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# Higgs Masses in a Renormalization-Group Improved Non-Minimal Model of Supersymmetry

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**Abstract:** We give a systematic study of Higgs masses in a non-minimal model of supersymmetry with renormalization group improvement for extended values of the parameters like  $\tan\beta$ ,  $M_C$  (the mass of charged Higgs) and  $Q$  (the squark mass scale), in the context of LHC experiments. Several new and interesting results are obtained.

**Keywords:** Higgs Mass, Non-Minimal Model of Supersymmetry, Renormalization Group, LHC Experiment

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## 1. Introduction

Supersymmetry is a very important tool for studying physics beyond standard model [1]. It is being vigorously pursued for detecting supersymmetric particles in nature. The supersymmetric particles include the supersymmetric partners of ordinary particles and supersymmetric Higgs bosons. The minimal supersymmetric standard model (MSSM) [2], for example, contain two CP-even neutral Higgs bosons, a CP-odd neutral Higgs boson and two charged Higgs bosons. The relaxation of any of the assumptions of MSSM leads to non-minimal model (NMSSM). Ellis et al [3] gave non-minimal model by inclusion of a single Higgs field whose vacuum expectation value determined the mixing of the two Higgs doublet of the minimal supersymmetric standard model. These authors studied the spectrum and couplings of Higgs bosons in this extended model and compared them with those in the minimal model. In their extensive work [3], Ellis et al analyzed the possible production mechanisms and phenomenological signatures of the different Higgs bosons at the TeV-scale.

On the other hand, Pandita [4] has calculated the dominant one-loop radiative corrections arising from quark-squark loops to the mass squared matrix of the CP-even Higgs bosons in a non-minimal supersymmetric standard model containing two Higgs doublets and a Higgs singlet chiral superfield using one-loop effective potential approximation. He evaluated upper and lower bounds on the radiatively corrected masses of all the scalar Higgs bosons as a function

of parameters of the model. He found that the one-loop radiative corrections were substantial only for the lightest Higgs boson of the model. He also calculated an absolute upper bound on the mass of the radiatively corrected lightest Higgs boson and compared it with the corresponding bound in the MSSM. Other interesting works on Higgs in NMSSM exist in the literature [5].

The present study is different from previous study [4] in some aspects. For example, we do not consider  $A_t = A_b = 0$  but  $A_t$  and  $A_b$  are given by non-zero values according to the Renormalization group improvement [6]. We consider the value of  $m_t$  given by the experimental value of 174 GeV. We consider dependence of the Higgs masses on the values of  $\tan\beta$  which varies from 0-50, unlike the previous work [4]. Moreover the charged Higgs mass is varied from 0-1000 GeV and supersymmetry breaking scale is considered to be very large (1 TeV).

The paper is organized as follows: Section 2 gives the expressions of Higgs masses in non-minimal supersymmetric standard model. In section 3 we give the renormalization group improvement [6] that is to be incorporated into the expression for the Higgs mass. The results and discussions are given in section 4. The conclusions are given in section 5.

## 2. Higgs masses in Non-Minimal Supersymmetric Standard Model

Following ref. [4], the radiatively corrected mass-squared matrix for the CP-even mass eigenstates are given by

$$\frac{1}{2} \left[ \frac{\partial^2 V_1}{\partial \psi_i \partial \psi_j} \right]_{v_1, v_2, x} = \begin{bmatrix} m_z^2 \cos^2 \beta & -m_z^2 \frac{\sin 2\beta}{2} + 2\lambda^2 v_1 v_2 & 2\lambda^2 v_1 x - \lambda k v_2 x \\ -m_z^2 \frac{\sin 2\beta}{2} + 2\lambda^2 v_1 v_2 & m_z^2 \sin^2 \beta & 2\lambda^2 v_2 x - \lambda k v_1 x \\ 2\lambda^2 v_1 x - \lambda k v_2 x & 2\lambda^2 v_2 x - \lambda k v_1 x & 4k^2 x^2 - k A_k x - \lambda k v_1 v_2 \end{bmatrix} + \begin{bmatrix} \frac{\lambda x}{2} \tan \beta & -\frac{\lambda x}{2} & -\frac{\lambda v_2}{2} \\ -\frac{\lambda x}{2} & \frac{\lambda x}{2} \cot \beta & -\frac{\lambda v_1}{2} \\ -\frac{\lambda v_2}{2} & -\frac{\lambda v_1}{2} & \frac{\lambda v_1 v_2}{2x} \end{bmatrix} \Delta + \frac{3g^2}{16\pi^2 M_W^2} \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} \\ \Delta_{12} & \Delta_{22} & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} \end{bmatrix} \quad (1)$$

Where  $\psi_i \equiv \text{Re}H_i$  for  $i=1,2$  and  $\psi_3 \equiv \text{Re}N$ . The eigenvalues of this matrix are the Higgs square masses  $m_{S_1}^2, m_{S_2}^2$  and  $m_{S_3}^2$ . The eigenvalues of  $\tilde{t}$  and  $\tilde{b}$  mass squared matrices are given by

$$m_{\tilde{t}_{1,2}}^2 = m_t^2(\mu) + \frac{1}{2}(m_Q^2 + m_U^2) + \frac{1}{4}m_z^2 \cos 2\beta \pm \sqrt{\left[ \frac{1}{2}(m_Q^2 - m_U^2) + \frac{1}{12}(8m_w^2 - 5m_z^2) \cos 2\beta \right]^2 + m_t^2(\mu)(A_t + \lambda x \cot \beta)^2} \quad (2)$$

$$m_{\tilde{b}_{1,2}}^2 = m_b^2(\mu) + \frac{1}{2}(m_Q^2 + m_D^2) - \frac{1}{4}m_z^2 \cos 2\beta \pm \sqrt{\left[ \frac{1}{2}(m_Q^2 - m_D^2) - \frac{1}{12}(4m_w^2 - m_z^2) \cos 2\beta \right]^2 + m_b^2(\mu)(A_b + \lambda x \tan \beta)^2} \quad (3)$$

Where the approximation  $m_Q = m_U = m_D = m_{SUSY}$  and the relations

$$A_\Sigma = A_\lambda + kx$$

$$v^2 = v_1^2 + v_2^2 \simeq (246)^2 \quad (4)$$

$$r = \frac{x}{v}$$

$$m_c^2 = m_w^2 - \lambda^2(v_1^2 + v_2^2) + \lambda(A_\lambda + kx) \frac{2x}{\sin 2\beta} \quad (5)$$

and

$$A_\lambda = (m_c^2 - m_w^2 + \lambda^2 v^2) \frac{\sin 2\beta}{2\lambda x} - kx \quad (6)$$

are used.

### 3. RG-improvement of Yukawa Couplings

The renormalization group (RG) equations for the top-quark and bottom-quark Yukawa couplings are given by [7]:

$$16\pi^2 \frac{dh_t}{dt} = h_t \left[ \frac{9}{2} h_t^2 + \frac{1}{2} h_b^2 - \sum_i C_i^{(t)} g_i^2 \right] \quad (7)$$

$$16\pi^2 \frac{dh_b}{dt} = h_b \left[ \frac{9}{2} h_b^2 + \frac{1}{2} h_t^2 - \sum_i C_i^{(b)} g_i^2 \right] \quad (8)$$

where  $t = \ln \mu$ ,  $h_j = \text{Yukawa coupling}$ ,  $C_i^{(t)} = \left( \frac{17}{20}, \frac{9}{4}, 8 \right)$  and  $C_i^{(b)} = \left( \frac{1}{4}, \frac{9}{4}, 8 \right)$ ,

$$i = Y, 2L, 3C.$$

We now introduce, in brief, the method of solution of RG-equations [6] which has been used in our analysis. For  $\mu > m_t$ , the integration of equations (7,8) gives

$$h_t(\mu) = \frac{h_t(m_t)}{A_t} e^{\frac{9}{2} I_t + \frac{1}{2} I_b} \quad (9)$$

$$h_b(\mu) = \frac{h_b(m_t)}{A_b} e^{\frac{1}{2} I_t + \frac{9}{2} I_b} \quad (10)$$

where

$$I_t = \int_{\ln m_t}^{\ln \mu} \frac{h_t^2(m_t)}{16\pi^2} dt = \frac{1}{16\pi^2} h_t^2(m_t) \ln \frac{\mu}{m_t} \quad (11)$$

$$I_b = \int_{\ln m_t}^{\ln \mu} \frac{h_b^2(m_t)}{16\pi^2} dt = \frac{1}{16\pi^2} h_b^2(m_t) \ln \frac{\mu}{m_t} \quad (12)$$

$$m_t(\mu) = \frac{m_t(m_t)}{A_t} e^{\frac{9}{2} I_t + \frac{1}{2} I_b} \quad (13)$$

$$m_b(\mu) = \frac{m_b(m_t)}{A_b} e^{\frac{1}{2} I_t + \frac{9}{2} I_b} \quad (14)$$

and

$$A_f = \prod \left( \frac{\alpha_i(\mu)}{\alpha_i(m_t)} \right)^{c_i/2a_i} \quad (15)$$

The functions  $A_f$  ( $f = t, b$ ) are obtained by integrating out the gauge coupling contributions in equations (7,8) and keeping terms up to one loop out of the two loop approximation

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i(m_t)} - \frac{a_i}{2\pi} \ln \frac{\mu}{m_t} \quad (16)$$

$$a_i = \begin{pmatrix} 21 \\ 5 \\ -3 \\ 5 \end{pmatrix} \quad (17)$$

Here

$$c_i^{(t)} = \left( \frac{17}{20}, \frac{9}{4}, 8 \right) \text{ and } c_i^{(b)} = \left( \frac{1}{4}, \frac{9}{4}, 8 \right)$$

For a given  $\tan \beta$  ( $= \frac{v_2}{v_1}$ ), the  $h_t$  and  $h_b$  Yukawa couplings

are computed at  $\mu = m_t$  using the following relations

$$h_t(m_t) = \frac{m_t(m_t)}{174 \sin\beta} \quad (18)$$

$$h_b(m_t) = \frac{m_b(m_t)}{174 \cos\beta} \quad (19)$$

The equations (9-19) are additional improvisation to the earlier work of ref.[4] giving the RG-modified Higgs masses.

## 4. Results and Discussions

The numerical values of parameters chosen by us in this model are as follows:

$$\lambda = 2, k = 1, x = 80, \mu = 200, Q = 1000 \text{ GeV}.$$

In our systematic study, the variation of Higgs boson masses  $m_{s1}, m_{s2}$  and  $m_{s3}$  with  $\tan\beta$  are studied for a wide range of variation of  $\tan\beta (= 1 - 50)$  for a particular value of  $M_c (= 200 \text{ GeV})$ . We find that  $m_{s1}$  can have a minimum value of about 200 GeV and a maximum value of about 390 GeV. The corresponding values of  $m_{s2}$  ( $m_{s3}$ ) can be about 260 (550) GeV and 390 (900) GeV. Interestingly, we also find that the minimum values of  $m_{s1}$  and  $m_{s2}$  corresponds to  $\tan\beta \approx 1$ . For this value of  $\tan\beta$ ,  $m_{s3}$  is about 900 GeV. The highest values of  $m_{s1}$  and  $m_{s2}$  correspond to  $\tan\beta = 25 - 50$ . For large values of  $\tan\beta$ , the value of  $m_{s3}$  is about 550 GeV.

Next the variation of  $m_{s1}, m_{s2}$  and  $m_{s3}$  with  $M_c (= 0 - 1000 \text{ GeV})$  for  $\tan\beta = 1.5$  and  $Q = 1 \text{ TeV}$  are studied for the parameters chosen in ref. [4] (viz.  $\lambda = 0.87, k = 0.63, r = 0.1, \mu = 200, Q = 1000 \text{ GeV}$ ) after RG-improvement to the model. The minimum value of  $m_{s1}$  is about 25 GeV ( $M_c = 100 \text{ GeV}$ ) and maximum value of  $m_{s1}$  is 180 GeV for  $\tan\beta = 1.5$  and  $Q = 1 \text{ TeV}$ . For  $m_{s2}$  the minimum value is about 250 GeV ( $M_c = 250 \text{ GeV}$ ) and for  $m_{s3}$  the minimum value is 780 GeV ( $M_c = 100 \text{ GeV}$ ) for same  $\tan\beta$  and  $Q$ .

These results suggest that RG-improvements on  $m_b(\mu)$  and  $m_t(\mu)$  give rather large values of  $m_{s1}$  compared to the predictions on the upper limit ( $\sim 150-160 \text{ GeV}$ ) in NMSSM [8, 9, 10]. Besides this, the upper bound on the mass of the lightest Higgs boson studied in this paper is larger than that in SM [11] and MSSM [12]. However, the studies considered in this paper will hopefully be verified in the future LHC experiments.

## 5. Conclusions

This simple-minded study includes the effect of RG-

improvement to the Higgs masses in NMSSM [4]. Unlike the previous studies [3,4], one considers here the experimental value of the top quark mass and an appropriate value of the squark mass scale. Moreover the variation of  $m_{s1}, m_{s2}$  and  $m_{s3}$  with  $\tan\beta$  are also studied here. This study also includes the parameters given in ref.[4]. These analyses give some new results for large  $M_c$  and  $\tan\beta$ .

Thus the RG-improvements in the Yukawa couplings of the b- and t-quarks introduce certain interesting features in the analyses of the Higgs masses in non-minimal supersymmetric standard model. We find that the Higgs masses are now much heavier than the previous results and are most likely to be discovered in the mass range covered by the LHC.

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