cdot\mathbf{a}^T=\sum_{i=1}^{4} w_i\cdot a_i$ shows that the teacher is slightly strict or pessimistic, because (s)he puts more weights on the lower performances.

The well-known Ordered Weighted Averaging (OWA) operator [16, 18] is a very important and widely used method. During over twenty years, the OWA operator has been well used and developed in a large variety of theoretical areas and real applications [1, 3–4, 7–10, 12, 14–19]. By using a measurement called orness/andness [16, 18] (or attitudinal character), OWA operators can effectively model the decision-making preferences from absolutely optimistic (corresponding to orness 1) to absolutely pessimistic (corresponding to orness 0). Generally, an OWA operator is a weighting vector $\mathbf{w}=(w_1,\cdots,w_n)$, which could be used to aggregate an input vector $\mathbf{b}=(b_1,\cdots,b_n)$ (ordered in magnitude, i.e., $b_1 \geq b_2 \geq \cdots \geq b_n$) and then we can get the aggregation result $F(\mathbf{b},\mathbf{w}) = \sum_{i=1}^{n} b_i w_i$.

If the teacher wishes to evaluate the comprehensive performance of a group of students, we need a more general frame to still involve the teacher's preferences into the model. In this situation, however, we need to consider two times of the teacher's performance: one is for single student's performances on different tests; the other is for the evaluation preference for different students. In this study, we will develop a reasonable and effective model to model this dual preferences frame and later we will add an additional dimension over time variable; and we will still use the OWA weights and orness, but in time environment the terminologies will be correspondingly called time OWA and t-orness [3] as later we will discuss. And in this study we will introduce some well-known special OWA weights into educational practices and show their reasonability in our tridimensional evaluation model.

The remainder of this study is organized as follows: Section 2 discusses the aggregation method for several students' performances and several subjects using two dimensional OWA operators; the detailed processes of this method including five major steps are also presented. Section 3 proposes the time dimension for long time evaluation model using Time Induced OWA model. Section 4 summarizes the main result of this study.

2. Two Dimensional OWA Weights with Given Preference Pair in Educational Evaluation

Mathematically, OWA operators are nothing but the Choquet integrals [2] with symmetric capacity. However, as we will see, the recursive OWA operators [15] and Time Induced OWA operators [17] are not convenient to be seen as Choquet integrals.

Definition 1. [16] An OWA operator of dimension n is a mapping $F:(-\infty,+\infty)^n \rightarrow (-\infty,+\infty)$, which has an associated weighting vector $\mathbf{w}=(w_1,w_2,\ldots,w_n)$ satisfying the following properties

$$\sum_{j=1}^{n} w_j = 1; \qquad 0 \leq w_j \leq 1; \qquad j=1,2,\ldots,n$$

and such that

$$F_{\mathbf{w}}(a_1,a_2,\ldots,a_n) = \sum_{j=1}^{n} w_j \cdot a_{\sigma(j)}, \tag{1}$$

where $\sigma:\{1,\ldots,n\} \rightarrow \{1,\ldots,n\}$ is a permutation such that $a_{\sigma(i)} \geq a_{\sigma(j)}$ whenever $i<j$.

Definition 2. [16] The degree of "*orness*" associated with this operator is defined as

$$\text{orness}(\mathbf{w}) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i = \sum_{i=1}^{n}\frac{n-i}{n-1}w_i. \tag{2}$$

The measure of "*andness*" associated with an OWA operator is the complement of its "*orness*", and is defined as

$$\text{andness}(\mathbf{w}) = 1 - \text{orness}(\mathbf{w})$$

or

$$\text{andness}(\mathbf{w}) = \frac{1}{n-1}\sum_{i=1}^{n}(i-1)w_i = \sum_{i=1}^{n}\frac{i-1}{n-1}w_i.$$

Proposition 1. [16] The *max*, *min* and *average* operator correspond to $\mathbf{w}^* = (1,0,\ldots,0)$, $\mathbf{w}_* = (0,\ldots,0,1)$ and $\mathbf{w}_A = (1/n, 1/n, \ldots, 1/n)$, respectively, and $\text{orness}(\mathbf{w}^*) = 1$, $\text{orness}(\mathbf{w}_*) = 0$ and $\text{orness}(\mathbf{w}_A) = 1/2$.

Remark In this paper, the max can represent the oldest data/input/score and conversely the min can represent the most recent one.

The next proposition shows one particular reverse property related to the *orness* degree of two OWA operators that are reverse of each other.

Proposition 2. [16] (*Reverse Property*) For any OWA weighting vector $\mathbf{w}=(w_1,w_2,\ldots,w_n)$, $\text{orness}(\mathbf{w}) = \alpha$, then for the reverse of $\mathbf{w}$: $\mathbf{w}' = (w_1', w_2', \ldots, w_n') = (w_n, w_{n-1}, \ldots, w_1)$, $\text{orness}(\mathbf{w}') = 1-\alpha$.

In educational evaluation practice, suppose for one comprehensive test of several subjects M_j $(j=1,\ldots,p)$, student A_i $(i=1,\ldots,q)$ obtains p performances (represented as a vector) for different subjects: s_{ij}. Thus, we can form a $q \times p$ performance matrix $S=(s_{ij})$; and each row of S, $\mathbf{s}_i = (s_{i1},\ldots,s_{ip})$ represents score/performance vector of student A_i, while each column of S, $\mathbf{t}_j = (t_{1j},\ldots,t_{qj})$ represents the performances of all the students for only one certain subject M_j. If the teacher's attitude between different subjects is α and his preference for all students is β, then we given select two OWA operators $\mathbf{u}^{(p)}$ and $\mathbf{v}^{(q)}$ with orness α and β, respectively, for him. In this study, the selection for $\mathbf{u}^{(p)}$ and $\mathbf{v}^{(q)}$ is arbitrary.

Therefore, we can firstly aggregate $\mathbf{s}_i = (s_{i1},\ldots,s_{ip})$ under the same OWA weighting vector $\mathbf{u}^{(p)}$, i.e., $F_{\mathbf{u}^{(p)}}(\mathbf{s}_i)$; and then aggregate score vectors for different students $\mathbf{f} = \left(F_{\mathbf{u}}(\mathbf{s}_1),\ldots,F_{\mathbf{u}}(\mathbf{s}_q)\right)$ with OWA weighting vector $\mathbf{v}^{(q)}$, i.e., $F_{\mathbf{v}^{(q)}}(\mathbf{f})$.

For example, suppose we have the performance matrix with 3×4:

$$S = \begin{bmatrix} 6 & 6 & 10 & 8 \\ 8 & 10 & 9 & 8 \\ 7 & 8 & 6 & 5 \end{bmatrix}.$$

And the teacher's two attitudinal characters for subjects and students are $\alpha = 2/3$ and $\beta = 3/4$, respectively. We can select two monotonic OWA weighting vectors $\mathbf{u}^{(4)} = (0.4, 0.3, 0.2, 0.1)$ and $\mathbf{v}^{(3)} = (0.6, 0.3, 0.1)$, with orness values $orness(\mathbf{u}^{(4)}) = 2/3$ and $orness(\mathbf{v}^{(3)}) = 3/4$. Note that these two attitudes in practice may represent the teacher is slightly optimistic or with some confidence.

Consequently, we compute to obtain:

$$F_{\mathbf{u}^{(4)}}(\mathbf{s}_1) = (0.4)(10) + (0.3)(8) + (0.2)(6) + (0.1)(6) = 8.2;$$

$$F_{\mathbf{u}^{(4)}}(\mathbf{s}_2) = (0.4)(10) + (0.3)(9) + (0.2)(8) + (0.1)(8) = 9.1;$$

$$F_{\mathbf{u}^{(4)}}(\mathbf{s}_3) = (0.4)(8) + (0.3)(7) + (0.2)(6) + (0.1)(5) = 7.$$

Thus, $\mathbf{f} = (8.2, 9.1, 7)$, and we have

$$\begin{aligned} F_{\mathbf{v}^{(3)}}(\mathbf{f}) &= (0.6)(9.1) + (0.3)(8.2) + (0.1)(7) \\ &= 5.46 + 2.46 + 0.7 = 8.62 \end{aligned}.$$

As a comprehensive result, we obtain score 8.62 as the evaluated performance of those three students with four different subjects. By comparison to tradition weighted arithmetical mean, this result involves useful preferences and experiences of first line teachers. Therefore, a much more comprehensive evaluation for a collection (say, for all students of a class) is obtained.

The detailed process including 5 major steps of this method is as follows.

The detailed process of Two dimensional OWA evaluation:

Step 1: Determine the $q \times p$ performance matrix $S = (s_{ij})$ of objects being evaluated. Each row of *S*, $\mathbf{s}_i = (s_{i1}, ..., s_{ip})$ represents score/performance vector of student A_i, while each column of *S*, $\mathbf{t}_j = (t_{1j}, ..., t_{qj})$ represents the performances of all the students for only one certain subject M_j.

Step 2: Determine preference degree (orness) α with respect to different subjects and an OWA vector $\mathbf{u}^{(p)}$ such that $orness(\mathbf{u}^{(p)}) = \alpha$.

Step 3: Determine preference degree (orness) β with respect to different students and an OWA vector $\mathbf{v}^{(q)}$ such that $orness(\mathbf{v}^{(q)}) = \beta$.

Step 4: Aggregate $\mathbf{s}_i = (s_{i1}, ..., s_{ip})$ under the same OWA weighting vector $\mathbf{u}^{(p)}$ to obtain $F_{\mathbf{u}^{(p)}}(\mathbf{s}_i)$ ($i = 1, ..., q$).

Step 5: Aggregate score vectors for different students $\mathbf{f} = (F_{\mathbf{u}}(\mathbf{s}_1), ..., F_{\mathbf{u}}(\mathbf{s}_q))$ with OWA weighting vector $\mathbf{v}^{(q)}$ to obtain $F_{\mathbf{v}^{(q)}}(\mathbf{f})$. Then, $F_{\mathbf{v}^{(q)}}(\mathbf{f})$ is the final comprehensive evaluation result.

3. Comprehensive Time Induce OWA Merge for Long Time Evaluation

The Time Induced OWA (*IOWA*) operator was proposed by Yager in [17]. The IOWA operator is used to aggregate pairs of the form (t_i, a_i) ($i = 1, 2, \cdots, n$). Within these pairs, t_i is called the *order-inducing value/variable* and a_i is called the *argument value/variable* [3]. Note that a_i s are the elements that need to be aggregated, rather than t_i s.

Therefore, for *n* pairs of the form (t_i, a_i) ($i = 1, 2, \cdots, n$), an Time IOWA operator of dimension n is a mapping $F_{\mathbf{w}} : (-\infty, +\infty)^n \to (-\infty, +\infty)$, which also has an associated weighting vector $\mathbf{w} = (w_1, w_2, \ldots, w_n)$ satisfying the following properties

$$\sum_{j=1}^{n} w_j = 1; \qquad 0 \le w_j \le 1; \qquad j = 1, 2, \ldots, n$$

such that

$$F_{\mathbf{w}}\left[(t_1, a_1), (t_2, a_2), \cdots, (t_n, a_n)\right] = \sum_{j=1}^{n} w_j b_j = \mathbf{w}\mathbf{b}^T,$$

where $\mathbf{b} = (b_1, b_2, \ldots, b_n)$ is simply the reordered form of $\mathbf{a} = (a_1, a_2, \ldots, a_n)$, which corresponds to the linear ordering relation "$\prec$" in the linearly ordered set (or chain): $C = (\{t_1, t_2, \ldots, t_n\}, \prec)$.

For example (see [3]), suppose $(t_1, a_1) = (2001, 3)$, $(t_2, a_2) = (2002, 1)$, $(t_3, a_3) = (2003, 2)$, $(t_4, a_4) = (2004, 4)$, t_i represent years, the time-ordered arguments vector to be aggregated is not $\mathbf{b} = (4, 3, 2, 1)$, but $\mathbf{b} = (3, 1, 2, 4)$. In time series, the newly arrived argument is always (t_{n+1}, a_{n+1}), which means that we only need to add a_{n+1} to the rightmost side of the original time-ordered arguments vector $\mathbf{b}^{(n)} = (a_1, a_2, \cdots, a_n)$ to obtain the new time-ordered arguments vector $\mathbf{b}^{(n+1)} = (a_1, a_2, \cdots, a_n, a_{n+1})$.

Suppose every month we have a comprehensive test with several subjects for students, using the methods aforementioned in Section 2, we can obtain the evaluation result for a certain month, say, M_k $(k = 1, 2, ...)$. If we use the similar method for different *k*, and with the different selection for $\mathbf{u}^{(p)}$ and $\mathbf{v}^{(q)}$, we can compute the scores for every month as s_k $(k = 1, 2, ...)$. In addition, this process can be endless; that is, we can use a similar way to well-known exponential smooth weights for every month. Therefore, we can timely know the current learning and teaching status in each organization unit in school. In this sense, we need a recursive model like exponential smooth method. In this study, we will use the recursive OWA weights [15] to aggregate the performances of each month.

Based on Stancu Polynomials, Singh et al. proposed the *Stancu OWA operators* [12]. For an OWA operator of dimension i, $\mathbf{u}^{(i)}=(u_{i1},u_{i2},\ldots,u_{ii})$ $(i=1,2,\ldots)$, if its weights satisfy the condition (an empty product denotes 1)

$$u_{ij}=\frac{\binom{i-1}{i-j}\prod_{s=0}^{i-j-1}(t+s\alpha)\prod_{s=0}^{j-2}(1-t+s\alpha)}{\prod_{s=0}^{i-2}(1+s\alpha)} \tag{3}$$

with $t\in[0,1]$, $\alpha\geq 0$ being two parameters, then it is called a *Stancu OWA operator*.

Proposition 3. [4] Left Recursive OWA operators is a particular case of Stancu OWA operators with two equal parameters, i.e., $t=\alpha$.

We also need to review some newly finding results [5].

Proposition 4. For any Recursive OWA operator (explicit form) $\mathbf{w}^{(n)}=(w_1^{(n)},w_2^{(n)},\cdots,w_n^{(n)})$ $(n=2,3,\ldots)$ with constant orness α

$$w_i^{(n)}=\frac{1}{i-1}\left[(i-2)+\frac{1-\alpha}{\alpha}\right]\cdot w_{i-1}^{(n)},\ i=2,3,\ldots,n$$

Proposition 5. For any two families of Recursive OWA operators with the same dimension: $\mathbf{w}^{(n)}=(w_1^{(n)},w_2^{(n)},\cdots,w_n^{(n)})$ and $\mathbf{v}^{(n)}=(v_1^{(n)},v_2^{(n)},\cdots,v_n^{(n)})$ $(n=2,3,\ldots)$, such that $orness(\mathbf{w}^{(n)})=\alpha$ and $orness(\mathbf{v}^{(n)})=\beta$. If $\alpha>\beta$, then $\frac{w_i^{(n)}}{w_{i-1}^{(n)}}<\frac{v_i^{(n)}}{v_{i-1}^{(n)}}$ $(i=2,3,\ldots,n)$.

Proposition 6. For any Recursive OWA operator (explicit form) $\mathbf{w}^{(n)}=(w_1^{(n)},w_2^{(n)},\cdots,w_n^{(n)})$ $(n=2,3,\ldots)$ with the constant orness α,

if $\alpha>0.5$ then $w_i^{(n)}<w_{i-1}^{(n)}$, $i=2,3,\ldots,n$

if $\alpha<0.5$ then $w_i^{(n)}>w_{i-1}^{(n)}$, $i=2,3,\ldots,n$

For Recursive OWA operator, when a new argument s_{n+1} (represent the comprehensive score in month n+1), arrives at the rightmost of the original arguments vector $\mathbf{s}^{(n)}$ to form:

$$\mathbf{s}^{(n+1)}=[\mathbf{s}^{(n)},s_{n+1}]=[s_1,\cdots,s_n,s_{n+1}],$$

if we wish to keep the time orness α unchanged (time andness $1-\alpha$ also unchaged), we can use the following formula [20] to obtain the new aggregation result:

$$\text{Let } k^{(n+1)}=\frac{1-\alpha}{1+(n-1)\alpha} \tag{4}$$

then $F_{\mathbf{w}^{(n+1)}}(\mathbf{s}^{(n+1)})=(1-k^{(n+1)})F_{\mathbf{w}^{(n)}}(\mathbf{s}^{(n)})+k^{(n+1)}\cdot s_{n+1}$ has the same underlying time orness α (or time andness $1-\alpha$). In other words, it can be shown that the *implicit* OWA operator $\mathbf{w}^{(n+1)}$ obtained repeatedly by this formula has the property: orness$(\mathbf{w}^{(n+1)})=\alpha$ for any $n=2,3,\ldots$, and $\mathbf{w}^{(n+1)}$ needs not to be explicitly expressed.

If one needs to continue the evaluation problem for the next month, and by using (4), we can use the recursive method to evaluate each month's comprehensive performance of all students based on the historical performance and the new one – this process is very similar to the exponential smooth process. However, the traditional ES model can not involve the time attitude of teachers/decision-makers.

For example, suppose the teacher's time attitude degree (time andness) is 3/4, and suppose $\mathbf{s}^{(2)}=(s_1,s_2)$ being the two scores of the first month and the second month, then $\mathbf{w}^{(2)}=(w_1^{(2)},w_2^{(2)})=(1/4,\ 3/4)$, and we have $F_{\mathbf{w}^{(2)}}(\mathbf{s}^{(2)})=w_1^{(2)}s_1+w_2^{(2)}s_2$. A new month passed, and a new score s_3 arrives at the rightmost of $\mathbf{s}^{(2)}$ to form $\mathbf{s}^{(3)}=(s_1,s_2,s_3)$, and we wish to keep the time andness degree 3/4 unchanged for $\mathbf{w}^{(3)}$, then we can create $\mathbf{w}^{(3)}$ by the following recursive way:

$$\begin{aligned}\mathbf{w}^{(3)}&=[\frac{2}{5}\mathbf{w}^{(2)},\ \frac{3}{5}]=[\frac{2}{5}(w_1^{(2)},w_2^{(2)}),\ \frac{3}{5}]=[\frac{2}{5}(1/4,\ 3/4),\ \frac{3}{5}]\\&=[0.1,0.3,0.6]=[w_1^{(3)},w_2^{(3)},w_3^{(3)}]\end{aligned}$$

and then

$$\begin{aligned}F_{\mathbf{w}^{(3)}}(\mathbf{s}^{(3)})=\mathbf{w}^{(3)}\cdot(\mathbf{s}^{(3)})^T&=\frac{2}{5}\mathbf{w}^{(2)}\cdot(\mathbf{s}^{(2)})^T+\frac{3}{5}s_3\\&=\frac{2}{5}F_{\mathbf{w}^{(2)}}(\mathbf{s}^{(2)})+\frac{3}{5}s_3\end{aligned}$$

if the comprehensive score of Month4 is s_4, then $F_{\mathbf{w}^{(4)}}(\mathbf{s}^{(4)})=\frac{1}{2}F_{\mathbf{w}^{(3)}}(\mathbf{s}^{(3)})+\frac{1}{2}s_4$. And we can find that the underlying "explicit" I-OWA operator $\mathbf{w}^{(4)}=[0.05,0.15,0.3,0.5]$ with time orness 1/4 (time andness 3/4).

With this similar recursive process, we can very conveniently provide the comprehensive scores of those students at every time nodes. Therefore, we can real time monitor the learning and teaching performance of every month node, while keep the involved time attitude unchanged.

4. Conclusions

In this study, we introduced the two dimensional OWA aggregation function into pedagogical evaluation practice. With the proposed models, we can obtain a comprehensive performance for and $p\times q$ score matrix. The model can involve the experiences and preferences of first line teachers and thereby more useful and reaonsable. Furthermore, we also considered a long time educational evaluation model which uses Stancu OWA operators and therefore quite convenient and reasonable. The model involving time attitude of teacher also can monitor the long time teaching and learning status in teaching organizations and schools.

References

[1] O. Aristondo, J. L. García-Lapresta, C. Lasso de la Vega, R. A. Marques Pereira, Classical inequality indices, welfare and illfare functions, and the dual decomposition, Fuzzy Sets Syst. 228 (2013) 114–136.

[2] M. Grabisch, J. L. Marichal, R. Mesiar, E. Pap, Aggregation Functions, Cambridge University Press 2009, ISBN: 1107013429.

[3] L. Jin, Some properties and representation methods for Ordered Weighted Averaging operators, Fuzzy Sets Syst. 261 (2015) 60–86.

[4] L. Jin, G. Qian, OWA Generation Function and some adjustment methods for OWA operators with Application, IEEE Trans. Fuzzy Syst. 24 (1) (2016) 168–178.

[5] L. Jin, M. Kalina, G. Qian, Discrete and continuous recursive forms for OWA operators, (submitted paper)

[6] A. Kolesárová, R. Mesiar, On linear and quadratic constructions of aggregation functions, Fuzzy Sets Syst. 268 (2014) 1–14.

[7] T. Leon, P. Zuccarello, G. Ayala, E. de Ves, J. Domingo, Applying logistic regression to relevance feedback in image retrieval systems, Pattern Recognit. 40 (2007) 2621–2632.

[8] X. W. Liu, S. L. Han, Orness and parameterized RIM quantifier aggregation with OWA operators: A summary, Int. J. Approx. Reasoning 48 (2008) 77–97.

[9] J. M. Merigo, A. M. Gil-Lafuente, The induced generalized OWA operator, Inf. Sci. 179 (6) (2009) 729–741.

[10] R. Mesiar, A. Stupňanová, R. R. Yager, Generalizations of OWA operators, IEEE Trans. Fuzzy Syst. 23 (6) (2015) 2154–2162.

[11] R. Mesiar, E. Pap, Aggregation of infinite sequences, Inf. Sci. 178 (2008) 3557–3564.

[12] A. K. Singh, A. Kishor, N. R. Pal. Stancu OWA Operator, IEEE Trans. Fuzzy Syst. 23 (4) (2015) 1306–1313.

[13] J. Špirková, Weighted operators based on dissimilarity function, Inf. Sci. 281 (2014) 172–181.

[14] V. Torra, The weighted OWA operator, Int. J. Intell. Syst. 12 (1997) 153–166.

[15] L. Troiano, R. R. Yager, Recursive and iterative OWA operators, Int. J. Uncertainty Fuzziness Knowl.-Based Syst. 13(6) (2005) 579–599.

[16] R. R. Yager, On ordered weighted averaging aggregation operators in multicriteria decision making, IEEE Trans. Syst. Man Cybern. 18 (1) (1988) 183–190.

[17] R. R. Yager, Time series smoothing and OWA aggregation, IEEE Trans. Fuzzy Syst. 16 (4) (2008) 994–1007.

[18] R. R. Yager, Families of OWA operators, Fuzzy Sets Syst. 59 (1993) 125–143.

[19] R. R. Yager, J. Kacprzyk, G. Beliakov, Recent Developments on the Ordered Weighted Averaging Operators: Theory and Practice, Springer-Verlag, Berlin, 2011.

[20] R. R. Yager, R. Mesiar, On the Transformation of Fuzzy Measures to the Power Set and its Role in Determining the Measure of a Measure, IEEE Trans. Fuzzy Syst. 23 (4) (2015) 842–849.