

**Methodology Article**

# Analysis of a Vector-Potential Representation of Electromagnetic Radiation Quantum

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**Abstract:** On the basis of the generalized coordinates use the opportunity of a clear representation of electromagnetic radiation quantum is shown. It is established that equation Lagrange in a classical variant passes in the wave equation for vector - potential, and at quantization in Schrodinger equation for a quantum of electromagnetic radiation in space of the generalized coordinates. The solution of Schrodinger equation is given. It is shown that in space of the generalized coordinates the vacuum energy is a constant, not dependent on the changing parameter of a quantum - its frequencies, and the length of a quantum is exponential falls with increase in volumetric density of its energy.

**Keywords:** Electromagnetic Field, Quantum, Lagrange Function, Generalized Coordinates, Schrodinger Equation, Wave Function

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## 1. Introduction

The opportunity of clear representation of an electromagnetic radiation quantum always excited physicists [1]. How the light quantum is located in space? Whether it has the beginning and the end? Apparently, nevertheless has but then why in a spectrum of a quantum is only one frequency?

Attempts to find answers to these questions have led physicists to opinion what evidently to present quantum in Euclidian space it is impossible. In [2] is underlined that the concept of the photon coordinates at all has no physical sense. However, the elementary description of a light quantum in Euclidian space can be received with the help of Schrodinger's equation in this space. Using the formula for the Hamiltonian of a quantum as  $H = ck$ , where  $k$  there is module of a quantum impulse,  $c$  – a light velocity in vacuum, we can find the operational form of the Schrodinger's equation as  $\hat{E}\psi = c\hat{k}\psi$ , where  $\psi$  – a quantum wave function in Euclidian space. Using the following designations of operators [3]  $\hat{E} = i\hbar \frac{\partial}{\partial t}$  and  $\hat{k}_X = -i\hbar \frac{\partial}{\partial X}$  (for example, for the quantum flying along an axis  $X$ ) we shall find  $\frac{\partial \psi}{\partial t} + c \frac{\partial \psi}{\partial X} = 0$ . Information value of the found equation is very small since the solution of this equation can be any

function of a kind  $\psi = \psi(X - ct)$ . For example, it is impossible to carry out a normalization of this wave function.

There is a question, whether there is coordinate space in which photon (quantum) shall evidently present? We shall consider this question in more detail having starting the classical description of electromagnetic radiation.

## 2. The Generalized Coordinates

It is known that the volumetric density of electromagnetic field energy in vacuum can be presented as [4]:

$$w = T + U = \frac{E^2 + H^2}{8\pi} \quad (1)$$

where  $E$  there is a vector of electric field strength,  $H$  – a vector of magnetic field strength,  $T = \frac{E^2}{8\pi}$  and  $U = \frac{H^2}{8\pi}$ .

The Lagrangian of a free electromagnetic field (at absence of a charges and currents) looks like [4]:

$$l = T - U = \frac{E^2 - H^2}{8\pi} \quad (2)$$

For the Lagrangian  $l$  the equation of Euler - Lagrange [2] is correct:

$$\frac{d}{dt} \left( \frac{\partial l}{\partial \dot{\mathbf{q}}} \right) = \frac{\partial l}{\partial \mathbf{q}} \quad (3)$$

where  $\dot{\mathbf{q}}$  there is a vector of generalized velocity,  $\mathbf{q}$  - a vector of generalized coordinate. We shall note that the generalized arguments in the equation (3) can be not connected to mechanical velocities and Euclidean coordinates.

As the generalized speed we shall accept the electromagnetic field strength  $\dot{\mathbf{q}} = \mathbf{E}$ . It is allowable since the general formula [4] is correct:

$$w = \dot{\mathbf{q}} \frac{\partial l}{\partial \dot{\mathbf{q}}} - l = \mathbf{E} \frac{\partial l}{\partial \mathbf{E}} - l = \mathbf{E} \frac{2\mathbf{E}}{8\pi} - \frac{E^2 - H^2}{8\pi} = \frac{E^2 + H^2}{8\pi} \quad (4)$$

Let's find the generalized coordinate. We shall assume  $\mathbf{q} = f(\mathbf{E}, \mathbf{H})$ . In this case the equation (3) will be transformed to the kind:

$$\frac{d\mathbf{E}}{dt} = \mathbf{E} \frac{\partial \mathbf{E}}{\partial \mathbf{q}} - \mathbf{H} \frac{\partial \mathbf{H}}{\partial \mathbf{q}} \quad (5)$$

Passing in (5) from full derivatives to partial derivatives we have:

$$\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{E}}{\partial \mathbf{q}} \dot{\mathbf{q}} = \mathbf{E} \frac{\partial \mathbf{E}}{\partial \mathbf{q}} - \mathbf{H} \frac{\partial \mathbf{H}}{\partial \mathbf{q}} \quad (6)$$

Reducing in (6) the identical terms we shall find:

$$\frac{\partial \mathbf{E}}{\partial t} = -\mathbf{H} \frac{\partial \mathbf{H}}{\partial \mathbf{q}} \quad (7)$$

Using the Maxwell's equation  $\text{rot} \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$  [4] for the free electromagnetic field we shall receive:

$$d\mathbf{q} = -\frac{1}{c} \frac{\mathbf{H} d\mathbf{H}}{\text{rot} \mathbf{H}} \quad (8)$$

We used the initial conditions  $\mathbf{q} = 0$  at  $\mathbf{H} = 0$ . It is allowable since in the right part (8) in the numerator at  $\mathbf{H} \rightarrow 0$  is magnitude higher order of a minority than in the denominator.

Solving the equation (8) we receive the generalized coordinate:

$$\mathbf{q} = -\frac{1}{c} \int \frac{\mathbf{H} d\mathbf{H}}{\text{rot} \mathbf{H}} \quad (9)$$

We have found that the generalized coordinate depends only on a magnetic field strength  $\mathbf{q} = f(\mathbf{H})$ .

We research physical sense of the generalized coordinate.

Using the Maxwell equation  $\text{rot} \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}$  it is possible to

write down  $\text{rot} \frac{\partial \mathbf{q}}{\partial t} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}$  or  $\text{rot} \mathbf{q} = -\frac{\mathbf{H}}{c}$ . Taking into account  $\mathbf{H} = \text{rot} \mathbf{A}$  we shall find  $\mathbf{q} = -\frac{\mathbf{A}}{c}$ . Thus, as the generalized coordinate it is used the vector-potential  $\mathbf{A}$  with

the opposite sign (normalized on the velocity of light in used system of units).

Before to carry out the further transformations we shall show that Euler-Lagrange's equation (3) at used the generalized velocity and the generalized coordinate passes to the wave equation for vector-potential  $\mathbf{A}$ .

Let's transform the right part of the equation (3):

$$\frac{\partial l}{\partial \mathbf{q}} = \frac{\partial (T - U)}{\partial \mathbf{q}} = -\frac{\partial U}{\partial \mathbf{q}} = -\frac{\mathbf{H}}{4\pi} \frac{\partial \mathbf{H}}{\partial \mathbf{q}} = \frac{c}{4\pi} \text{rot} \mathbf{H} = -\frac{c}{4\pi} \Delta \mathbf{A} \quad (10)$$

There are used the determination of the vector-potential  $\mathbf{H} = \text{rot} \mathbf{A}$  also a known formula of vector analysis  $\text{rot} \text{rot} \mathbf{A} = \text{grad} \text{div} \mathbf{A} - \Delta \mathbf{A}$ , and also Coulomb's gauge  $\text{div} \mathbf{A} = 0$ .

The transformations of the equation (3) left part with use  $\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}$  (the scalar potential is equal to zero as charges are absent) for a free electromagnetic field result:

$$\frac{d}{dt} \left( \frac{\partial l}{\partial \dot{\mathbf{q}}} \right) = \frac{d}{dt} \left( \frac{\partial l}{\partial \mathbf{E}} \right) = \frac{1}{4\pi} \frac{d\mathbf{E}}{dt} = -\frac{1}{4\pi c} \frac{\partial^2 \mathbf{A}}{\partial t^2} \quad (11)$$

Equating (10) and (11) we shall find the wave equation for vector-potential of a free electromagnetic field  $\Delta \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = 0$ .

### 3. Quantization of an Electromagnetic Field

We carry out quantization of the electromagnetic field on the method suggested in [5].

For the basis we shall take the Lagrange's equation (3) having written down it as:

$$\frac{\partial}{\partial t} \left( \frac{\partial l}{\partial \dot{\mathbf{q}}} \right) + \dot{\mathbf{q}} \frac{\partial}{\partial \mathbf{q}} \left( \frac{\partial l}{\partial \dot{\mathbf{q}}} \right) = \frac{\partial l}{\partial \mathbf{q}} \quad (12)$$

Let's transform (12) taking into account  $\dot{\mathbf{q}} = \mathbf{E}$  and  $\frac{\partial l}{\partial \dot{\mathbf{q}}} = \frac{\mathbf{E}}{4\pi}$ :

$$\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial}{\partial \mathbf{q}} \left( \frac{E^2}{2} \right) = -4\pi \frac{\partial U}{\partial \mathbf{q}} \quad (13)$$

For integration the equation (13) on the generalized coordinate  $\mathbf{q}$  we use complex potential of the generalized velocity:

$$s = s_0 + \frac{\hbar}{i} s_1 \quad (14)$$

where use of the reduced Planck's constant  $\hbar$  will be proved further.

Let's determine the real part of potential:

$$\frac{1}{4\pi} \mathbf{E} = \frac{\partial s_0}{\partial \mathbf{q}} \quad (15)$$

Integrating once the equation (13) we find:

$$\frac{\partial s_0}{\partial t} + \frac{E^2}{8\pi} = -U \quad (16)$$

The constant of integration is accepted equal to zero that is reached by a choice of the initial level of potential.

The equation (16) is the Hamilton – Jacobi's equation [6] therefore the size  $s_0$  can be assumed as the real part of volumetric density of the action  $s_0 = \int l dt$ .

Using (15) it is possible to write down function  $T$  and to determine function  $U$  as:

$$T = 2\pi \left( \frac{\partial s_0}{\partial \mathbf{q}} \right)^2 \text{ and } U = -s_1 \quad (17)$$

In this case the equation (16) looks like:

$$\frac{\partial s_0}{\partial t} + 2\pi \left( \frac{\partial s_0}{\partial \mathbf{q}} \right)^2 - s_1 = 0 \quad (18)$$

We shall introduce by analogy to [7] the wave function of photon as  $\Psi = \exp\left(i \frac{s}{\hbar}\right)$ , where  $s = \int l dt$  there is volumetric density of action in space of the generalized coordinates.

The formula (14) can be assumed as first two components of the volumetric density of action  $s = s(t, \mathbf{q})$  expansion into a degrees  $\frac{\hbar}{i}$  series. We shall notice that units of measurements of the Planck's constant and the volumetric density of action  $s = s(t, \mathbf{q})$  must correspond to units of measurements of the generalized coordinates.

Taking into account (14) we shall transform wave function to the kind:

$$\Psi = \exp\left(i \frac{s}{\hbar}\right) = \exp(s_1) \exp\left(\frac{i}{\hbar} s_0\right) = |\Psi| \exp\left(\frac{i}{\hbar} s_0\right) \quad (19)$$

where is  $|\Psi| = \exp(s_1)$ .

## 4. Schrodinger's Equation for a Light Quantum

We shall show that the equation:

$$i\hbar \frac{\partial \Psi}{\partial t} + 2\pi\hbar^2 \frac{\partial^2 \Psi}{\partial q^2} + \ln|\Psi| \Psi = 0 \quad (20)$$

it is possible to suppose as the Schrodinger's equation for light quantum.

It is simple to translate the equation (20) in the Euclidian space. For this purpose we use the formula associating the

generalized coordinate with vector - potential  $\mathbf{q} = -\frac{\mathbf{A}}{c}$ .

Substituting it in (20) we shall find:

$$i\hbar \frac{\partial \Psi}{\partial t} + 2\pi(\hbar c)^2 \frac{\partial^2 \Psi}{\partial A^2} + \ln|\Psi| \Psi = 0 \quad (21)$$

Let's notice that in the equation (21) the fourth degree so-called Planck's charge  $e_p = \sqrt{\hbar c}$  generated from fundamental physical constants is used.

Therefore the equation (21) can be written down as:

$$i\hbar \frac{\partial \Psi}{\partial t} + 2\pi e_p^4 \frac{\partial^2 \Psi}{\partial A^2} + \ln|\Psi| \Psi = 0 \quad (22)$$

Thus the Planck's charge concerns not to the charged particle, and to a photon. The probability to find out a particle with Planck's charge is smallest. A role of the Planck's charge is another. It represents as though photon "memory" that the photon has arisen due to a charges and currents.

Let's note that in the equation (22) is present only two constants: Planck's constant describing energy of a photon and Planck's charge reflecting principle of occurrence of a photon its genesis. The "memory" what size was of the particle charge generated a photon it does not remain.

In spite of the fact that the equation (20) is nonlinear this nonlinearity takes place only in space of the generalized coordinates. Nonlinearity of the Schrodinger's equation (20) is consequence of the nonlinear dependence on parameters (9) the generalized coordinate. In Euclidean coordinates the process of electromagnetic quantum propagation has linear character. For example, in quantum-mechanical systems a linear principle of superposition [8] is correct.

First of all we shall prove that the equation (20) is equivalent to the equation (18).

Using the formula  $\Psi = \exp\left(i \frac{s}{\hbar}\right)$  we shall find:

$$\frac{\partial \Psi}{\partial t} = \frac{i}{\hbar} \Psi \frac{\partial s}{\partial t} \quad (23)$$

The second derivative on the generalized coordinate:

$$\frac{\partial^2 \Psi}{\partial q^2} = -\frac{1}{\hbar^2} \Psi \left( \frac{\partial s}{\partial q} \right)^2 + \frac{i}{\hbar} \Psi \frac{\partial^2 s}{\partial q^2} \quad (24)$$

Substituting (23) and (24) in (20) we shall find:

$$\frac{\partial s}{\partial t} + 2\pi \left( \frac{\partial s}{\partial q} \right)^2 - 2\pi\hbar i \frac{\partial^2 s}{\partial q^2} = \ln|\Psi| \quad (25)$$

Using (14) we have:

$$\frac{\partial s_0}{\partial t} + 2\pi \left( \frac{\partial s_0}{\partial q} \right)^2 - s_1 - \hbar i \left( \frac{\partial s_1}{\partial t} + 4\pi \frac{\partial s_0}{\partial q} \frac{\partial s_1}{\partial q} + 2\pi \frac{\partial^2 s_0}{\partial q^2} \right) = 2\pi\hbar^2 \left( \left( \frac{\partial s_1}{\partial q} \right)^2 + \frac{\partial^2 s_1}{\partial q^2} \right) \quad (26)$$

Ignoring the second order of magnitude on  $\hbar^2$  and equating to zero separately real and imaginary parts of the equation (26) we come to the equation (18), and also to the equation:

$$\frac{\partial s_1}{\partial t} + 4\pi \frac{\partial s_0}{\partial q} \frac{\partial s_1}{\partial q} + 2\pi \frac{\partial^2 s_0}{\partial q^2} = 0 \quad (27)$$

Let's substitute in the equation (28) formula (15) and also we shall take into account  $\dot{\mathbf{q}} = \mathbf{E}$ :

$$\frac{\partial s_1}{\partial t} + \dot{\mathbf{q}} \frac{\partial s_1}{\partial \mathbf{q}} + \frac{1}{2} \frac{\partial \dot{\mathbf{q}}}{\partial \mathbf{q}} = 0 \quad (28)$$

Taking into account  $\frac{\partial s_1}{\partial t} = \frac{1}{2|\Psi|^2} \frac{\partial |\Psi|^2}{\partial t}$  and

$\frac{\partial s_1}{\partial \mathbf{q}} = \frac{1}{2|\Psi|^2} \frac{\partial |\Psi|^2}{\partial \mathbf{q}}$  we shall find:

$$\frac{\partial |\Psi|^2}{\partial t} + \frac{\partial \dot{\mathbf{q}} |\Psi|^2}{\partial \mathbf{q}} = 0 \quad (29)$$

If the equation (29) is equation of continuity the size  $\mathbf{j} = \dot{\mathbf{q}} |\Psi|^2$  is current density of size  $|\Psi|^2$  for a photon in space of the generalized coordinates. There is a problem of the size  $|\Psi|^2$  interpretation. The equation (20) is Schrodinger's equation for a photon i.e. according to this equation it is supposed that the photon is some particle having certain volume in space of generalized coordinates. Each element of the photon can be in the certain element of a space with some probability. Therefore  $|\Psi|^2$  it is possible to assume as some density of probability for a photon element be present in the given place of a space of the generalized coordinates. The equation (29) reflects the following position: change of the density of probability of a photon element presence in some volume of space of the generalized coordinates generates a current of this density of probability through the volume border.

Thus the equations (18) and (29) are equivalents to the Schrodinger's equation (20) for a photon of an electromagnetic radiation.

## 5. Solving of the Schrodinger's Equation for a Light Quantum

Let's find the solution of the Schrodinger's equation (20) for unit quantum.

We use the stationary solution of the equation (20) as [9]:

$$\Psi = f(\mathbf{kq} - \omega t) \exp[i(\mathbf{r}\mathbf{q} - \delta t)] \quad (30)$$

where are constants  $\mathbf{r}$  and  $\delta$ , and also function  $|\Psi| = f(\mathbf{kq} - \omega t)$  still unknown. Stationary we name the solution which during evolution does not change the form.

Substituting (30) in (20) we shall receive:

$$2\pi\hbar^2 k^2 \frac{d^2 f}{d\xi^2} + i\hbar \frac{df}{d\xi} (-\omega + 4\pi\hbar\mathbf{k}\mathbf{r}) + f\hbar(\delta - 2\pi\hbar r^2) + f \ln f = 0 \quad (31)$$

In the equation (31) differentiation is carried out on the variable  $\xi = \mathbf{kq} - \omega t$ . As function  $f = |\Psi|$  represents real size the equation (31) should not have imaginary members. Having accepted  $\omega = 4\pi\hbar\mathbf{k}\mathbf{r}$  we shall receive:

$$2\pi\hbar^2 k^2 \frac{d^2 f}{d\xi^2} + f\hbar(\delta - 2\pi\hbar r^2) + f \ln f = 0 \quad (32)$$

The solution of the equation (32) we search as:

$$f = C_1 \exp\left[\frac{C_2(\mathbf{kq} - \omega t)^2}{2}\right] \quad (33)$$

where  $C_1$  and  $C_2$  there are constants. Having substituted (33) in (32) we find:

$$2\pi\hbar^2 k^2 C_2 + \hbar\delta - 2\pi\hbar^2 r^2 + \ln C_1 + \left(2\pi\hbar^2 k^2 C_2 + \frac{1}{2}\right) C_2 \xi^2 = 0 \quad (34)$$

Last component in the equation (34) there is infinitely grows at tending for example time  $t \rightarrow \infty$ , and the fixed generalized coordinate  $\mathbf{q}$  that is physically impossible. Therefore the expression in brackets should be equal to zero.

Hence  $C_2 = -\frac{1}{4\pi(\hbar k)^2}$ . Then:

$$C_1 = \exp\left(2\pi(\hbar r)^2 - \hbar\delta + \frac{1}{2}\right) \quad (35)$$

Thus the solution of the equation (20) can be written down according to (30) and (33) as:

$$\Psi = \exp\left(2\pi(\hbar r)^2 - \hbar\delta + \frac{1}{2}\right) \exp\left[-\frac{(\mathbf{kq} - \omega t)^2}{8\pi(\hbar k)^2}\right] \exp[i(\mathbf{r}\mathbf{q} - \delta t)] \quad (36)$$

We shall designate the quantum speed in vacuum (the speed of the enveloping curve wave) in space of the generalized coordinates as  $\mathbf{c} = \frac{\omega}{\mathbf{k}}$ . Using earlier found formula  $\omega = 4\pi\hbar\mathbf{k}\mathbf{r}$  we shall receive  $\mathbf{c} = 4\pi\hbar\mathbf{r}$ . Hence wave function (36) can be written down as:

$$\Psi = \exp\left(\frac{c^2}{8\pi} - \hbar\delta + \frac{1}{2}\right) \exp\left[-\frac{(\mathbf{q} - \mathbf{c}t)^2}{8\pi\hbar^2}\right] \exp\left[i\left(\frac{\mathbf{c}\mathbf{q}}{4\pi\hbar} - \delta t\right)\right] \quad (37)$$

For calculation we use the initial moment of time  $t = 0$ , and also we shall assume the quantum frequency  $\delta$  such that equality  $c^2 = 8\pi\hbar\delta - 4\pi$  was carried out. Besides we shall designate  $\mathbf{z} = \frac{\mathbf{q}}{2\hbar}$ . In this case the formula (37) becomes simpler:

$$\Psi = \exp\left[-\frac{z^2}{2\pi}\right] \exp\left[i\left(\frac{\mathbf{c}\mathbf{z}}{2\pi}\right)\right] \quad (38)$$

Let's carry out the analysis of dimensions. From the equation (20) there is follows  $[\hbar] = [t] = [q]$ . From (37) we have  $[ct] = [q]$ . Thus quantum speed in space of the generalized coordinates is size dimensionless.

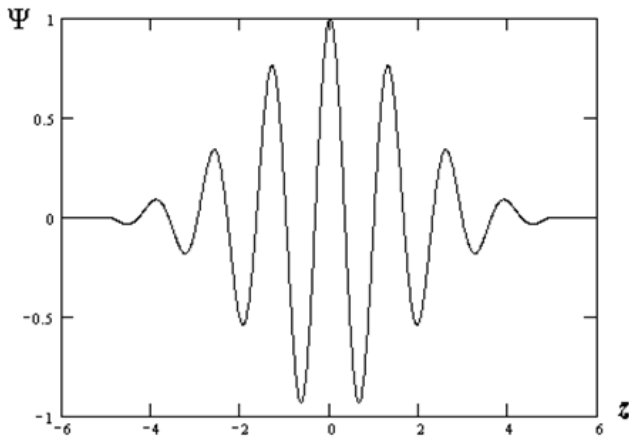


Figure 1. The quantum wave function in space of the generalized coordinates.

In fig. 1 the calculation of quantum wave function of electromagnetic radiation executed under the formula (38) at speed of the quantum propagation  $c = 30$  is shown.

There is a question how in space of the generalized coordinates the graph of wave function corresponds with the graph in Euclidean space?

As it has been shown earlier as the generalized coordinate the vector-potential is actually used. We shall bring an auxiliary attention to the question. How the kind of an electrostatics laws changes at transition to coordinate - scalar potential  $\phi$ ? Obviously the strength of a solitary charge  $e$  field in this case is equal  $E = \frac{1}{e}\phi^2$ . The kind of the law

changed. If in Euclidean space the strength falls in inverse proportion to a square of coordinate, in space of scalar potential the strength grows with growth of coordinate.

Hence, transition to the generalized coordinates should cardinaly change the kind of quantum wave function. The description of quantum in space of the generalized coordinates apparently is more adequate than in Euclidean space.

The Schrodinger's equation (20) is the soliton equation [9]. Unfortunately, now it is known only the one-soliton solution (37) of this equation. The finding two-solitons solution of the equation (20), apparently, will allow solve a problem of a quantum entanglement (a quantum teleportation) [10]. Only thus it is possible to find out character of the waves connection in the two-wave solution of the quantum equation (20).

## 6. Problem of a Vacuum and Quantum Length

Let's find volumetric density of quantum energy. Using (17) we shall receive:

$$w = T + U = 2\pi \left(\frac{\partial s_0}{\partial \mathbf{q}}\right)^2 - s_1 \quad (39)$$

In due time the quantum only moves in space of the generalized coordinates, not changing the form. Therefore taking into account (19) and using in the formula  $\mathbf{q} = \mathbf{c}t$  we

shall find  $s_1 = \ln|\Psi| = 2\pi(\hbar r)^2 - \hbar\delta + \frac{1}{2}$ .

Comparing (19) and (36) we shall find  $s_0 = \hbar(\mathbf{r}\mathbf{q} - \delta t)$ .

Hence the size  $T$  is equal  $T = 2\pi \left(\frac{\partial s_0}{\partial \mathbf{q}}\right)^2 = 2\pi(\hbar r)^2$ .

Thus the full volumetric density of quantum energy is equal:

$$w = \hbar\delta - \frac{1}{2} \quad (40)$$

According to (40) it is possible to assume that the size  $\hbar\delta$  there is sum of volumetric density of the quantum energy and volumetric density of the vacuum energy in space of the generalized coordinates. Then the constant  $w_v = \frac{1}{2}$  there is a

vacuum energy in space of the generalized coordinates. We shall note that as against Euclidean spaces in space of the generalized coordinates the volumetric density of the vacuum energy is constant.

Apparently, one of the reasons of evident representation impossibility of a light quantum in the Euclidean space consists that the quantum energy  $\hbar\omega$  (we use the standard designation of quantum frequency) is propagated in the vacuum which energy depends on the quantum energy  $\frac{\hbar\omega}{2}$ .

Therefore it is impossible to set an initial level of reading of quantum energy. At the beginning and at the end of quantum the quantum energy apparently is lost (transformed, dissolved, etc.) into the vacuum energy that deprives quantum of the evident representation.

It has been earlier marked that  $|\Psi|^2$  it is possible to interpret as some density of probability of a quantum element presence in the given place of space of the generalized coordinates.

Therefore for the size  $|\Psi|^2$  the following normalizing equation is correctly:

$$\frac{\int_{-\infty}^{+\infty} |\Psi|^2 d\mathbf{q}}{\int_{\frac{\mathbf{q}_0}{2}}^{\frac{\mathbf{q}_0}{2}} d\mathbf{q}} = \frac{2}{\mathbf{q}_0} \int_0^{+\infty} |\Psi|^2 d\mathbf{q} = 1 \quad (41)$$

where  $\mathbf{q}_0$  there is a quantum length in space of the generalized coordinates.

Substituting in (41) the formula for  $|\Psi| = \exp\left(2\pi(\hbar r)^2 - \hbar\delta + \frac{1}{2}\right) \exp\left[-\frac{(\mathbf{q} - \mathbf{c}t)^2}{8\pi\hbar^2}\right]$  that follows from (37) we shall find the quantum length under condition  $t = 0$ :

$$q_0 = 2 \int_0^{+\infty} |\Psi|^2 d\mathbf{q} = 2 \exp 2 \left( 2\pi (\hbar r)^2 - \hbar \delta + \frac{1}{2} \right) \int_0^{+\infty} \exp \left[ -\frac{\mathbf{q}^2}{4\pi\hbar^2} \right] d\mathbf{q} \quad (42)$$

Hence, the module of the quantum length is equal:

$$q_0 = 2\pi\hbar \exp 2 \left( 2\pi (\hbar r)^2 - \hbar \delta + \frac{1}{2} \right) \quad (43)$$

At finding (43) the formula  $\int_0^{\infty} \exp(-x^2) dx = \frac{\sqrt{\pi}}{2}$  is used.

Using the formula (40) for the volumetric density of a quantum energy, and also  $\mathbf{c} = 4\pi\hbar\mathbf{r}$ , see (37) we shall find the quantum length as:

$$q_0 = 2\pi\hbar \exp 2 \left( \frac{c^2}{8\pi} - w \right) \quad (44)$$

From (44) follows that quantum length in space of the generalized coordinates quantized. Besides the quantum length is exponential falls with increase in volumetric density of the quantum energy  $w$ . It apparently this dependence is correct and for the Euclidian spaces.

In fig. 2 dependence of the relative quantum length  $\frac{q_0}{2\pi\hbar}$  on the volumetric density of the quantum energy  $w$  is shown at speed of a quantum  $c = 30$ .

At  $w \rightarrow \infty$  the length of quantum tend to zero. If to assume that the length of quantum is more  $q_0 \geq 2\pi\hbar = h$  should be  $\frac{c^2}{8\pi} \geq w$  ( $h = 2\pi\hbar$  is the minimal length of a quantum equal to Planck's constant). Thus, the maximal volumetric density of the quantum energy is equal  $w_{\max} = \frac{c^2}{8\pi}$ . At  $c = 30$  the maximal volumetric density of the quantum energy is  $w_{\max} = 35.81$ . All calculations are made in space of the generalized coordinates.

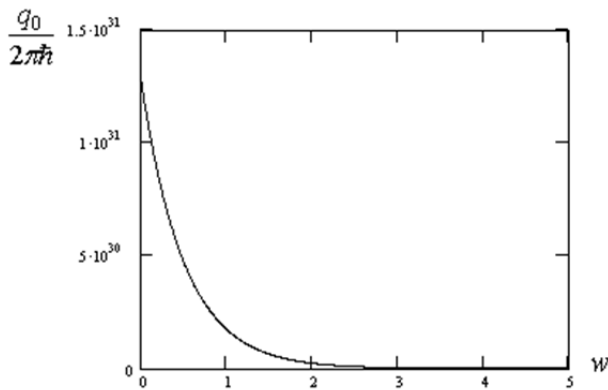


Figure 2. Dependence of the relative quantum length on volumetric density of its energy.

## 7. Conclusions

There is an opportunity of evident representation of an electromagnetic radiation quantum in space of the generalized coordinates.

The analysis shows that in a basis of the Schrodinger's equation for a light quantum the Euler – Lagrange's equation lays from which at the classical approach it is possible to receive the classical wave equation for vector - potential, and at quantization of an electromagnetic field the Schrodinger's equation for a wave function of quantum in space of the generalized coordinates.

In space of the generalized coordinates Schrodinger's equation has nonlinear character that is consequence of nonlinear dependence of the generalized coordinate from parameters.

In space of the generalized coordinates energy of vacuum is a constant not dependent on the changing parameter of quantum - its frequencies, and the quantum length exponential falls with increase in volumetric density of its energy.

The two-solitons solution of Schrodinger's equation for a photon, apparently, will allow solve a problem of a quantum entanglement (a quantum teleportation).

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