SciencePG
Science Publishing Group

The Application in the Portfolio of China's A-share Market with Fama-French Five-Factor Model and the Robust Median Covariance Matrix

Xinming Chen[1], Peng Song[2], Ke Gao[3], Yankuo Qiao[4]

[1]The School of Government, Central University of Finance and Economics, Beijing, China

[2]The School of Statistics and Mathematics, Central University of Finance and Economics, Beijing, China

[3]The School of Finance and Taxation, Central University of Finance and Economics, Beijing, China

[4]The School of Business, The State University of New Jersey, Newark, USA

Email address:

songpeng1101@126.com (Peng Song)

Abstract: In the traditional portfolio model, investors calculate the expected return of assets and the covariance matrix for optimal asset allocation. This paper divides market sentiment period into three states and selectes the securities in the Chinese stock market to construct portfolios. We implement both the Fama-French five-factor model and the robust median covariance matrix approach for predicting the expected return of the selected stocks and portfolio optimization respectively. Then we compare the performance of the portfolio constructed by the Fama-French three-factor model with that by the traditional covariance matrix in different market sentiment periods. The empirical results indicates that the performance of the portfolio constructed by the Fama-French five-factor model is more sensitive to the fluctuation of stock market sentiment, and that the robust median covariance matrix approach tends to have relatively stable portfolio return, while ineffective in the bull market. The main contribution of this paper is having empirically tested different model combinations in portfolio theory using the data of Chinese market where market sentiment has unique impact. To some extent, this paper provides a reference to the portfolio strategy.

Keywords: Fama-French Five-Factor Model, Robust Median Covariance Matrix, Application of Portfolio

1. Introduction

Nobel laureate Markowitz (1952) quantified the return and risk of assets, and pioneered the modern portfolio theory [1]. Since then, optimizing return given a certain level of risk has become the essential problem of mordern portfolio theory. Given a fixed number of assets, the portfolio optimization is generally achieved by allocating assets in such a way that minimizing risk for a certain target return or maximizing return at a certain level of risk. The optimal portfolio weights (asset matching weights) are therefore decided in the allocation process. Obviously, the choice of input variables and their robustness will directly affect the accuracy of the asset matching weight.

Since Sharpe (1964) first proposed the capital asset pricing model (CAPM), the research of asset pricing has been focusing on the improvement of the factor models (APT) [2]. CAPM is an elegant, single factor model, in which the market (systemic) risk is the only risk source. The underlying logic of CAPM is that the investors shall be compensated solely by bearing the market risk. Although the model provides innovative insights and intuitions, its validity has been undermined over time after 1970s because of the recognition of emerging market anomalies in later studies. Ross (1976) extended the scope of CAMP model by proposing arbitrage pricing theory (APT) which had become the theoretical framework for explaining anomalies [3]. Compared with the CAPM, the APT theory develops a multi-factor model, in which some risk factors in the model may not have solid economic explanation. Fama and French (1992) added the size factor and the value factor to the CAPM,

formulating the three-factor model [4]. The three-factor model assumes that the risk premium of asset returns is not only determined by market risk, but also related to size effect and book value ratio. The statistical insignificance of the regression intercept indicates that risk-award return of stock can be fully explained by the above three factors. Notably, both the CAPM model and the Fama-French three-factor model are based on the Efficient Market Hypothesis. However, in the latter part of the last century, as more complex market conditions emerge, the development of behavioral finance theory provides a new perspective for asset pricing theory. Proposed by the behavioral finance theory, Carhart (1995) further diversified factor construction dimensions by adding the momentum factor to the three-factor model, constituting a four-factor model [5].To better explain the existent mispricing among the current market stock returns, Fama and French (2015) supplemented the original three-factor model by adding the profitability factor and the investment factor, forming the five-factor model, and further enhancing the Model's ability to accurately measure and reasonably explain asset returns [6].Some scholars have studied the applications of the factor model to the china stock market. Zhang et al. (2012) empirically tested a three-factor model in China's A-share market and concluded that it fits Chinese market [7]. In comparison with the three-factor model, Zhao et al. (2016) applied the five-factor model to China's A-share market and demonstrated that although the effects of profitability factor and investment factor are not significant, the model accuracy increased slightly [8]. On top of Zhao's work, this paper will carry on an empirical test on the application of the Fama and French multi-factor model for pricing portfolios in China's A-share market.

Constructing robust estimator of covariance matrix is an important topic in modern portfolio theory. In the end of the 20th century, many literatures documented the instability in portfolio optimization. However, the majority of research concentrated on the sensitivity and uncertainty of mean-variance approaches in solving the portfolio optimization problems. Campbell (1980) broaden the scope of the research by pioneering the Campbell Covariance Estimator for robust estimation of covariance matrices [9]. Visuri et al. (2000) proposed a non-parametrical robust estimation method based on the sign and rank of covariance matrices [10]. Rousseeuw et al. (2004) proposed a minimum covariance determinant, bringing the idea of iteration into the robust estimation process of covariance matrix [11]. The estimation is solved by iterating and minimizing the Mahalanobis distance, and then calculating weighted average. Kim and White (2004) questioned the robustness of traditional method for constructing covariance matrices due to the sensitivity of sample mean to skewness and kurtosis caused by widely existing outliers in real world financial data recorded in extreme cases [12]. Therefore, Ergun (2011) introduced the idea of quantile estimation into the covariance matrices construction [13]. He argued that this method is sufficiently robust against outliers in typical real world data. Huo et al. (2012) implemented the robust quantile idea in the estimation of covariance matrices by replacing mean with median to

obtain the robust Cov-median Estimator [14]. In addition, the comparison with the aforementioned robust estimation method of covariance matrix done in his study supported the superiority of robust median covariance matrix.

To sum up, this paper applies the Fama-French five-factor model to portfolio optimization, and introduces a robust median covariance matrix to reduce the instability of the portfolio performance. We are going to practically implement the investment strategy by attempting to construct a portfolio with the highest sharp ratio. In addition, for comparison purpose, we calculate the expected returns and covariance matrices from historical data for given assets using the Fama-French three-factor model and other traditional methods. By comparing those portfolios in a comprehensive way, the article contributes to portfolio management strategies in multiple states of China's stock market.

2. Method

The framework of modern portfolio theory was first introduced by Markowitz in 1952. He proposed that investors need to go through two stages in the decision making process of investment. In the first stage investors select which assets to invest, and then in the second stage they shall allocate the selected assets efficiently. The portfolio theory aims at solving the problem of asset allocation in the second stage. In this paper, we use the factor model under arbitrage pricing framework to price the expected return of selected asset, solving the problem in the first stage, and implement the robust median covariance matrix to approximate the variance-covariance matrix of the asset relevance, supporting the optimal allocation in the second stage. On top of the combined approaches above, we can theoretically determine excellent asset allocation weight in China's stock market.

2.1. Portfolio Model

Portfolio theory, in essence, is a theory of selection, aiming for obtaining diversified portfolio of less correlated assets. Given n different assets, We define the return vector as $R = (r_i, r_2, r_3 \dots r_n)^T$, and define covariance matrix as $\Sigma = (\sigma_{ij})_{n \times n}$,among which $\sigma_{ij} = cov(r_i, r_j)$,Representing portfolio weights by W, i.e., $W = (w_1, w_2, w_3 \dots w_n)^T$,the optimization formulation of the portfolio is：

$$\begin{cases} max\, E(W^T R) \\ s.t.\, W^T \Sigma W \leq \sigma_0^2 \\ \sum_{i=1}^n w_i = 1 \end{cases} \qquad (1)$$

or

$$\begin{cases} min\, W^T \Sigma W \\ s.t.\, E(W^T R) \geq r_0 \\ \sum_{i=1}^n w_i = 1 \end{cases} \qquad (2)$$

If short selling is not allowed, then $w_i \geq 0$.

By maximizing the expected return given any risk level or vice versa, the portfolio efficient frontier can be identified. The portfolio with highest Sharpe Ratio can be located along

the capital market line (CML), which is a tangent line to the efficient frontier, dominating the frontier and passing through risk free rate on the y axis, as showed below in Figure 1,
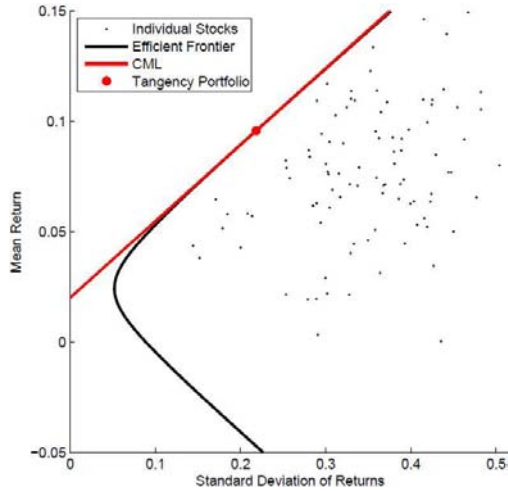

Figure 1. The efficient frontier of the portfolio.

2.2. Fama-French Factor Model.

Fama and French introduced the factors of SMB and HML

on top of the CAPM, and proposed the famous three-factor model:

$$E(r_i) - r_f = \alpha_i + \beta_i[E(r_M) - r_f] + S_i SMB + H_i HML \quad (3)$$

in which $E(r_i)$ is the expected return of the portfolio; r_f represents the risk-free rate of interest; $E(r_m)$ denotes the expected return rate for the whole market portfolio; SMB is the size factor, representing the difference of portfolio returns between firms with low market value and high market value, and HML is the value factor, representing the difference of portfolio returns between the firms with higher book-to-market ratio and those with lower growth ratio.

The three-factor model has been a prevalent model in asset pricing domain and has attracted extensive attention from both scholars and investors. Inspired by the model, many scholars began to consider investment from the perspective of company management, instead of the traditional point of view about asset pricing. Fama and French further introduced the factors measuring profitability and investment level of the company into factor pricing, formulating a five-factor model.

$$E(r_i) - r_f = \alpha_i + \beta_i[E(r_M) - r_f] + S_i SMB + H_i HML + R_i RMW + C_i CMA \quad (4)$$

Compared with the three-factor model, the newly introduced profitability factor (RMW) is constructed by categorizing the listed companies into two groups, Robust and Weak, according to the rate of operating profit. Fama proposed that the companies with higher profit margins could distribute more dividend to shareholders and therefore the expected return of the stock should be higher. The RMW factor is then constructed by the historic excess return of robust group stocks over weak group stocks.

Analogously, listed companies can be categorized into two groups, Conservative and Aggressive, in accordance with total assets growth rate. Fama argued that high investment level will reduce the contemporaneous level of total assets, and increase the uncertainty in the future, together having a negative impact on the investors' purchase incentives. The investment level factor is then built by the excess return of the conservative group over that of the aggressive group.

2.3. Robust Median Covariance Matrix

In the process of the traditional covariance estimation, define two stochastic processes,

$$\{x_t\}_{t=1\dots T} \text{ and } \{y_t\}_{t=1\dots T},$$

The covariance formula can be expressed as follow,

$$C = E[(x_t - \mu_x)(y_t - \mu_y)] \quad (5)$$

in which the population expectations of the two variables are represented respectively as,

$$\mu_x = E(x_t) \text{ and } \mu_y = E(y_t).$$

For estimation purpose, the sample mean value is substituted for the population mean, and the estimated covariance can be obtained as follow.

$$\hat{C} = \hat{E}[(x_t - \bar{x})(y_t - \bar{y})] \quad (6)$$

$\hat{E}(.)$ denotes the solution of the sample mean.

The traditional method for estimating the covariance is based on the sample mean and is susceptible to the outliers. Denote outliers in two stochastic processes as m_x and m_y respectively. Assume that outliers occurred in at $[\tau T], \tau \in (0,1)$; outliers occurred in Y at $[sT], s \in (0,1)$. Therefore the covariance matrix can be estimated by the formula below.

$$\hat{C} = \hat{C}_0 + \frac{m_x}{T}(x_{[sT]} - \bar{x}) + \frac{m_y}{T}(y_{[\tau T]} - \bar{y}) + \frac{m_x m_y}{T^2} + \frac{m_x m_y}{T} \times 1_{[[sT]=[\tau T]]} \quad (7)$$

$\hat{C}_0$ represents the estimate of the covariance without outliers. The formula implies that after taking adding the outliers into the calculation, the estimation error is determined by three parts: sample size, deviation scale and outliers. In the case that the outliers of both variables occur at the same time,

the error becomes larger.

In the presence of outliers, the quantile is more robust than the mean. Based on the idea of quantile robustness, Huo et al (2012) suggested a robust median covariance measure, i.e.,

$$C_R = M[(x_t - k_x)(y_t - k_y)] \qquad (8)$$

in which $k_x = Median(x_t)$ represents the population median of two variables, and $k_y = Median(y_t)$ represents the median of the joint distribution. Median covariance estimates were obtained by substituting the median of the samples for the population median as follow.

$$\hat{C}_R = \hat{M}[(x_t - \hat{k}_x)(y_t - \hat{k}_y)] \qquad (9)$$

In fact, the robust median covariance estimation method is not only more robust than traditional methods, but also more understandable and more applicable. Traditional estimation methods rely heavily on the underlying assumption that the first moment of the distribution shall exist, which restrict the compatibility of the model in the real world application. For example, if random variables follows Cauchy distribution, whose first moment doesn't exist, the traditional estimation method fails apart and the robust median estimate remain effective. In fact, the robust method can be applied to any potential distribution.

When this method is applied to a multidimensional variable (assuming there are n variables), the variance-covariance matrix of multidimensional variables can be obtained as below.

$$\hat{\Sigma}_R = (\hat{C}_{Rij})_{n \times n} \qquad (10)$$

3. Result

In this section, the Fama-French five-factor model and the robust median covariance matrix are applied to the portfolio. In comparison with the test results of the three-factor model with the historical data, we explore the performance of the portfolio in different strategies and propose some insights on portfolio management.

3.1. Data Selection

China's A-share market began to split share structure reform in 2005, and the degree of marketization of China's stock market system became higher and the market became more efficient after the reform; During the global financial crisis of 2008, the China's A-share stock index fell from 6100 points to 1600 points, experiencing a year of bear market which is followed by up to 6 years' fluctuation; In the latter half of 2014 to early 2015, influenced by the "area along the way" and other policies, A-shares ushered in bull market lasting around a year. In order to explore and compare the performance of the model in three market sentiments, this empirical test will be conducted in three time periods: the first time period is from January 2008 to December 2008, representing the low market sentiment (bear market) period, a period of 246 trading days in total; the second time period is from July 2013 to June 2014, representing the period of stable market sentiment (fluctuation), a scope of 244 trading days; the third time period is from July 2014 to June 2015, representing the high market sentiment period (bull market), a time span of total 245 trading days. According to Markowitz's two-stage portfolio theory, stock selection is required before the model characterizes the investment behavior.

In order to better diversify the non-systematic risk and reduce the correlation between the stock returns, we select stock in different industries. Wind information slices the A-share market into 10 industries: energy, materials, industry, optional consumption, daily consumption, healthcare, finance, information technology, telecommunication services and public utilities. Striping out ST and ST * stocks, in general we only pick stocks with good fundamentals from each sector, which are not delisted more than 15 days in the tested period. The selection covers stocks listed both in Shanghai and Shenzhen, and stocks with different market cap. Wind All A index is used as the benchmark index.

3.2. Process Data

In order to utilize the pricing power of Fama-French factor model to the full extent, it is essential to select appropriate proxies for factors to avoid measurement error. We choose the return rate of Wind A index as the market return to construct market factor; We use commercial bank's three months daily lump sum deposit interest rate during the test period to proxy for Risk-free rate. In order to improve the accuracy of proxy for the size factor in China's stock market, we use April 30, the last day for annual report disclosure of listed company, as a settlement time. The market value is then calculated by stock price multiplying shares outstanding. To narrow the gap between the number of stocks in the Big Group and in the Small group, we categorize stocks in accordance with the median size in Shanghai stock market due to the fact that the majority of stocks traded in Shenzhen are small-cap stocks. In the construction of the *HML* factor, the book value is measured by shareholders' equity; the corresponding market value is measured by the market cap after the last trading day in December, since this financial indicator is an annual index. The firms with corresponding book-to-market ratio calculated from above is then ordered and split into top 30%, middle 40% and last 30% respectively. The construction of profitability factor (*RMW*) is analogous to that of the *HML* factor, except that the stocks are divided into two groups according to the median of operating profit obtained from financial statements. Consistent with profitability factor, investment mode factor (*CMA*) is constructed using indicators, total assets growth rate, from financial statements.

3.3. Empirical Analysis Method

Fama and French focused on the selection and validation of factors when they formulate the factor model. The portfolio is constructed, using monthly data, according to the cross effects of factors in the model. Fama and French argued that it takes time for investors in the market to judge and absorb information released from financial statement. In other words, it takes time for stock prices to reflect revealed information. Empirically the whole process takes about 3 months, so the annual rolling test spans from July to June next year. Our goal is to use the Fama-French factor model to price assets and build portfolios in different states of market sentiment. In the selection of the starting point for each period, our second and third test period are consistent with the specifications in Fama and French's paper.

However, in order to have a complete state of market sentiment (Bear Market), we identify the first period from the first trading day of 2008 to the last trading day of 2008. In addition, in order to get sufficient data for empirical test, we use daily data instead of monthly data for each test period which is around one trading year. Furthermore, considering the changes in market information and transaction costs, we adjust the positions of the portfolio every three month, which is for updating information for recalculation of the expected return, covariance matrix, and asset allocation weights. The optimal asset allocation weights are solved by maximizing Sharpe Ratio.

3.4. Empirical Results

During the three test periods, the selected stock is shown in Table 1.

Table 1. The codes and names of the stocks.

2008.1-2008.12		2013.7-2014.6		2014.7-2015.6	
601699	LUANHUANNENG	600583	HAIYOUGONGCHENG	000059	HUAJINGUFEN
000751	XINYEGUFEN	002010	CHUANHUAGUFEN	600547	SHANDONGHUAJIN
600350	SHANDONGGAOSU	601186	ZHONGGUOTIEJIAN	600018	SHANGANGJIETUAN
600690	QINGDAOHAIER	002024	SUNINGYUNSHANG	000651	GELIDIANQI
000729	YANJINGPIJIU	600519	GUIZHOUMAOTAI	600887	YILIGUFEN
600085	TONGRENTANG	600276	HENGRUNYAOYE	000538	YUNNNABAIYAO
600030	ZHONGXINZHENGQUAN	600036	ZHAOSHANGYINHANG	000002	WANKEA
600601	FANGZHENGKEJI	002027	FENZHONGCHUANMEI	000063	ZHONGXINGTONGXUN
600050	ZHONGGUOLIANTONG	600804	PENGBOSHI	600050	ZHONGGUOLIANTONG
601991	DATANGFADIAN	600011	HUANENGGUOJI	600027	HUADIANGUOJI

Table 2 presents the performance measures of the portfolio, showing the average performance of the portfolio during the test period. FF3 stands for the three-factor model, FF5 stands for the five-factor model, Con denotes the traditional covariance matrix, and denotes the median Variance matrix.

Table 2. The performance of the portfolio.

	Period 1			Period 2			Period 3		
	Return	standard deviation	Sharpe ratio	Return	standard deviation	Sharpe ratio	Return	standard deviation	Sharpe ratio
Wind	-0.01380	0.02838	-0.71231	-0.00950	0.01165	-0.82191	-0.00285	0.03751	-0.07801
FF3Con	-0.00501	0.03850	-0.29680	-0.00930	0.02352	-0.39841	-0.00799	0.01480	-0.54434
FF3Rob	-0.01405	0.02556	-0.80074	-0.00930	0.01939	-0.48307	-0.008456	0.01366	-0.62450
FF5Con	-0.01580	0.03757	-0.59119	-0.00912	0.01045	-0.87928	-0.00780	0.01623	-0.48490
FF5Rob	-0.01496	0.03233	-0.66095	-0.00916	0.00878	-1.05180	-0.00822	0.01331	-0.62289

In the test period of Bear Market, investors are selling stocks and the Sharp ratio is negative, indicating the capital appreciation rate from the stock market is lower than the risk-free rate. Compared with the FF5 portfolio, the FF3-Rob portfolio has a lower volatility and a higher cumulative return on average. As showed in the Figure 2, the overall performance of FF3 is slightly better than that of FF5, and the portfolio volatility using the robust median covariance matrix is lower than that of traditional method. The cumulative return time series, as shown in Figure 3, demonstrate that Rob does reduce the volatility of the portfolio under the same pricing model for expected return. The advantage of FF3 is not significant under the same estimation method of covariance matrix. It is worth noting that the portfolio constructed using FF3-Con obtained a positive cumulative return of around 5% at the end of the test.
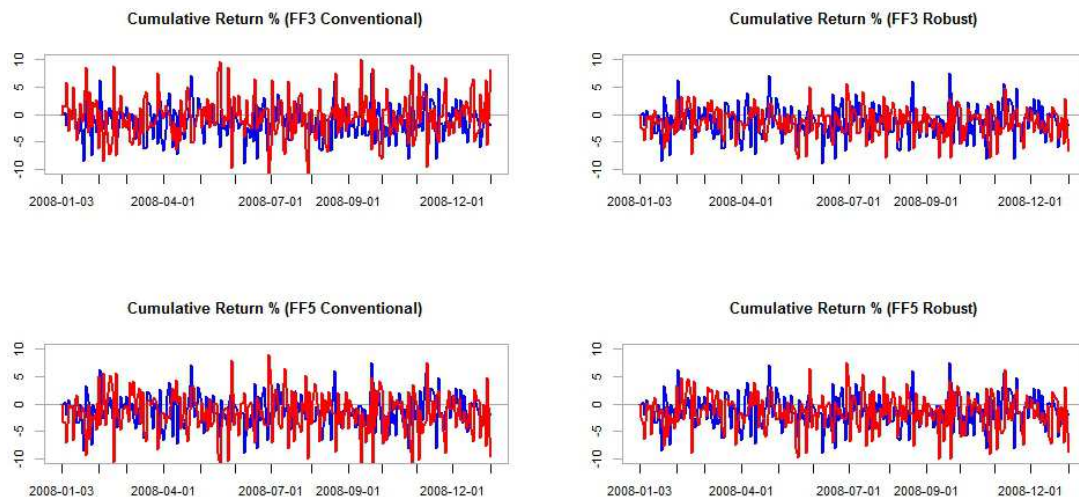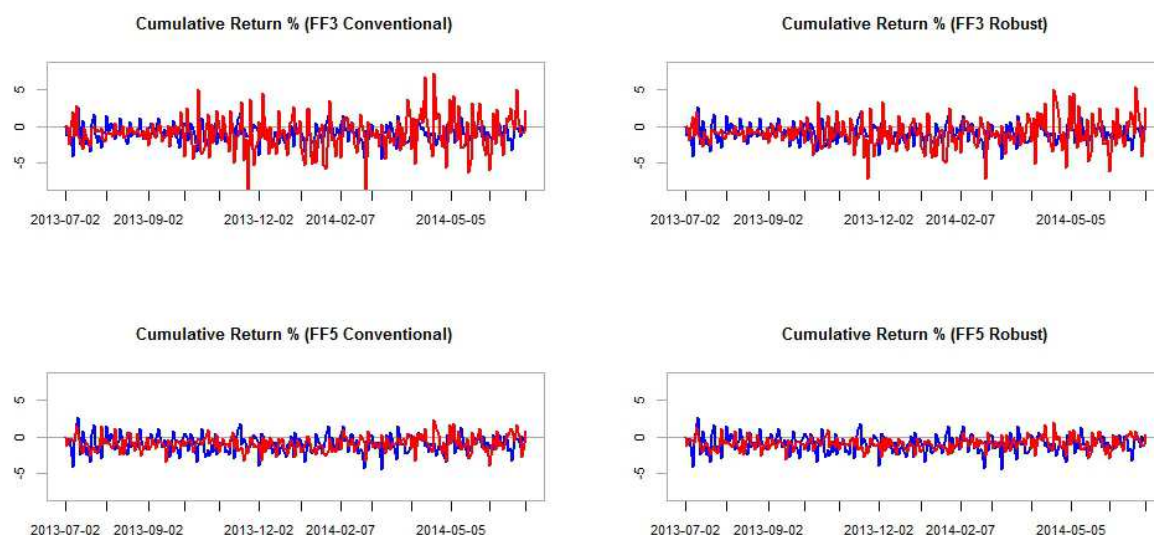

Figure 2. The cumulative return of the first period.

***Figure 3.** The cumulative return of the second period.*

In the test period of the high market sentiment when capital increasingly rushed into the stock market, the volatility of the four portfolios is distinctly lower compared to the overall market index. However, the portfolios are less profitable than the overall market index, implying the lower risk with the less return. The performance of the four portfolios is almost the same, while the application of Rob can slightly reduce the volatility, with a slight sacrifice of return accordingly. The portfolio of FF5-Con can achieve a higher Sharpe ratio while

the advantage is quite limited. The cumulative return time series, as shown in Figure 4, demonstrate that the overall market index is fluctuating sharply, especially in the end of the test period around June 2015, during which a short market crash happened. The time series of returns of the four constructed portfolio is fluctuating, although not quite observable, in different degrees. In the early stage of each test, the performance is rather equivalent to market index, while getting more table later in the test hovering around 0%.
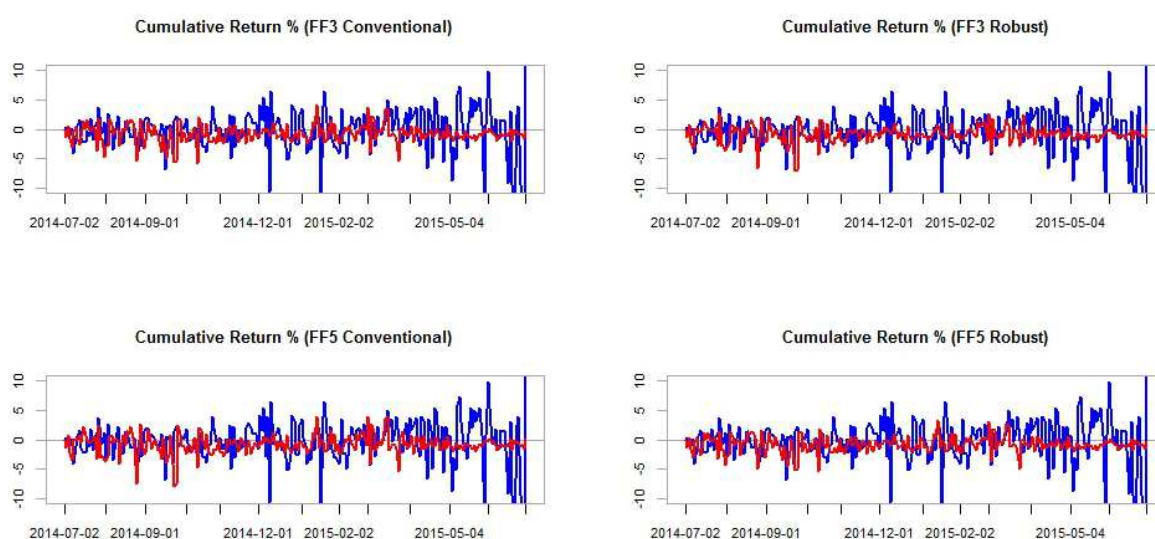

***Figure 4.** The cumulative return of the third period.*

4. Conclusion

This paper applies the Fama-French five-factor model and the robust median to the portfolio management under the framework of the traditional portfolio theory. The empirical test is carried out using China's A share market data in three distinguished period representing different level of the market sentiment. In comparison with the performance of different factor models and covariance matrices, we reach the following basic conclusions:

1) Compared to the Fama-French three-factor model, the profitability of portfolio constructed by selecting stocks based on the Fama-French five-factor model is more sensitive to market sentiment. The performance of the portfolio constructed by Fama-French three-factor model is stronger than that of the Fama-French five-factor model in depressed market sentiment. The combination of Fama-French three-factor model and traditional covariance matrices (FF3) could generate a higher positive accumulated return. In the state of stable market, the both portfolios constructed by the

Fama-French five-factor model and by Fama-French five-factor model can obtain roughly equivalent return to the market index. However, the portfolio return obtained by Fama-French five-factor model is less volatile. In addition, the portfolio constructed using the Fama-French five-factor model and the robust median covariance matrix (FF5-Rob) is the most robust one among all the four portfolios. In the period of high market sentiment with capital increasingly flowing into the stock market, the cumulative portfolio returns by the Fama-French three-factor model and by the Fama-French five-factor model are consistent, both lower than the market-wide index returns.

2) Using robustness median covariance matrix can reduce the volatility of the portfolio returns, but the effect of the hige market sentiment is not obvious.

In conclusion, the Fama-French multi-factor model is based on the Efficient Market Hypothesis. When the market sentiment is not stable, the stock return is more justified by investor behavior influenced by market sentiment factor, which may not be easily measured but definitely effective. As the number of factor increases, it's not surprise that the explanatory power of multi-factor models will increase in some degree. However, the Fama-French multi-factor model does not take into account the investor's emotional factors, which leads to mispricing or malfunction of the model. In other words, when market is current in an extreme sentiment state, emotional factor becomes much more important in the pricing model and therefore an approximate multi-factor pricing model without recognizing the dominance of emotional factor cannot be robust empirically. The robustness covariance matrix method can reduce the volatility of the investment portfolio in general cases, but the achievement comes at a cost of decreasing return accordingly, implying that the simple but overarching philosophy of the coexistence of the risk and the profit still holds in the equity investment domain.

Acknowledgements

References

[1] Markowitz H. Portfolio selection [J]. The journal of finance, 1952, 7(1): 77-91.

[2] Ross S A. The arbitrage theory of capital asset pricing [J]. Journal of economic theory, 1976, 13(3): 341-360.

[3] Sharpe W F. Capital asset prices: A theory of market equilibrium under conditions of risk [J]. The Journal of Finance, 1964, 19(3): 425-442.

[4] Fama E F, French K R. The cross‐section of expected stock returns [J]. the Journal of Finance, 1992, 47(2): 427-465.

[5] Carhart M M. Survivor bias and persistence in mutual fund performance [D]. University of Chicago Graduate School of Business, 1995.

[6] Fama E F, French K R. Incremental variables and the investment opportunity set [J]. Journal of Financial Economics, 2015, 117(3): 470-488.

[7] Zhang Xiecheng. The study on the China's A-share market with Fama-French three-factor model [D]. Fudan university, 2012.

[8] Zhao Shengmin, Honglei, Zhang Kai. Does Fama-French Five Factor Model Outperform Three Factor Model? Evidence from China's A-Share Market [J]. Nankai Economic Studies, 2016(2):41-59.

[9] Campbell N A. Robust procedures in multivariate analysis I: Robust covariance estimation [J]. Applied statistics, 1980: 231-237.

[10] Visuri S, Koivunen V, Oja H. Sign and rank covariance matrices [J]. Journal of Statistical Planning and Inference, 2000, 91(2): 557-575.

[11] Rousseeuw P J, Van Aelst S, Van Driessen K, et al. Robust multivariate regression [J]. Technometrics, 2012.

[12] Kim T H, White H. On more robust estimation of skewness and kurtosis [J]. Finance Research Letters, 2004, 1(1): 56-73.

[13] Ergun A T. Skewness and Kurtosis Persistence: Conventional vs. Robust Measures [C] //Midwest Finance Association 2012 Annual Meetings Paper. 2011.

[14] Huo L, Kim T H, Kim Y. Robust estimation of covariance and its application to portfolio optimization [J]. Finance Research Letters, 2012, 9(3): 121-134.