International Journal of Information and Communication Sciences
2016; 1(2): 22-29
http://www.sciencepublishinggroup.com/j/ijics
doi: 10.11648/j.ijics.20160102.12

Decision Wave Equation and Block Diagram of Electromagnetoelastic Actuator Nano- and Microdisplacement for Communications Systems

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To cite this article:
S. M. Afonin. Decision Wave Equation and Block Diagram of Electromagnetoelastic Actuator Nano- and Microdisplacement for Communications Systems. *International Journal of Information and Communication Sciences*. Vol. 1, No. 2, 2016, pp. 22-29. doi: 10.11648/j.ijics.20160102.12

Received: August 2, 2016; **Accepted**: August 12, 2016; **Published**: September 6, 2016

Abstract: Decision wave equation, structural-parametric model and block diagram of electromagnetoelastic actuators are obtained, its transfer functions are bult. Effects of geometric and physical parameters of electromagnetoelastic actuators and external load on its dynamic characteristics are determined. For calculation of communications systems with piezoactuators the block diagram and the transfer functions of piezoactuators are obtained.

Keywords: Electromagnetoelastic Actuators, Piezoactuator, Block Diagram, Transfer Functions

1. Introduction

The application of precise electromechanical actuators based on electromagnetoelasticity (piezoelectric, piezomagnetic, electrostriction, and magnetostriction effects) is promising in nanotechnology, nanobiology, power engineering, microelectronics, astronomy for large compound telescopes, antennas satellite telescopes and adaptive optics equipment for precision matching, compensation of temperature and gravitation deformations, and atmospheric turbulence via wave front correction. Precise electromechanical actuators for communications systems operate within working loads providing elastic deformations of actuators. Piezoelectric actuator (piezoactuator) - piezomechanical device intended for actuation of mechanisms, systems or management based on the piezoelectric effect, converts electrical signals into mechanical movement or force [1 − 25].

The piezoactuator of nanometric movements operates based on the inverse piezoeffect, in which the motion is achieved due to deformation of the piezoelement when an external electric voltage is applied to it. Piezoactuators for drives of nano- and micrometric movements provide a movement range from several nanometers to tens of microns, a sensitivity of up to 10 nm/V, a loading capacity of up to 1000 N, the power at the output shaft of up to 100 W, and a ransmission band of up to 1000 Hz. The investigation of static and dynamic characteristics of a piezoactuator of nano- and micrometric movements as the control object is necessary for calculation the piezodrive for control systems of nano- and micrometric movements. At the nano- and microlevels, piezoactuators are used in linear nano- and microdrives and micropumps. Piezoactuators provide high stress and speed of operation and return to the initial state when switched off; they have very low displacements - less than 1%. Piezoactuators are used in the majority of nanomanipulators for scanning tunneling microscopes (STMs), scanning force microscopes (SFMs), and atomic force microscopes (AFMs). Nanorobotic manipulators with nano- and microdisplacements with piezoactuators based are a key component in nano- and microdisplacement nanorobotic systems. The main requirement for nanomanipulators is to guarantee the positioning accurate to nanometers, control systems are intended not only for nanomanipulations but also for nanoassembly, nanomeasurements, and nanomanufacturing [1 − 6].

By solving the wave equation with allowance for the corresponding equations of the piezoeffect, the boundary conditions on loaded working surfaces of a piezoactuator, and the strains along the coordinate axes, it is possible to construct a structural parametric model of the piezoactuator. The transfer functions and the parametric structure scheme of the piezoactuator are obtained from a set of equations describing the corresponding structural parametric model of the piezoelectric actuator for communications systems.

2. Decision Wave Equation and Structural-Parametric Model of Electromagnetoelastic Ectuator

Deformation of the piezoactuator corresponds to its stressed state. If the mechanical stress T is created in the piezoelectric element, the deformation S is formed in it. There are six stress components T_1, T_2, T_3, T_4, T_5, T_6, the components T_1 - T_3 are related to extension-compression stresses, T_4 - T_6 to shear stresses.

The matrix state equations [7] connecting the electric and elastic variables for polarized ceramics have the form

$$\mathbf{D} = \mathbf{dT} + \boldsymbol{\varepsilon}^T \mathbf{E}, \tag{1}$$

$$\mathbf{S} = \mathbf{s}^E \mathbf{T} + \mathbf{d}^t \mathbf{E}. \tag{2}$$

Here, the first equation describes the direct piezoelectric effect, and the second - the inverse piezoelectric effect; $\mathbf{S}$ is the column matrix of relative deformations; $\mathbf{T}$ is the column matrix of mechanical stresses; $\mathbf{E}$ is the column matrix of electric field strength along the coordinate axes; $\mathbf{D}$ is the column matrix of electric induction along the coordinate axes; $\mathbf{s}^E$ is the elastic compliance matrix for $E = \text{const}$; $\boldsymbol{\varepsilon}^T$ is the matrix of dielectric constants for $T = \text{const}$; $\mathbf{d}^t$ is the transposed matrix of the piezoelectric modules.

Polarized ceramics PZT represents the piezoelectric texture, there are five independent components s_{11}^E, s_{12}^E, s_{13}^E, s_{33}^E, s_{55}^E in the elastic compliance matrix for polarized piezoelectric ceramics

$$\mathbf{s}_{ij}^E = \begin{pmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{12}^E & s_{11}^E & s_{13}^E & 0 & 0 & 0 \\ s_{13}^E & s_{13}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{55}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(s_{11}^E - s_{12}^E) \end{pmatrix}.$$

In this case, we can write the transposed matrix of the piezoelectric modules $\mathbf{d}^t$ as

$$\mathbf{d}_{ij}^t = \begin{pmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The matrix of dielectric constants $\boldsymbol{\varepsilon}^T$ has the form

$$\boldsymbol{\varepsilon}^T = \begin{pmatrix} \varepsilon_{11}^{\mathrm{T}} & 0 & 0 \\ 0 & \varepsilon_{22}^{\mathrm{T}} & 0 \\ 0 & 0 & \varepsilon_{33}^{\mathrm{T}} \end{pmatrix}.$$

The direction of the polarization axis P, i.e., the direction along which polarization was performed, is usually taken as the direction of axis 3.

The equation of electromagnetoelasticity of the actuator [7] has the form

$$S_i = s_{ij}^{E,H,\Theta} T_j + d_{mi}^{H,\Theta} E_m + d_{mi}^{E,\Theta} H_m + \alpha_i^{E,H} \Delta\Theta, \tag{3}$$

where S_i is the relative deformation along the axis i, E is the electric field strength, H is the magnetic field strength, Θ is the temperature, $s_{ij}^{E,H,\Theta}$ is the elastic compliance for $E = \text{const}$, $H = \text{const}$, $\Theta = \text{const}$, T_j is the mechanical stress along the axis j, $d_{mi}^{H,\Theta}$ is the piezomodule, i.e., the partial derivative of the relative deformation with respect to the electric field strength for constant magnetic field strength and temperature, i.e., for $H = \text{const}$, $\Theta = \text{const}$, E_m is the electric field strength along the axis m, $d_{mi}^{E,\Theta}$ is the magnetostriction coefficient, H_m is the magnetic field strength along the axis m, $\alpha_i^{E,H}$ is the coefficient of thermal expansion, $\Delta\Theta$ is deviation of the temperature Θ from the value $\Theta = \text{const}$, i = 1, 2, … , 6, j = 1, 2, … , 6, m = 1, 2, 3.

When the electric and magnetic fields act on the electromagnetoelastic actuator separately, we have the respective electromagnetoelasticity equations [7] as the equation of inverse piezoelectric effect:

$S_3 = d_{33} E_3 + s_{33}^E T_3$ for the longitudinal deformation when the electric field along axis 3 causes deformation along axis 3,

$S_1 = d_{31} E_3 + s_{11}^E T_1$ for the transverse deformation when the electric field along axis 3 causes deformation along axis 1,

$S_5 = d_{15} E_1 + s_{55}^E T_5$ for the shift deformation when the electric field along axis 1 causes deformation in the plane perpendicular to this axis, as the equation of magnetostriction:

$S_3 = d_{33} H_3 + s_{33}^H T_3$ for the longitudinal deformation when the magnetic field along axis 3 causes deformation along axis 3,

$S_1 = d_{31} H_3 + s_{11}^H T_1$ for the transverse deformation when the magnetic field along axis 3 causes deformation along axis 1,

$S_5 = d_{15} H_1 + s_{55}^H T_5$ for the shift deformation when the magnetic field along axis 1 causes deformation in the plane perpendicular to this axis.

To illustrate this, we consider piezoelasticity problems. Let us consider the longitudinal piezoelectric effect in a piezoelectric actuator shown in Fig. 1, which represents a piezoelectric plate of thickness δ with the electrodes deposited on its faces perpendicular to axis 3, the area of which is equal to S_0.

The equation of the inverse piezoelectric effect [6, 7] for the longitudinal strain in a voltage-controlled piezoactuator has the following form:

$$S_3 = d_{33}E_3(t) + s_{33}^E T_3(x,t)\,, \tag{4}$$

Here, $S_3 = \partial\xi(x,t)/\partial x$ is the relative displacement of the cross section of the piezoactuator, d_{33} is the piezoelectric modulus for the longitudinal piezoelectric effect, $E_3(t) = U(t)/\delta$ is the electric field strength, $U(t)$ is the voltage between the electrodes of actuator, δ is the thickness, s_{33}^E is the elastic compliance along axis 3, and T_3 is the mechanical stress along axis 3.

The equation of equilibrium for the forces acting on the piezoactuator (piezoelectric plate) can be written as

$$T_3 S_0 = F + M\frac{\partial^2\xi(x,t)}{\partial t^2}$$

where F is the external force applied to the piezoactuator, S_0 is the cross section area and M is the displaced mass.

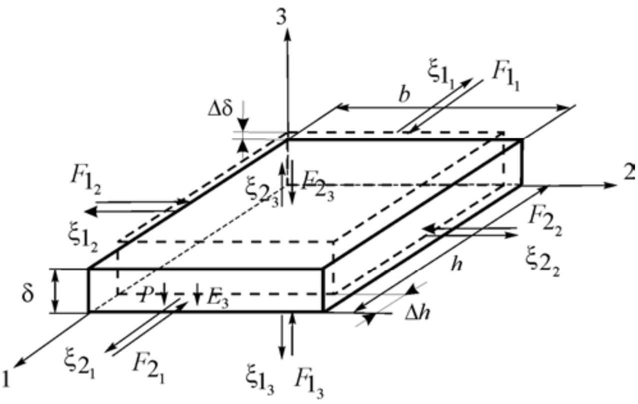

Fig. 1. *Piezoactuator.*

For constructing a structural parametric model of the voltage-controlled piezoactuator, let us solve simultaneously the wave equation, the equation of the inverse longitudinal piezoelectric effect, and the equation of forces acting on the faces of the piezoactuator.

Calculations of the piezoactuators are performed using a wave equation [5−7] describing the wave propagation in a long line with damping but without distortions, which can be written as

$$\frac{1}{(c^E)^2}\frac{\partial^2\xi(x,t)}{\partial t^2} + \frac{2\alpha}{c^E}\frac{\partial\xi(x,t)}{\partial t} + \alpha^2\xi(x,t) = \frac{\partial^2\xi(x,t)}{\partial x^2}\,, \tag{5}$$

where $\xi(x,t)$ is the displacement of the section of the piezoelectric plate, x is the coordinate, t is time, c^E is the sound speed for $E = \text{const}$, α is the damping coefficient that takes into account the attenuation of oscillations caused by the energy dissipation due to thermal losses during the wave propagation.

Using the Laplace transform, we can reduce the original problem for the partial differential hyperbolic equation of type (5) to a simpler problem for the linear ordinary differential equation [8, 9] with the parameter of the Laplace operator p.

Applying the Laplace transform to the wave equation (5)

$$\Xi(x,p) = L\{\xi(x,t)\} = \int_0^\infty \xi(x,t)e^{-pt}dt\,, \tag{6}$$

and setting the zero initial conditions,

$$\xi(x,t)\big|_{t=0} = \frac{\partial\xi(x,t)}{\partial t}\bigg|_{t=0} = 0\,. \tag{7}$$

As a result, we obtain the linear ordinary second-order differential equation with the parameter p written as

$$\frac{d^2\Xi(x,p)}{dx^2} - \left[\frac{1}{(c^E)^2}p^2 + \frac{2\alpha}{c^E}p + \alpha^2\right]\Xi(x,p) = 0\,, \tag{8}$$

with its solution being the function

$$\Xi(x,p) = Ce^{-x\gamma} + Be^{x\gamma}\,, \tag{9}$$

where $\Xi(x,p)$ is the Laplace transform of the displacement of the section of the piezoelectric actuator, $\gamma = p/c^E + \alpha$ is the propagation coefficient.

C and B are constant coefficients. Determining these coefficients from the boundary conditions as

$$\Xi(0,p) = \Xi_1(p) \text{ for } x = 0$$

$$\Xi(\delta,p) = \Xi_2(p) \text{ for } x = \delta$$

Then, the constant coefficients

$$C = \left(\Xi_1 e^{\delta\gamma} - \Xi_2\right)/[2\mathrm{sh}(\delta\gamma)],\quad B = -\left(\Xi_1 e^{-\delta\gamma} - \Xi_2\right)/[2\mathrm{sh}(\delta\gamma)].$$

Then, the solution (9) of the linear ordinary second-order differential equation can be written as

$$\Xi(x,p) = \{\Xi_1(p)\mathrm{sh}[(\delta - x)\gamma] + \Xi_2(p)\mathrm{sh}(x\gamma)\}/\mathrm{sh}(\delta\gamma)\,. \tag{10}$$

The equations for the forces operating on the faces of the piezoelectric actuator plate are as follows:

$$T_3(0,p)S_0 = F_1(p) + M_1p^2\Xi_1(p) \qquad \text{for} \quad x = 0\,, \tag{11}$$

$$T_3(\delta,p)S_0 = -F_2(p) - M_2p^2\Xi_1(p) \qquad \text{for} \quad x = \delta\,,$$

where $T_3(0,p)$ and $T_3(\delta,p)$ are determined from the equation of the inverse piezoelectric effect.

For $x = 0$ and $x = \delta$, we obtain the following set of equations for determining stresses in the piezoactuator:

$$T_3(0,p) = \frac{1}{s_{33}^E}\frac{d\Xi(x,p)}{dx}\bigg|_{x=0} - \frac{d_{33}}{s_{33}^E}E_3(p)\,, \tag{12}$$

$$T_3(\delta,p) = \frac{1}{s_{33}^E}\frac{d\Xi(x,p)}{dx}\bigg|_{x=\delta} - \frac{d_{33}}{s_{33}^E}E_3(p)\,.$$

Equations (11) yield the following set of equations for the structural parametric model of the piezoactuator:

$$\Xi_1(p) = \left[1/\left(M_1 p^2\right)\right] \cdot \left\{-F_1(p) + \left(1/\chi_{33}^E\right)\left[d_{33}E_3(p) - [\gamma/\mathrm{sh}(\delta\gamma)]\left[\mathrm{ch}(\delta\gamma)\Xi_1(p) - \Xi_2(p)\right]\right]\right\}, \quad (13)$$

$$\Xi_2(p) = \left[1/\left(M_2 p^2\right)\right] \cdot \left\{-F_2(p) + \left(1/\chi_{33}^E\right)\left[d_{33}E_3(p) - [\gamma/\mathrm{sh}(\delta\gamma)]\left[\mathrm{ch}(\delta\gamma)\Xi_2(p) - \Xi_1(p)\right]\right]\right\},$$

where $\chi_{33}^E = s_{33}^E / S_0 = \delta/\left[m\left(c^E\right)^2\right]$, m is the mass of the piezoactuator.

Figure 2 shows the parametric block diagram of a voltage-controlled piezoactuator corresponding to the set of equations (13) supplemented with an external circuit equation $U(p) = U_0(p)/(RC_0 p + 1)$, where $U_0(p)$ is the supply voltage, R is the resistance of the external circuit, and C_0 is the static capacitance of the piezoactuator.

The equation of the inverse piezoelectric effect [6, 7] for the transverse strain in the voltage-controlled piezoactuator

$$S_1 = d_{31}E_3(t) + s_{11}^E T_1(x,t), \quad (14)$$

where $S_1 = \partial\xi(x,t)/\partial x$ is the relative displacement of the cross section of the piezoactuator along axis 1, d_{31} is the piezoelectric modulus for the transverse piezoelectric effect, s_{11}^E is the elastic compliance along axis 1, and T_1 is the stress along axis 1.

The wave equation of the piezoactuator can be written as equation (5). Then, the solution of the linear ordinary differential equation (8) can be written as (9), where the constants C and B for this solution are determined from the boundary conditions as

$$\Xi(0,p) = \Xi_1(p) \text{ for } x = 0,$$

$$\Xi(l,p) = \Xi_2(p) \text{ for } x = h,$$

$$C = \left(\Xi_1 e^{h\gamma} - \Xi_2\right)/[2\mathrm{sh}(h\gamma)], \quad B = -\left(\Xi_1 e^{-h\gamma} - \Xi_2\right)/[2\mathrm{sh}(h\gamma)].$$

Then, the solution (9) can be written as

$$\Xi(x,p) = \left\{\Xi_1(p)\mathrm{sh}[(h-x)\gamma] + \Xi_2(p)\mathrm{sh}(x\gamma)\right\}/\mathrm{sh}(h\gamma). \quad (15)$$

The equations of forces acting on the faces of the piezoelectric actuator are as follows:

$$T_1(0,p)S_0 = F_1(p) + M_1 p^2 \Xi_1(p) \quad \text{for } x = 0, \quad (16)$$

$$T_1(h,p)S_0 = -F_2(p) - M_2 p^2 \Xi_2(p) \quad \text{for } x = h,$$

where $T_1(0,p)$ and $T_1(h,p)$ are determined from the equation of the inverse piezoelectric effect. Thus, we obtain the following set of equations for mechanical stresses in the piezoactuator at $x = 0$ and $x = h$

$$T_1(0,p) = \frac{1}{s_{11}^E}\left.\frac{d\Xi(x,p)}{dx}\right|_{x=0} - \frac{d_{31}}{s_{11}^E}E_3(p), \quad (17)$$

$$T_1(h,p) = \frac{1}{s_{11}^E}\left.\frac{d\Xi(x,p)}{dx}\right|_{x=h} - \frac{d_{31}}{s_{11}^E}E_3(p).$$

The set of equations (16) for mechanical stresses in piezoactuator yields the following set of equations describing the structural parametric model of piezoactuator for the transverse piezoelectric effect

$$\Xi_1(p) = \left[1/\left(M_1 p^2\right)\right] \cdot \left\{-F_1(p) + \left(1/\chi_{11}^E\right)\left[d_{31}E_3(p) - [\gamma/\mathrm{sh}(h\gamma)]\left[\mathrm{ch}(h\gamma)\Xi_1(p) - \Xi_1(p)\right]\right]\right\}, \quad (18)$$

$$\Xi_2(p) = \left[1/\left(M_2 p^2\right)\right] \cdot \left\{-F_2(p) + \left(1/\chi_{11}^E\right)\left[d_{31}E_3(p) - [\gamma/\mathrm{sh}(h\gamma)]\left[\mathrm{ch}(h\gamma)\Xi_2(p) - \Xi_1(p)\right]\right]\right\},$$

where $\chi_{11}^E = s_{11}^E / S_0 = h/\left[m\left(c^E\right)^2\right]$. Taking into account generalized electromagnetoelasticity equation (3), we obtain the following system of equations describing the generalized structural-parametric model of the electromagnetoelastic actuator for the communications systems:

$$\Xi_1(p) = \left[1/\left(M_1 p^2\right)\right] \cdot \left\{-F_1(p) + \left(1/\chi_{ij}^\Psi\right)\left[\nu_{mi}\Psi_m(p) - [\gamma/\mathrm{sh}(l\gamma)]\left[\mathrm{ch}(l\gamma)\Xi_1(p) - \Xi_2(p)\right]\right]\right\}, \quad (19)$$

$$\Xi_2(p) = \left[1/\left(M_2 p^2\right)\right] \cdot \left\{-F_2(p) + \left(1/\chi_{ij}^\Psi\right)\left[\nu_{mi}\Psi_m(p) - [\gamma/\mathrm{sh}(l\gamma)]\left[\mathrm{ch}(l\gamma)\Xi_2(p) - \Xi_1(p)\right]\right]\right\},$$

where $\nu_{mi} = \begin{cases} d_{33}, d_{31}, d_{15} \\ g_{33}, g_{31}, g_{15} \\ d_{33}, d_{31}, d_{15} \end{cases}$, $\Psi_m = \begin{cases} E_3, E_1 \\ D_3, D_1 \\ H_3, H_1 \end{cases}$,

$s_{ij}^\Psi = \begin{cases} s_{33}^E, s_{11}^E, s_{55}^E \\ s_{33}^D, s_{11}^D, s_{55}^D \\ s_{33}^H, s_{11}^H, s_{55}^H \end{cases}$, $c^\Psi = \begin{cases} c^E \\ c^D \\ c^H \end{cases}$, $\gamma = \begin{cases} \gamma^E \\ \gamma^D \\ \gamma^H \end{cases}$, $l = \begin{cases} \delta \\ h \\ b \end{cases}$,

$\chi_{ij}^\Psi = s_{ij}^\Psi / S_0$,

then parameters Ψ of the control for the electromagnetoelastic actuator: E for voltage control, D for current control, H for magnetic field strength control. Figure 3 shows the generalized parametric block diagram of the electromagnetoelastic actuator corresponding to the set of equations (19).

Generalized structural-parametric model (19) of the electromagnetoelastic actuator after algebraic transformations provides the transfer functions of the electromagnetoelastic actuator for communications systems in the form of the ratio of the Laplace transform of the displacement of the transducer face and the Laplace transform of the corresponding force at zero initial conditions. The joint solution of equations (19) for the Laplace transforms of displacements of two faces of the electromagnetoelastic actuator yields

$$\Xi_1(p) = W_{11}(p)\Psi_m(p) + W_{12}(p)F_1(p) + W_{13}(p)F_2(p), \quad (20)$$

$$\Xi_2(p) = W_{21}(p)\Psi_m(p) + W_{22}(p)F_1(p) + W_{23}(p)F_2(p),$$

where the generalized transfer functions of the electromagnetoelastic actuator are

$$W_{11}(p) = \Xi_1(p)/\Psi_m(p) = \nu_{mi}\left[M_2\chi_{ij}^{\Psi}p^2 + \gamma \operatorname{th}(l\gamma/2)\right] / A_{ij},$$

$$\chi_{ij}^{\Psi} = s_{ij}^{\Psi}/S_0,$$

$$A_{ij} = M_1M_2\left(\chi_{ij}^{\Psi}\right)^2 p^4 + \left\{(M_1 + M_2)\chi_{ij}^{\Psi}/\left[c^{\Psi}\operatorname{th}(l\gamma)\right]\right\}p^3 + \left[(M_1 + M_2)\chi_{ij}^{\Psi}\alpha/\operatorname{th}(l\gamma) + 1/\left(c^{\Psi}\right)^2\right]p^2 + 2\alpha p/c^{\Psi} + \alpha^2,$$

$$W_{21}(p) = \Xi_2(p)/\Psi_m(p) = \nu_{mi}\left[M_1\chi_{ij}^{\Psi}p^2 + \gamma \operatorname{th}(l\gamma/2)\right] / A_{ij},$$

$$W_{12}(p) = \Xi_1(p)/F_1(p) = -\chi_{ij}^{\Psi}\left[M_2\chi_{ij}^{\Psi}p^2 + \gamma/\operatorname{th}(l\gamma)\right]/A_{ij},$$

$$W_{13}(p) = \Xi_1(p)/F_2(p) = W_{22}(p) = \Xi_2(p)/F_1(p) = \left[\chi_{ij}^{\Psi}\gamma/\operatorname{sh}(l\gamma)\right]/A_{ij},$$

$$W_{23}(p) = \Xi_2(p)/F_2(p) = -\chi_{ij}^{\Psi}\left[M_1\chi_{ij}^{\Psi}p^2 + \gamma/\operatorname{th}(l\gamma)\right]/A_{ij}.$$

Therefore, we obtain from equations (20) the generalized matrix equaion for the electromagnetoelastic actuator in the matrix form for communications systems

$$\begin{pmatrix}\Xi_1(p)\\ \Xi_2(p)\end{pmatrix} = \begin{pmatrix}W_{11}(p) & W_{12}(p) & W_{13}(p)\\ W_{21}(p) & W_{22}(p) & W_{23}(p)\end{pmatrix}\begin{pmatrix}\Psi_m(p)\\ F_1(p)\\ F_2(p)\end{pmatrix}. \quad (21)$$

Let us find the displacement of the faces the electromagnetoelastic actuator in a stationary regime for $\Psi_m(t) = \Psi_{m0}\cdot 1(t)$, $F_1(t) = F_2(t) = 0$ and inertial load. The static displacement of the faces the electromagnetoelastic actuator $\xi_1(\infty)$ and $\xi_2(\infty)$ can be written in the following form:

$$\xi_1(\infty) = \lim_{t\to\infty}\xi_1(t) = \lim_{\substack{p\to 0\\ \alpha\to 0}} pW_{11}(p)\Psi_{m0}/p = \nu_{mi}l\Psi_{m0}\,(M_2 + m/2)/(M_1 + M_2 + m), \quad (22)$$

$$\xi_2(\infty) = \lim_{t\to\infty}\xi_2(t) = \lim_{\substack{p\to 0\\ \alpha\to 0}} pW_{21}(p)\Psi_{m0}/p = \nu_{mi}l\Psi_{m0}\,(M_1 + m/2)/(M_1 + M_2 + m), \quad (23)$$

$$\xi_1(\infty) + \xi_2(\infty) = \lim_{t\to\infty}(\xi_1(t) + \xi_2(t)) = \nu_{mi}l\Psi_{m0}, \quad (24)$$

where m is the mass of the electromagnetoelastic actuator, M_1, M_2 are the load masses.

Let us consider a numerical example of the calculation of static characteristics of the piezoactuator from piezoceramics PZT under the longitudinal piezoelectric effect at $m \ll M_1$ and $m \ll M_2$. For $d_{33} = 4\cdot 10^{-10}$ m/V, $U = 500$ V, $M_1 = 10$ kg and $M_2 = 40$ kg we obtain the static displacement of the faces of the piezoactuator $\xi_1(\infty) = 160$ nm, $\xi_2(\infty) = 40$ nm, $\xi_1(\infty) + \xi_2(\infty) = 200$ nm.

The static displacement the faces of the piezoactuator for the transverse piezoelectric effect and inertial load at $U(t) = U_0\cdot 1(t)$, $E_3(t) = E_{30}\cdot 1(t) = (U_0/\delta)\cdot 1(t)$ and $F_1(t) = F_2(t) = 0$ can be written in the following form:

$$\xi_1(\infty) = \lim_{t\to\infty}\xi_1(t) = \lim_{\substack{p\to 0\\ \alpha\to 0}} pW_{11}(p)(U_0/\delta)/p = d_{31}(h/\delta)U_0\,(M_2 + m/2)/(M_1 + M_2 + m), \quad (25)$$

$$\xi_2(\infty) = \lim_{t\to\infty}\xi_2(t) = \lim_{\substack{p\to 0\\ \alpha\to 0}} pW_{21}(p)(U_0/\delta)/p = d_{31}(h/\delta)U_0\,(M_1 + m/2)/(M_1 + M_2 + m), \quad (26)$$

$$\xi_1(\infty) + \xi_2(\infty) = \lim_{t\to\infty}(\xi_1(t) + \xi_2(t)) = d_{31}(h/\delta)U_0. \quad (27)$$

The static displacement of the faces of the piezoactuator for the transverse piezoelectric effect and inertial load at $m \ll M_1$ and $m \ll M_2$

$$\xi_1(\infty) = \lim_{t\to\infty}\xi_1(t) = \lim_{\substack{p\to 0\\ \alpha\to 0}} pW_{11}(p)(U_0/\delta)/p = d_{31}(h/\delta)U_0\,M_2/(M_1 + M_2), \quad (28)$$

$$\xi_2(\infty) = \lim_{t\to\infty}\xi_2(t) = \lim_{\substack{p\to 0\\ \alpha\to 0}} pW_{21}(p)(U_0/\delta)/p = d_{31}(h/\delta)U_0\,M_1/(M_1 + M_2). \quad (29)$$

Let us consider a numerical example of the calculation of static characteristics of the piezoactuator from piezoceramics PZT under the transverse piezoelectric effect at $m \ll M_1$ and $m \ll M_2$. For $d_{31} = 2.5\cdot 10^{-10}$ m/V, $h = 4\cdot 10^{-2}$ m, $\delta = 2\cdot 10^{-3}$ m, $U = 200$ V, $M_1 = 10$ kg and $M_2 = 40$ kg we obtain the static displacement of the faces of the piezoelectric actuator $\xi_1(\infty) = 800$ nm, $\xi_2(\infty) = 200$ nm, $\xi_1(\infty) + \xi_2(\infty) = 1$ μm.

Let us consider the description of the piezoactuator for the longitudinal piezoelectric effect for one rigidly fixed face of the transducer at $M_1 \to \infty$, therefore, we obtain from equation (21) the transfer functions of the piezoactuator for the longitudinal piezoelectric effect in the following form:

$$W_{21}(p) = \Xi_2(p)/E_3(p) = d_{33}\delta/\left[M_2\delta\chi_{33}^{E}p^2 + \delta\gamma\operatorname{cth}(\delta\gamma)\right], \quad (30)$$

If $M_1 \to \infty$ and $M_2 = 0$, equation (30) yields an expression for the transfer function of unloaded piezoactuator under the longitudinal piezoelectric effect

$$W_{21}(p) = \Xi_2(p)/E_3(p) = d_{33}/[\gamma\operatorname{cth}(\delta\gamma)]. \quad (31)$$

Now, using equation (31), we write the expression for the transfer function of unloaded piezoactuator under the transverse piezoelectric effect at $M_1 \to \infty$ and $M_2 = 0$

$$W_{21}(p) = \Xi_2(p)/E_3(p) = d_{31}/[\gamma \mathrm{cth}(h\gamma)]. \qquad (32)$$

We write the resonance condition $\mathrm{ctg}(\omega h/c^E) = 0$.

This means that the piezoactuator is a quarter-wave vibrator with the resonance frequency $f_r = c^E/(4h)$.

The transfer function of an unloaded piezoactuator under the transversal piezoeffect with voltage control, when $M_1 \to \infty$ and $M_2 = 0$, has the form

$$W_2(p) = \Xi_2(p)/U(p) = \Xi_2(p)/[E_3(p)\delta] = (d_{31}h/\delta)/[h\gamma \mathrm{cth}(h\gamma)]. \qquad (33)$$

Accordingly, its frequency transfer function is described by the relation

$$W_2(j\omega) = \Xi_2(j\omega)/U(j\omega) = (d_{31}h/\delta)\mathrm{th}[h(j\omega/c^E + \alpha)]/[h(j\omega/c^E + \alpha)]. \qquad (34)$$

From equation (34) using the expression $\mathrm{th}(j\omega h/c^E) = j\mathrm{tg}(\omega h/c^E)$, where j is the imaginary unit, and the relations $\mathrm{th}(\alpha h) \to \alpha h$, where $\alpha h >> \alpha^2 h^2$ when $\alpha h \to 0$, we calculate the peak movement amplitude ξ_{2mr} the piezoactuator end at the resonance frequency under the voltage amplitude U_m in the form $\xi_{2mr} = d_{31}U_m k_r h/\delta = \xi_{2m} k_r$, where $\xi_{2m} = d_{31}U_m h/\delta$ is the amplitude of the movement in the piezoactuator in the static mode and $k_r = \xi_{2m\kappa}/\xi_{2m} = 8f_r/(\pi\alpha c^E)$ is the coefficient of the piezoactuator normalized relative to the movement in the static state at the resonance frequency under a single voltage amplitude in the dynamical and static modes.

The calculations conducted for a piezoelectric actuator of industrial piezoceramics PZT under the transversal piezoeffect and voltage control, where $d_{31} = 2.5 \cdot 10^{-10}$ m/V, $h = 4 \cdot 10^{-2}$ m, $\delta = 2 \cdot 10^{-3}$ m, $U_m = 20$ V, $k_r = 30$, $c^E = 3 \cdot 10^3$ m/s, yielded at the resonance frequency $f_r = 18.75$ kHz a maximum movement amplitude of $\xi_{2mr} = 3$ μm, under a value in the static mode $\xi_{2m} = 100$ nm. The experimental and calculated values for the piezoactuator are in agreement up to an accuracy of 5%.

Let us consider the static responses of the piezoactuator under the longitudinal piezoeffects. Let us determine the value of the The static displacement of the face of the piezoactuator $\xi_2(\infty)$ in the static regime for $U(t) = U_0 \cdot 1(t)$ and $F_2(t) = 0$ or $F_2(t) = F_0 \cdot 1(t)$ and $U(t) = 0$.

Accordingly, the static displacement $\xi_2(\infty)$ of the piezoactuator under the longitudinal piezoeffect in the form

$$\xi_2(\infty) = \lim_{t\to\infty} \xi_2(t) = \lim_{p\to 0} pW_2(p)U_0/p = \lim_{\substack{p\to 0\\ \alpha\to 0}} d_{33}U_0 \mathrm{th}(\alpha\delta)/(\alpha\delta) = d_{33}U_0, \qquad (35)$$

$$\xi_2(\infty) = \lim_{p\to 0} pW_{23}(p)F_0/p = -\lim_{\substack{p\to 0\\ \alpha\to 0}} [\delta^2 F_0 \mathrm{th}(\alpha\delta)]/[m(c^E)^2 \alpha\delta] = -\delta s_{33}^E F_0/S_0. \qquad (36)$$

Let us consider a numerical example of the calculation of static characteristics of the piezoactuator under the longitudinal piezoeffects. For $d_{33} = 4 \cdot 10^{-10}$ m/V, $U = 500$ V, we obtain $\xi_2(\infty) = 200$ nm. For $\delta = 6 \cdot 10^{-4}$ m, $s_{33}^E = 3.5 \cdot 10^{-11}$ m^2/N, $F_0 = 1000$ N, $S_0 = 1.75 \cdot 10^{-4}$ m^2, we obtain $\xi_2(\infty) = -120$ nm. The experimental and calculated values for the piezoactuator are in agreement to an accuracy of 5%.

Let us consider the operation at low frequencies for the piezoactuator with one face rigidly fixed so that $M_1 \to \infty$ and $m << M_2$. Representing $W_{21}(p)$ and $W_{23}(p)$ as

$$W_{21}(p) = \Xi_2(p)/E_3(p) = d_{33}\delta/[M_2\delta\chi_{33}^E p^2 + \delta\gamma \mathrm{cth}(\delta\gamma)], \qquad (37)$$

$$W_{23}(p) = \Xi_2(p)/F_2(p) = -\delta\chi_{33}^E/[M_2\delta\chi_{33}^E p^2 + \delta\gamma \mathrm{cth}(\delta\gamma)]. \qquad (38)$$

Using the approximation of the hyperbolic cotangent by two terms of the power series in transfer functions (37) and (38), at $m << M_2$ we obtain the following expressions the transfer functions in the frequency range of $0 < \omega < 0{,}01c^E/\delta$

$$W_{21}(p) = \Xi_2(p)/E_3(p) = d_{33}\delta/(T_t^2 p^2 + 2T_t\xi_t p + 1), \qquad (39)$$

$$W_{23}(p) = \Xi_2(p)/F_2(p) = -(s_{33}^E\delta/S_0)/(T_t^2 p^2 + 2T_t\xi_t p + 1), \qquad (40)$$

$$T_t = (\delta/c^E)\sqrt{M_2/m} = \sqrt{M_2/C_{33}^E}, \quad \xi_t = (\alpha\delta/3)\sqrt{m/M_2},$$
$$C_{33}^E = S_0/(s_{33}^E\delta) = 1/(\chi_{33}^E\delta).$$

where T_t is the time constant and ξ_t is the damping coefficient, C_{33}^E - is the is rigidity of the piezoactuator under the longitudinal piezoeffect.

3. Results and Discussions

Taking into account equation of generalized electromagnetoelasticity (piezoelectric, piezomagnetic, electrostriction, and magnetostriction effects) and decision wave equation we obtain a generalized block diagram of electromagnetoelastic actuator Figure 3 for the communications systems. The results of constructing a generalized structural-parametric model and a generalized block diagram of electromagnetoelastic actuator [2-6] for the longitudinal, transverse and shift deformations are shown in Figure 3. Block diagram piezoactuator for longitudinal piezoeffect Figure 2 converts to generalized parametric block diagram of the electromagnetoelastic actuator Figure 3 with the replacement of the parameters

$$\Psi_m = E_3, \ \nu_{mi} = d_{33}, \ s_{ij}^\Psi = s_{33}^E, \ l = \delta.$$

Generalized structural-parametric model and generalized parametric block diagram of the electromagnetoelastic actuator after algebraic transformations provides the transfer functions of the electromagnetoelastic actuator for communications systems [9-25]. The piezoactuator with the transverse piezoelectric effect compared to the piezoactuator for the longitudinal piezoelectric effect provides a greater range of static displacement and less working force. The magnetostriction actuators provides a greater range of static working forces [12-14].

Using the solutions of the wave equation of the electromagnetoelastic actuator and taking into account the features of the deformations along the coordinate axes, it is possible to construct the generalized structural-parametric model, generalized parametric block diagram and the transfer functions of the electromagnetoelastic actuator for communications systems.

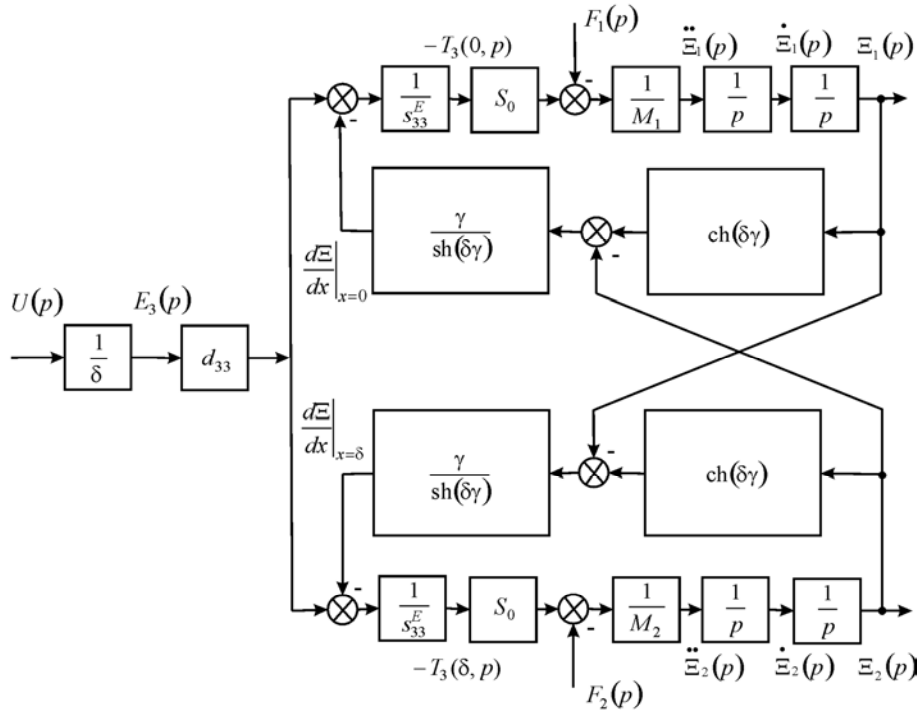

Fig. 2. *Parametric block diagram of the voltage-controlled piezoactuator for longitudinal piezoelectric effect.*

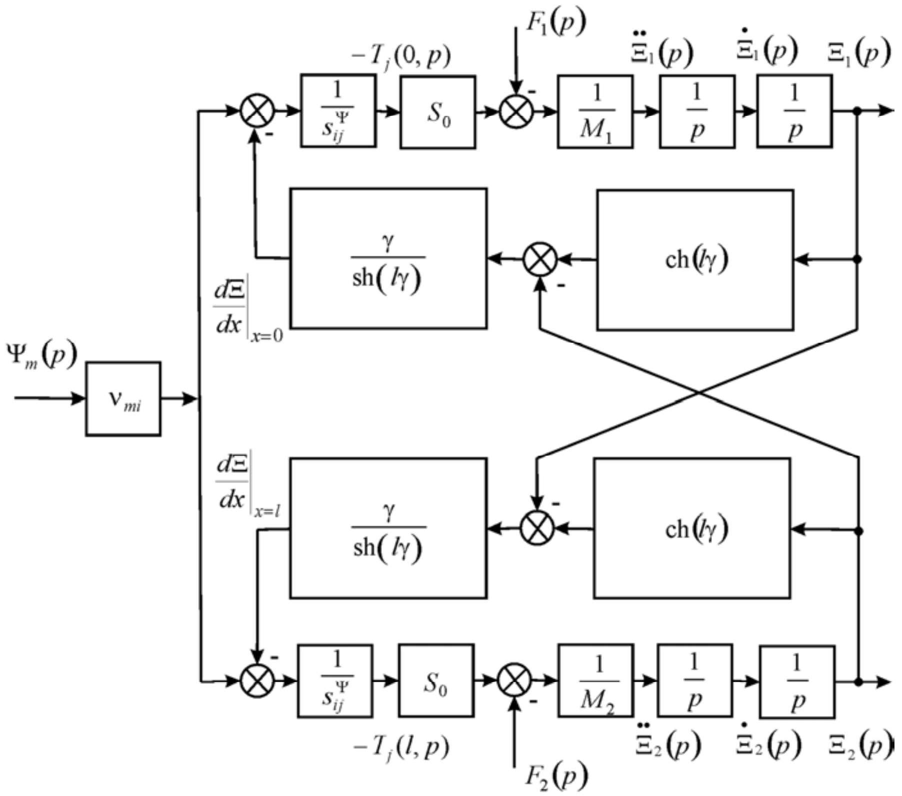

Fig. 3. *Generalized parametric block diagram of the electromagnetoelastic actuator.*

4. Conclusions

Thus, using the obtained solutions of the wave equation and taking into account the features of the deformations along the coordinate axes, it is possible to construct the generalized structural-parametric model and parametric block diagram of the electromagnetoelastic actuator and to describe its dynamic and static properties with allowance for the physical properties, the external load during its operation as a part of the communications systems.

The transfer functions and the parametric block diagrams of the piezoactuators for the transverse and longitudinal piezoelectric effects are obtained from structural parametric models of the piezoactuators for communications systems.

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