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# Modelling Kenyan Foreign Exchange Risk Using Asymmetry Garch Models and Extreme Value Theory Approaches

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**Abstract:** The Foreign Exchange Market in developing countries, Kenya being one of them is a key driving force for the development of a country economic growth. In the last decade, world financial markets have been characterized by significant instabilities and the currency exchange rate market is not an exception. As a consequence of the significant instabilities in the financial markets, this paper models the tail risk associated with the Kenya Shilling against the leading currencies, especially the one day ahead Value-at-Risk forecast in risk control, by using the two leading alternatives, the two-stage GARCH-EVT approach and the asymmetry GARCH models. In practice by applying the conditional Extreme Value Theory, the tail behaviour of the daily returns is modelled and thus the VaR while by using the asymmetry GARCH models, one models the whole distribution of the returns and thereafter estimates the Value at Risk. In addition to modelling the value at risk, we further examine the performance of the two leading alternatives with the daily log returns of leading currencies in the Kenyan Foreign Exchange market (US dollar, Sterling Pound and Euro) foreign currencies from the period January 2005 – August 2017 for trading days excluding weekends and holidays. The backtesting result indicate that the conditional Extreme Value Theory does not completely dominate the asymmetry GARCH models in estimating the VaR especially in the Sterling Pound and Euro Exchange Rates.

**Keywords:** Asymmetry GARCH Models, Value-at-Risk, Extreme Value Theory and Backtesting

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## 1. Introduction

The value of one country's currency converted in terms of the currency of another country is defined as the exchange rate. Financial markets worldwide have proved to be very turbulent. A major problem has risen in the past of how to measure and quantify risks especially in financial markets. Value-at-Risk (VaR) has been the popular measure of risks simply because of its simplicity among the risk measures available. Since the problem of quantifying the risk existed, the VaR as a method for measuring risk was put into practice and proposed in details by [1]. The worst loss that can occur over a given period of time in a given level of confidence is defined as Value-at-Risk according to [2].

There are basically two broad methods of estimating VaR namely; analytical and historical approach. In the former, an

assumption of a known distribution for the returns is made. Basically, because of the serial autocorrelation and non-constant variance of the returns, the distribution of the returns is modelled. For the historical approach, it avoids the assumption of the distribution and majors on the empirical distribution of the past information. The main drawback of this method is that of the appropriate window size to be used and the high dependence on the past information.

Many models have been proposed to estimate VaR across different regions and countries. In many literature, daily returns of assets are normally distributed but in reality they are heavy tailed and have a characteristic of skewness which may lead to the Value-at-Risk being overestimated and underestimated. The conditional Extreme Value Theory and GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models have gained popularity in the

recent past. It is in view of this that the study has implemented the models to explore the features of financial markets especially the Kenyan Foreign Exchange Market and to test how robust the conditional EVT model is.

Given that both conditional EVT and asymmetry GARCH models address the challenge of heavy tails in financial data and can be further used to forecast the VaR, the main aim of this work is to comprehend the major strengths and limitations of the Extreme Value Theory and GARCH models in estimating the value at risk. Our concern in this work is on the Kenyan Foreign Exchange market. A period spanning from January 2005 to August 2017 was used. Currencies used were the US Dollar, the Euro and the Sterling Pound. In the recent past, especially during the fiscal year 2015/2016, the Kenya Shilling depreciated against the US Dollar and the Euro mainly because of the tightening of the global financial market conditions and the slowdown in China. The Kenya Shilling strengthened against the Sterling Pound because of the uncertainty surrounding the United Kingdom exit from the European Union. In order to compare how the models performed in estimating the Value-at-Risk, backtesting tests are performed.

The other parts of the paper are categorized as: part 2 explores past works that exist, part 3 explores the GARCH family models and Extreme Value Theory and outlines basic calculation on how to do forecasting of the Value-at-Risk as per the Extreme Value Theory model and GARCH models. Part 4 outlines the Exchange rate used in this work. Part 5 gives the general results. Part 6 presents backtesting results. Lastly, part 7 provides a conclusion and recommendations for further studies.

## 2. Literature Review

GARCH-type model is the prevailing parametric approach used to model time-changing volatility of different distributions. The different GARCH models are conditioned on different distributions namely normal, the skewed student-t distribution and student-t distribution. However, the GARCH whose conditional distribution is normal has been heavily criticised since it under estimates the risk. Due to this shortfall of GARCH-normal the GARCH with t-distribution was proposed in order to capture the fat tails in finance. But the challenge with Student-t distribution is that it is symmetric, it does not reflect the asymmetric feature of the returns.

In order address the challenge of the symmetry GARCH models in financial markets, [3] fronted the Exponential GARCH. He hypothesized that the variance of asset returns responded differently to both positive and negative returns. The findings were that the hypothesis was true and in addition, he noted that there was a negative relationship between excess returns and the variance of stock markets. [4] Introduced another type of asymmetric model called the Giotis, Jagannathan and Runkle GARCH (GJR-GARCH) model.

A study by [2], noted that the prices of financial assets

often react more to “bad news than good news” and such condition contributes to leverage. [3], quantified the effects of good and bad news on volatility and found out that there was asymmetry in the volatility of stock markets. Specifically, they noted that there was a relationship between volatility and the mode of news. According to their study it was observed that bad news tend to create more volatility compared to the good news.

[4], modelled exchange rate volatility for USD/KES by applying different univariate GARCH models to the daily returns. The findings were that the best model in estimating and forecasting the Value-at-Risk was the APARCH and GJR-GARCH both on the student t distribution. They also compared the one-step-ahead VaR estimate from the asymmetric models with student t-distribution and concluded that AR (2) – APARCH (1, 1) model is superior in the estimating the one-step-ahead VaR.

In many fields, extreme value theory is well established. Many researchers have analysed the variations which are extreme and which financial markets are subjected to. [5] Showed that EVT mainly deals with the behaviour of the tail dependence of asset returns and is used in the modelling of the maxima of a random variable. [6], estimated the conditional quantile and conditional expected shortfall using the GARCH-EVT model for the Tunisian Stock Market. They fitted GARCH models to return data and the conditional Extreme Value Theory to model the tail behaviour. Comparison was made between the two methods and it was noted that the GARCH-EVT approach was the best performing model compared to the other methods.

[7], compared GARCH and EVT models in modelling Value-at-Risk for major countries stock market’s daily loss including United Kingdom, Hong Kong and United States and compared how the models performed. The backtesting results showed that the conditional Extreme Value Theory model performed in the same way to the GARCH model under the generalized error distribution. They further observed that the EGARCH model performed best in forecasting the Value-at-Risk.

## 3. Methodology

### 3.1. Value-at-Risk

The worst loss that can occur over a given period of time in a given level of confidence is defined as Value-at-Risk according to [2].

Proposals on models to estimate VaR across different regions and countries have been made in empirical literature. In this section, the asymmetry GARCH models and the GARCH-EVT methods of estimating Value at Risk are discussed.

#### 3.1.1. Standard GARCH Model

The GARCH (p, q) model is given by:

$$r_t = \mu + a_t, a_t = \sigma_t \varepsilon_t$$

where  $\delta_t$  is given by

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^p \alpha_i \alpha_{t-i}^2 + \sum_{i=1}^q \beta_i \delta_{t-i}^2$$

Where  $r_t$  are the logarithmic returns,  $\mu$  are the mean value of the returns,  $a_t$  are the innovations from the mean equation. In order to ensure stationary of loss series,  $\alpha_1 + \beta_1 < 1$ . The most commonly used GARCH model is the GARCH (1, 1) given by:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \alpha_{t-1}^2 + \beta_1 \delta_{t-1}^2$$

In order to have a positive variances every time, the following restrictions are imposed  $\alpha_0 > 0$ , and  $\alpha_1, \beta_1 \geq 0$ .

The standard GARCH model captures volatility clustering and heavy tails but they fail to model the leverage effect. In financial world to overcome this challenge, many asymmetry GARCH models have been proposed such as EGARCH model by [3] and Power GARCH model by [5]. the GJR-GARCH model by [4]

### 3.1.2. Asymmetric GARCH Models

[2], noted that the prices of financial assets often react more to bad news than good news and such a condition leads to leverage effect. To get the asymmetry property of volatility models were developed called asymmetry GARCH. In this paper, the following asymmetric models will be used to calculate the Value-at-Risk; Exponential GARCH, Glosten Jagannathan and Runkle GARCH model and the Power GARCH.

### 3.1.3. Exponential GARCH Model (EGARCH)

The EGARCH model for capturing the asymmetry nature of returns was developed to solve three main weaknesses of the symmetric GARCH model:

- i. Non sensitivity to asymmetric response of volatility to shocks
- ii. Parameter restrictions that ensures conditional variance positivity
- iii. Difficulty in measuring the persistence in stationary time series

The form of EGARCH (p, q) is given by:

$$\ln \delta_t^2 = \omega + \alpha_1 \left| \frac{x_{t-i}}{\sigma_{t-i}} \right| + \gamma_i \frac{x_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^q \beta_j \ln \sigma_{t-j}^2$$

Where  $\gamma$  is the asymmetric response parameter. In this paper we will use the simplest form of EGARCH which is the EGARCH (1, 1) model which is given by:

$$\ln \delta_t^2 = \omega + \alpha_1 \left| \frac{x_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{x_{t-1}}{\sigma_{t-1}} + \beta_1 \ln \sigma_{t-1}^2$$

### 3.1.4. GJR-GARCH (Glosten Jagannathan and Runkle GARCH) Model

[4] Proposed the GJR-GARCH model. The model takes into consideration that leverage effect based on the state of past innovation. Volatility equation of GJR-GARCH (p, q) is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i \frac{x_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \gamma_i I_{t-i} \frac{x_{t-i}}{\sigma_{t-i}}$$

Where  $\alpha, \beta$  and  $\gamma$  are constants, and  $I$  is a dummy variables. The parameters are assumed to be non-negative and that  $\frac{\alpha + \gamma + \beta}{2} < 1$

### 3.1.5. The Power GARCH Model (APARCH)

The APARCH (p, q) was introduced by [5]. In this model what is modelled is the standard deviation not the variance as in most GARCH models. The variance equation of APARCH (p, q) is given by:

$$\sigma_t^\delta = \alpha_0 + \sum_{i=1}^p \alpha_i (|a_{t-i}| + \gamma_i a_{t-i})^\delta + \sum_{j=1}^q \beta_j \sigma_{t-j}^\delta$$

Where  $\alpha_0 > 0, \sigma \geq 0, -1 \leq \gamma_i \leq 0, j = 1, 2, \dots, q, i = 1, 2, \dots, p, \beta_j, \gamma_i$  is the leverage parameter and  $\delta$  is the parameter for the power term.

### 3.2. Conditional Distributions

Since returns are not normal in distribution, the occurrence of the so-called fat-tails distributions warrants the use of alternative distributions such as the generalized error distribution, student t distribution and skewed student t-distribution. In this paper, the normal distribution, generalized error distribution and student t-distribution were implemented in the asymmetry GARCH models and the best error distribution selected based on the smallest BIC value.

#### 3.2.1. Normal Distribution

In the [6] seminal paper, the standard normal distribution is given by:

$$f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_t^2}{2}}$$

#### 3.2.2. Student-t Distribution

[7] Introduced the student t distribution whose form is:

$$f(z_t, t) = \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2}) \sqrt{\pi(v-2)}} \left(1 + \frac{z_t^2}{v-2}\right)^{-\frac{v+1}{2}}$$

Where  $\Gamma(\cdot)$  is the gamma function. Since student t distribution is not the sole distribution that accounts for fat tails in financial data, the generalized error distribution also does account for fat tails.

#### 3.2.3. Generalized Error Distribution

The Generalized error distribution also referred as the generalized Gaussian distribution described in the formula according to [8].

$$f(x) = \frac{\lambda \cdot s}{2 \cdot \Gamma\left(\frac{1}{s}\right)} \cdot \exp(-\lambda^s \cdot |x - \mu|^s)$$

Where  $\Gamma(z)$  the Euler Function,  $\lambda, s, \mu$  are the scale, shape and location parameters respectively. The suitable method for estimation of the generalized error distribution is the

maximum likelihood estimation, according to [8].

### 3.3. Extreme Value Theory

As opposed to the asymmetry GARCH models, the tail distribution of the returns is dealt with in the extreme value theory. Generally, two approaches have been proposed in modelling the extremes. One is the block maxima method proposed by [9]. In this method, it assumes that every maximum value in the blocks is an extreme value. The second method is the peak over threshold approach where an extreme value is defined as the observation exceeding a particular threshold. In this paper, we will follow the peak over threshold method.

In the peak over threshold method, extreme events are defined as the values that exceed a given threshold  $u$ . Let  $W$  be a random variable, then the distribution function of the excesses above a given threshold is given as:

$$F_u(y) = P[W - u \leq y | W > u]$$

Where  $y$  are the excess of  $W$  over threshold  $u$ . If  $W - u$  is greater than or equal to zero then the excess function according to [10] can be written as:

$$F_u(y) = \frac{F(y + u) - F(u)}{1 - F(u)}$$

For some underlying distribution  $F$  describing the entire time series.  $0 \leq y < w_F - u$  Where  $w_F - u$  is the right end point of  $F$ . We are mainly interested in estimating the extremes that is  $F_u$ . [10] and [11] showed that for a large class of underlying distribution function the conditional excess distribution function  $F_u(y)$  is well approximated, by the Generalized Pareto Distribution which describes the limit distribution of scaled excesses over high threshold

$$F_u(y) \approx G_{\xi, \beta}(y); u \rightarrow \infty$$

Where

$$G_{\xi, \beta}(y) = \begin{cases} 1 - (1 + \frac{\xi \beta}{y})^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1 - e^{-\frac{y}{\beta}}, \xi = 0 \end{cases}$$

For  $y \in [0, x_F - u]$  if  $\xi \geq 0$  and  $y \in [0, \frac{-\beta}{\xi}]$  if  $\xi < 0$ .

In peak over threshold method, the choice of a threshold is crucial. One of the methods of selecting a threshold is the graphical representation which makes use of the mean excess graph. In this paper, the mean excess function is implemented and the choice of the threshold,  $u$  is given by the value which the observed mean excess is approximately linear.

### 3.4. Estimation of Value-at-Risk

#### 3.4.1. Estimation under the Asymmetry GARCH

Here we discuss the estimation of Value-at-Risk and forecasting using asymmetry GARCH models discussed in preceding section. The calculation of the value at risk using the asymmetry GARCH models is obtained using the

equation:

$$VaR_t^p = \mu_t + q_d^r \sigma_t$$

Where  $\mu$  is the returns means,  $\sigma_t$  is the standard deviation and  $q_d^r$  represents quantile of the distribution at a level of confidence.

The forecast for one-day ahead variance for the EGARCH model is given by:

$$\ln \delta_{t+1}^2 = \omega + \alpha_i \left| \frac{x_{t-i+1}}{\sigma_{t-i+1}} \right| + \gamma_i \frac{x_{t-i+1}}{\sigma_{t-i+1}} + \beta_j \ln \sigma_{t-j+1}^2$$

The forecast for one-day ahead variance for the GJR-GARCH is:

$$\sigma_{t+1}^2 = \omega + \sum_{i=1}^p \alpha_i y_{t-i+1} + \sum_{j=1}^q \beta_j y_{t-j+1} + \gamma_i I_{t-i+1} y_{t-i+1}$$

Therefore to compute the one-step ahead VaR forecast under the assumed distribution, one computes the corresponding quantiles, which then are multiplied by the conditional standard deviation forecast hence:

$$VaR_{t+1}^p = \mu_t + q_d^r \sigma_{t+1}$$

#### 3.4.2. Estimation under Extreme Value Theory

The estimation of the Value-at-Risk follows the procedure of [12] which is implemented in this section. The two steps that were implemented in this paper are summarized as:

- i. Estimate a suitable GARCH-type process (in this study a GARCH (1, 1)) and extract its residuals.
- ii. Apply the Extreme Value Theory to the obtained residuals and use them to derive the VaR estimates.

Upon selection of an appropriate threshold, the residuals can then be modelled using the GPD and the VaR estimates calculated as:

$$VaR_q = u + \frac{\beta}{\xi} \left[ \left( \frac{n}{N_u} (1 - q)^{-\xi} \right) - 1 \right]$$

Where  $n$  is the observation totals and  $N_u$  are the observations above the threshold.

## 4. Data Description

To compare the robustness of the GARCH-EVT we chose the context of Kenyan Foreign Exchange Market as the base of analysis. The data employed was the 3172 daily observations of prices of the Kenyan Foreign Exchange Market covering the period from January 3rd, 2005 to August 4th, 2017. The in-sample estimation period sample size was 2672 observations and the rest 500 observations reserved for the out-of-sample period for backtesting purposes. The data excluded weekends and public holidays. The source of the data was the Central Bank of Kenya website. The currency exchange rates were transformed into daily log returns by applying:

$$R_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$

## 5. Empirical Result

Figure 1 shows the daily exchange rates raw prices for the Kenya Shilling versus US Dollar, Euro and Sterling Pound. As depicted in the figure, a decline is observed between 2005 and 2007. A sharp decline is observed in 2008 for all currencies because of the post-election violence that occurred then. Consistent instability is seen from 2008 to 2010 which is attributed to the violence that occurred after the elections

in 2007. Stability in the prices is observed from 2016 to July 2017 just before the general elections were held. It is worthwhile noting that the stability of the Sterling Pound in 2017 was greatly attributed to the Brexit campaigns that occurred in the United Kingdom. The daily logarithm returns in Figure 2 for each exchange currency, indicates periods of very high volatility and mostly volatility clustering. This illustrates the high volatility of developing country financial market.

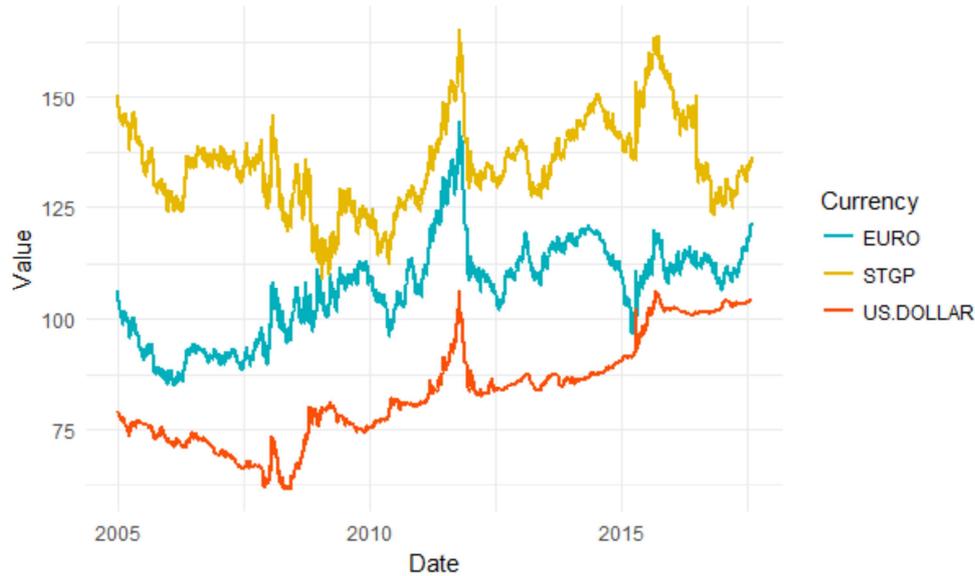


Figure 1. Time series plots for the daily exchange prices.

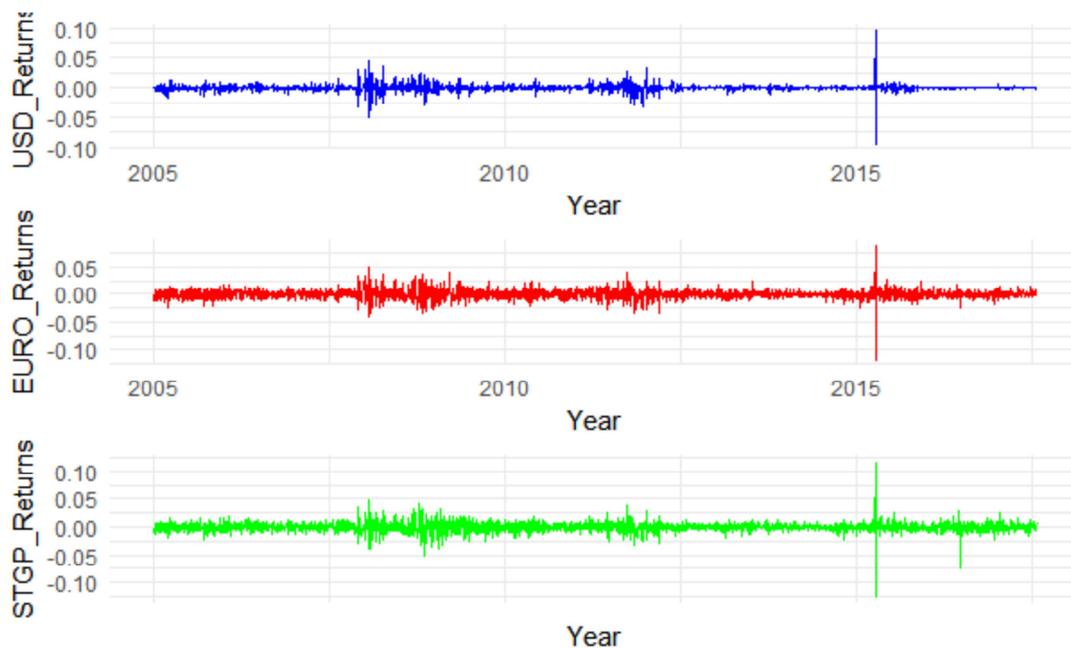


Figure 2. Time series plot for the daily returns for exchange rates.

As indicated in Table 1, an upward movement of the Foreign exchange market is depicted by the negative loss. In addition it is found out that Sterling Pound and Euro exchange rates experience more frequent negative shock, while the US Dollar exhibit more positive shock. The high

excess kurtosis observed across all exchange currencies confirm that fat tails in return distribution. The Jarque-Bera test for normality of the returns was performed. A p-value less than 0.05 gives a strong evidence of the returns being non-normal.

**Table 1.** Summary Statistics for Exchange Rate Returns.

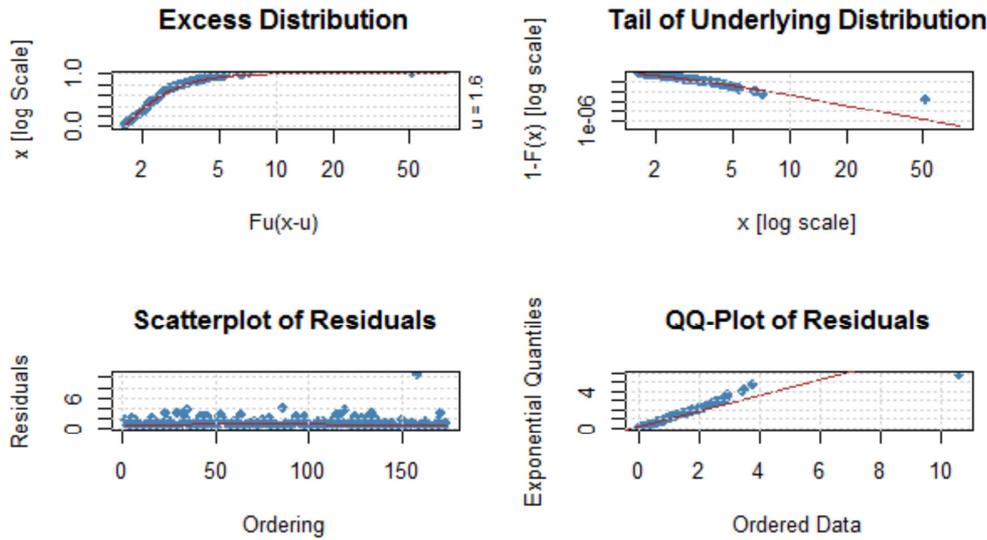
| Statistic   | US Dollar  | EURO       | Sterling Pound |
|-------------|------------|------------|----------------|
| Minimum     | -0.094419  | -0.117527  | -0.125491      |
| Mean        | 0.000087   | 0.000043   | -0.000031      |
| Maximum     | 0.095815   | 0.088406   | 0.116840       |
| Variance    | 0.000026   | 0.000062   | 0.000064       |
| Skewness    | 0.063995   | -0.415660  | -0.563409      |
| Kurtosis    | 87.728017  | 23.616311  | 38.177429      |
| Jarque-Bera | 1018511.55 | 73909.6092 | 193065.235     |
| JB p-value  | < 2.22e-16 | < 2.22e-16 | < 2.22e-16     |

**Table 2.** Optimal GARCH Model for the Exchange Rates.

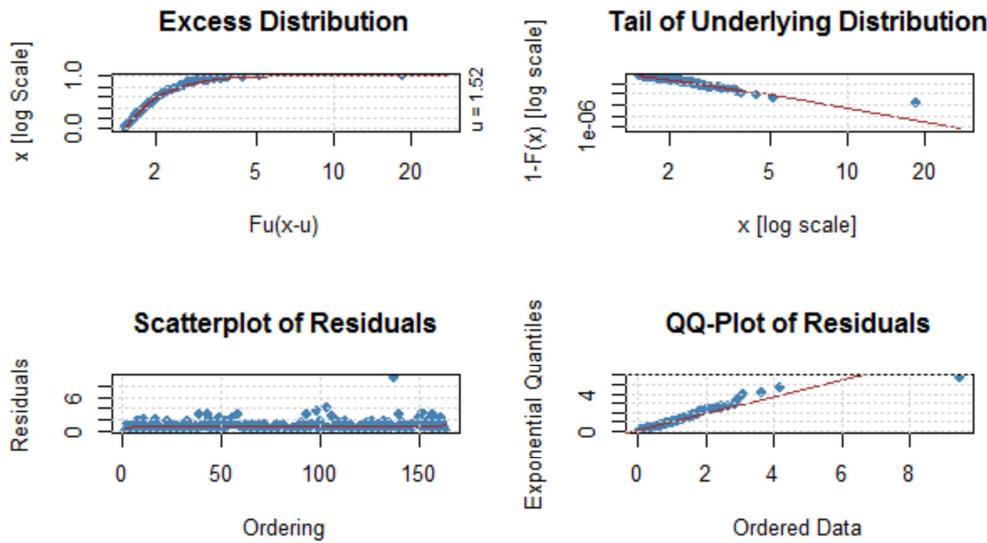
| Model         | USD/KSHS | STGP/KSHS | EURO/KSHS |
|---------------|----------|-----------|-----------|
| BIC           |          |           |           |
| EGARCH-normal | -8.8604  | -7.0372   | -7.0724   |
| EGARCH-t      | -9.2716  | -7.2579   | -7.2299   |
| EGARCH-ged    | -9.2751  | -7.2282   | -7.2093   |
| GJR-GARCH-n   | -8.8975  | -7.0894   | -7.1316   |
| GJR-GARCH-t   | -9.2629  | -7.2433   | -         |
| GJR-GARCH-ged | -6.1394  | -7.2228   | -7.2034   |
| APARCH-n      | -8.9181  | -7.0871   | -7.1220   |
| APARCH-t      | -9.3012  | -7.1120   | -         |
| APARCH-ged    | -6.3382  | -7.2265   | -         |

Table 2 presents optimal variance equation of the exchange rates. The optimal variance equation were chosen based on which error distribution had the minimum BIC value. Generally, the models portray a similar picture of volatility over time. The optimal GARCH model for the USD/KSHS is the APARCH model conditioned on the student-t distribution, for sterling pound and EURO exchange rate the optimal model is the EGARCH conditioned on the student t distribution.

In order to check if the excess distribution can be modelled by the GPD, Figures 3, 4 and 5 shows the empirical excess distribution along with the QQ-Plot of residuals. Across the exchange rates, the graphs manifest that the empirical excess distribution follows GPD implying that the exceedances can be modelled by Generalized Pareto Distribution.



**Figure 3.** Diagnostic Checks for KSHS/USD Exchange Rate.



**Figure 4.** Diagnostic Check for KSHS/STGP Exchange Rate.

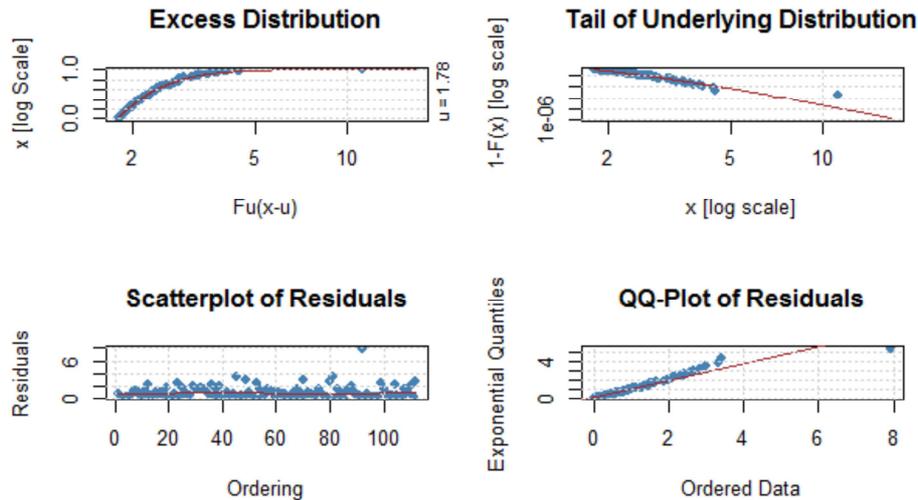


Figure 5. Diagnostic Checks for KSHS/EURO Exchange Rate.

Table 3, gives the estimated parameter and the respective thresholds for each exchange rate currency against the Kenya Shillings. The  $\xi$  represents the estimated parameters which determine the type of the distribution. Since it is positive for all currencies, these indicate that the distribution of the selected currencies belong to the Frechet distribution which is heavy-tailed, [11].

Table 3. Model Parameters for GPD.

| KSHS/ | Threshold ( $\tau$ ) | $\xi$     | $\beta$   | No. of Exceedances |
|-------|----------------------|-----------|-----------|--------------------|
| USD   | 1.60                 | 0.2636178 | 0.8764497 | 174                |
| STGP  | 1.52                 | 0.2215335 | 0.5245208 | 164                |
| EURO  | 1.78                 | 0.1530540 | 0.6037430 | 112                |

## 6. Dynamic Backtesting of the VaR Models

### 6.1. VaR Forecasting

The Value-at-Risk forecasts for the models and the forecasting results are presented in this part. Two types of backtesting criterion are implemented, conditional coverage test and the unconditional coverage test.

### 6.2. Unconditional and Conditional Coverage Test

Unconditional coverage test is based on the failure rates and was suggested by [13]. The test evaluates whether the given confidence level and the number of exceptions are consistent with each other. In order to implement this test the only information we require is the number of exceptions ( $y$ ), the confidence level ( $c$ ) and the number of observations ( $N$ ). The main aim is to check whether the observed failure rate is different from the failure rate given by the confidence level

The likelihood ratio statistic for the number of failures is given by:

$$LR_{uc} = -2\ln\left(\frac{(1-p)^{N-y}p^y}{\left[1-\frac{y}{N}\right]^{N-y}\left(\frac{y}{N}\right)^y}\right)$$

The correct model is the model that gives the right number of violations. A good model in finance is the one that gives an accurate number of violations over time and at the same time the violations being independent. In this regard, [14], developed the conditional coverage test which tests. This test checks if the number of violations is the same as the expected one, and also the independence of failures over time. The likelihood ratio statistic is given by:

$$LR_{cc} = -2 \ln \left( \frac{(1 - \pi)^{n_{00}+n_{01}} \pi^{n_{11}+n_{10}}}{(1 - \pi_0)^{n_{00}} \pi_0^{n_{01}} (1 - \pi_1)^{n_{10}} \pi_1^{n_{11}}} \right)$$

Where  $n_{ij}$  represents the number of observations with value  $i$  followed by  $j$  and  $n_{ij} = \frac{n_{ij}}{\sum n_{ij}}$  is the corresponding probability.

### 6.3. Backtesting Results

For all the exchange rates rolling window of 1000 is used to enable forecasting for the one day ahead VaR. In finance, the mostly commonly used level of confidence is 5%. Therefore all models are back tested at the 5% level of confidence. The violation results for all the competing models for each currency is presented in Table 4.

Table 4. Violation Test at 95% Confidence Level.

|                     | USD   | STGP  | EURO  |
|---------------------|-------|-------|-------|
| Expected Violations | 25    | 25    | 25    |
| APARCH-t            | 29(2) | -     | -     |
| EGARCH-t            | -     | 27(2) | 19(1) |
| sGARCH-EVT          | 32(1) | 29(1) | 22(2) |

The number in parentheses gives the rank of the models at 95% level

Results for the unconditional test for the competing models are presented in Table 5. The appropriate model is the one that fails to reject the null hypothesis, and this is evidenced by a higher p-value among the competing models. At 5%, for USD/KSHS, the standard GARCH-EVT has a higher p-value compared to the optimal APARCH model. For the

KSHS/EURO, the EGARCH model has a higher p-value compared to the standard GARCH-EVT, and finally, for KSHS/STGP the standard GARCH-EVT produced a higher p-value than the EGARCH model.

**Table 5.** Unconditional Coverage (UC) Test at 95% Confidence Level.

|            | KSHS/USD | KSHS/STGP | KSHS/EURO |
|------------|----------|-----------|-----------|
| EGARCH-t   | -        | 0.685     | 0.199     |
| APARCH-t   | 0.423    | -         | -         |
| sGARCH-EVT | 0.168    | 0.780     | 0.048     |

The conditional coverage test results are presented in Table 6. The number of violations being correctly identified and independent forms the null hypothesis. The model that does not reject the stipulated null is deemed the best model. The findings are that the EGARCH model with student t distribution is the best model for EURO and STGP while for US Dollar the model that stands out is the GARCH-EVT, because of the higher p-values. The outstanding conclusion from this test is that there is no strong evidence of complete dominance of the conditional EVT over the asymmetry GARCH models in modelling and forecasting the Value-at-Risk. In essence a higher p-value indicates a better performance in backtesting procedures. In accordance to that the Exponential GARCH model is the suitable model for STGP and EURO exchange rates and the conditional EVT is the best for USD exchange rate.

**Table 6.** Conditional Coverage (CC) Test at 95% Confidence Level.

|            | KSHS/USD | KSHS/STGP | KSHS/EURO |
|------------|----------|-----------|-----------|
| EGARCH-t   | -        | 0.842     | 0.417     |
| APARCH-t   | 0.703    | -         | -         |
| sGARCH-EVT | 0.809    | 0.802     | 0.319     |

## 7. Conclusion

To evaluate the Value-at-Risk estimates produced by asymmetry GARCH forecasts and GARCH-EVT was the main objective of this paper. The findings of the study are that the conditional Extreme Value Theory model deemed the best model in estimating the Value-at-Risk for the US Dollar exchange, while for the EURO and STGP exchange rate the EGARCH with t distribution is the best performing tool in estimating and forecasting Value-at-Risk. The backtesting result indicates that no sufficient evidence to pinpoint a complete superiority of the extreme value theory model over the asymmetry GARCH models.

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