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Model data fit comparison between DINA and G-DINA in cognitive diagnostic models

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Abstract: In this study, item and model data fit indices, calculated by DINA and G-DINA Models using the same sample and Q matrix, are analyzed. Fit indices for these two models from Cognitive Diagnostic Models are analyzed using 2LL, AIC and BIC statistics. Item fit indices are analyzed using residual correlations and probabilities. Analysis results showed G-DINA model had better fit results than DINA model. DINA model could give rather better results to estimate student profile in tests where higher level and progressive behaviors are used together. On the other hand, G-DINA model weights required attributes for an item when estimating student profile. Therefore in items requiring more than one attributes, contributions of attributes to probability that a student answers the item correctly are not equal. This provides an important advantage to testers to evaluate multiple choice items in assessing complex and prerequisite forming patterns.

Keywords: Cognitive Diagnostic Models, DINA Model, G-DINA Model, Model Fit Indices

1. Introduction

Cognitive Diagnostic Models (CDM) has gained ever increasing attention after "No Child Left Behind" Act of 2001 in USA. Main objective of this approach is to provide cognitive feedback about students to students, teachers, and families. Traditionally a test provides feedback either as total score or as sub-test level total scores. On the other hand, CDM can determine and provide feedback regarding each student's profile, which attributes student mastered and which attributes are non-mastered (Cheng, 2010). CDM Models present those results by calculating each item's relation with attributes assessed rather than total scores of the student taking the test.

Due to complexity of skill estimations regarding relations among items and attributes assessed in test and complexity of CDM structure, it is found out that some models remained at only theoretical stage and some other models are not practically of use. Therefore DINA among CDM models are most extensively studied model thanks to ease of application and interpretation. There are also several modified DINA models like G-DINA, HO-DINA, and NIDA. There is no definite restriction determining criteria as to which model should be used in which situation though there are plenty of studies in literature indicating in which situations those type modified models are used (DeCarlo, 2011; 2012; Embretson, 1998; Leighton &Gierl, 2007; de la Torre, Hong, & Deng, 2010). Therefore this study focuses on interpretations on data model fit of DINA model and one of its modified version G-DINA model.

1.1. DINA Model

DINA model is a latent class analysis developed by Haertel (1989). DINA model is closely related to Item Response Theory (IRT) (Haertel, E.H 1989). Nevertheless DINA model, differing from IRT models, does not assume continuous distribution of different magnitudes of skills of students. Instead, students are dichotomously assigned to small number of latent classes. DINA model classifies respondents into two dimensional classes for each attributes, that is, attributes student mastered are defined as a categorical variable rather than continues variable. First class is "Non-Mastery", namely class of respondents lacking prescribed trait, and the other is "Mastery", namely class of respondents possessing the prescribed trait. DINA model can be simply defined as follows: LetXij denotes response of respondent i to item j, and i= 1..…I and j= 1… J. Let us denote respondent's binary attributes vector as

$\alpha_i=\{\alpha_{ik}\}$, for k= 1,…,K when respondent's k^{th} entry is 1 it will denote k^{th}attribute possessed and when it is 0, not possessed (de la Torre, 2009a). Those denoted "attributes" here, may be defined as traits, competencies, task, sub-task, knowledge presentation, cognitive process or skill (Tatsuoka, 1995). CDM usually uses Q matrix coded as 1-0 and designed as JxK which was defined on the basis of response attributes in its calculations. (Embretson, 1984; Tatsuoka, 1985). In this matrix columns represent attributes and rows represent items. In this matrix q_{jk} entry denotes whether k^{th}attribute is required in correct answering j^{th} item. A Q matrix example is show in Table 1.

Table 1. *Q Matrix Example*

Items	α_1	α_2	α_3	α_4
1	1	0	0	0
2	0	1	0	0
3	1	0	1	0
4	0	1	1	1
5	1	0	1	1

Table 1 above indicates α1 attribute is required for answering item 1 correctly. Both α_1 and α_2 attributes are required for correct answering of item 3.

When attributes of DINA model respondents are determined, two additional parameters guess g and slip s parameters, are calculated for each item.

$s_j = P\left[Y_{ij} = 0|\eta_{ij} = 1\right]andg_j = P\left[Y_{ij} = 1|\eta_{ij} = 0\right]$,

s_j denotes an individual's probability of wrong answering item j who has the latent attributes (false positive probability) and g_j denotes individual's probability of correct answering who does not possess latent class attributes(correct positive probability). s_j parameter denotes slip and correct answering probability of individuals who had required attributes shall rise as parameter has lower values.

In DINA Model, function of correct answering probability of an individual who possesses all attributes is given by (de la Torre & Douglas, 2008):

$$P\left[Y_{ij} = 1\middle|\eta_{ij}, s_j, g_j\right] = (1 - s_j)^{\eta_{ij}} g_j^{1-\eta_{ij}}$$

Where P is probability that students possess all prescribed attributes to answer item correctly.$\eta_{ij}$is latent answer, specified by α and attribute for item i and a vector of q_j. Row of item j in Q matrix can be shown as:

$$\eta_{ij} = \prod_{k=1}^{K} \propto_{ik}^{q_{jk}}$$

DINA model assigns each students to a latent class which shows attributes student mastered and thus provides a cognitive profile of student according to attributes assessed by the test (de la Torre & Lee, 2010).

1.2. G-DINA Model

G-DINA model is a generalization of the DINA model. As many cognitive diagnostic models, this model is also based on JxK Q matrix. G-DINA model partitions latent classes into $2^{K_j^*}$ latent groups. Each latent group is reduced to an attribute vector represented by $\propto_{lj}^*$. Each latent group has probability of correct answering represented by $P(\propto_{lj}^*)$ (de la Torre, 2008a).

The original formulation of the G-DINA model based on $P(\propto_{lj}^*)$ can be decomposed into the sum of the effects due the presence of specific attributes and their interactions. Probability formula for G-DINA model is given by:

$$P(\alpha_{lj}^*) = \delta_{j0} + \sum_{k=1}^{K_j^*} \delta_{jk}\alpha_{lk} + \sum_{k'=k+1}^{K_j^*} \sum_{k=1}^{K_j^*-1} \delta_{jkk'}\alpha_{lk}\alpha_{lk'} \dots + \delta_{j12\dots K_j^*} \prod_{k=1}^{K_j^*} \alpha_{lk}$$

δ_{j0} = is the intercept for item j

δ_{jk} = is the main effect due to αk

$\delta_{jkk'}$= is the interaction effect due to αk and αk'

= is the interaction effect due to $\delta_{j12\dots K_j^*} = \alpha 1 \dots\dots \alpha_{K_j^*}$

Estimation codes of G-DINA are an implementation of EM algorithm. In analysis procedure, first $P(\propto_{lj}^*)$ values with standard errors are calculated, then posterior probabilities of skills are determined and latent classes of students and goodness of fit statistics for item and test are calculated according to those probabilities.

1.3. Outline

Purpose of this study is to determine DINA and G-DINA models and data relations and item data fit and provide guidelines for testers to select better model in the process of test development.

2. Method

2.1. Type of Study

Purpose of this study is to find out existing conditions to enable data fit comparison of those two models developed. Therefore this study may be seen as a descriptive research (Brown, 1999; Whitley, Kite, & Adams, 2012). Furthermore, this study can be viewed as theoretical research as well as it allows comparing model data fit between DINA and G-DINA models.

2.2. Work Group

In this study, real world data was used. For this study data pertaining to randomly selected 4677 examinees' answers from among 408 692 students taking 2008 OKS examination of grade 6 mathematics test was analyzed.

2.3. Assessment Tool

In this paper, Turkey 2008 OKS examination grade 6 mathematics test is used as assessment tool.Test is involving 16 questions. Guidelines set by Ministry of National Education Training and Education Council of Turkey for Grade 6 Mathematics Education program described 5 field of learning: "Numbers learning field", "Geometry learning field", "Measurement learning field", "Probability and Statistics learning field" and "Algebra learning field". Descriptive statistics for Assessment Tool is illustrated in Table 2

Table2. *Descriptive statistic for Sample*

N	4677
Mean	6.77
Standard Error	0.051
Median	6
Mod	6
Standard Deviation	3.48
Kurtosis	-0.40
Skewness	0.51
Range	16
Largest	0
Lowest	16
Reliability (Alpha)	0.76

2.4. Data Analysis

In this study, for data analysis, codes prepared for DINA and G-DINA model running under OX EDIT software were used. Q matrix defining the relation between items in assessment tool and attributes which was utilized in this study was prepared according to expert views.Based on 3 experts' opinion on primary school mathematics, a total of 16 items are associated to 4 attributes. Item and attributes relations according to expert views are shown in Table 3

Table 3.*Attributes Item Relation*

Attributes -Learning Fields	Items
Numbers	1,2,3,4,5,6,7,10,11,16
Geometry	4,5,8,9,10,12,13,14
Probability and Statistics	7,11,15
Algebra	3,7,8,11,16

A Q matrix based on Expert views is given in Table 4.

Table 4. *Q matrix for Assessment Tool*

Item	Numbers	Geometry	Probability	Algebra
1	1	0	0	0
2	1	0	0	0
3	1	0	0	1
4	1	1	0	0
5	1	1	0	0
6	1	0	0	0
7	1	0	1	1
8	0	1	0	1
9	0	1	0	0
10	1	1	0	0
11	1	0	1	1
12	0	1	0	0
13	0	1	0	0
14	0	1	0	0
15	0	0	1	0
16	1	0	0	1

Table 4 shows Experts associated 10, 8, 3 and 5 items with "Numbers learning field", "Geometry learning field"," Probability and Statistics learning field" and "Algebra learning field" respectively. Experts associated 8, 6 and 2 items with 1, 2 and 3 attributes respectively.

3. Findings

Analysis carried out by DINA model in this study, was completed in 20 iterations and g and s parameters related to items in assessment tool are obtained. DINA model parameters pertaining to assessment tool are provided in Table 5.

Table 5. *DINA model Parameters*

Item	Guess	Slip	1-s
1	0.10	0.56	0.44
2	0.33	0.38	0.62
3	0.31	0.03	0.98
4	0.17	0.45	0.56
5	0.58	0.02	0.99
6	0.18	0.52	0.48
7	0.29	0.51	0.49
8	0.15	0.56	0.45
9	0.26	0.15	0.86
10	0.37	0.20	0.81
11	0.19	0.66	0.34
12	0.17	0.78	0.22
13	0.17	0.47	0.54
14	0.36	0.20	0.81
15	0.28	0.02	0.99
16	0.24	0.24	0.76

Examining values for DINA model s and g parameters related to items reveals g value is varying between 0.10 and 0.58. Values for s parameter related to items vary between

0.02 and 0.78. Mean values for parameters are 0.26 and 0.35 for g and s respectively. Wenmin (2006) noted that lower s and g values were indication for a difficult test. Test is found to be more difficult than average when s and g parameters for items were considered.

De la Torre (2008,) De la Torre (2009) and Wenmin (2006) concluded that 1-s values closer to 0 would indicate a misrepresentation of attributes for items defined by Q matrix. In this respect, it means it is interpreted as an indicator for the rate of agreement between Q matrix and items pertaining to assessment tool. It is observed that l-s values pertaining to items varied between 0.22 and .99. Mean for l-s values was calculated as 0.64. In this respect, correct association of items with required attributes to answer correctly by Q matrix was indicated.

In this study, G-DINA model analysis calculated parameters pertaining to items and test in 80 iterations. Item parameters calculated for G-DINA are given in Table 6.

Table 6. *G-DINA Item Parameters*

1	**δ**	0	1						
	p	0.15	0.65						
2	**δ**	0	1						
	p	0.38	0.77						
3	**δ**	00	10	01	11				
	p	0.76	0.93	0.15	0.99				
4	**δ**	00	10	01	11				
	p	0.17	0.85	0.26	0.81				
5	**δ**	00	10	01	11				
	p	0.47	0.99	0.96	0.99				
6	**δ**	0	1						
	p	0.21	0.68						
7	**δ**	000	100	010	001	110	101	011	111
	p	0.27	0.99	0.25	0.34	0.01	0.99	0.23	0.76
8	**δ**	00	10	01	11				
	p	0.18	0.25	0.15	0.59				
9	**δ**	0	1						
	p	0.22	0.81						
10	**δ**	00	10	01	11				
	p	0.27	0.83	0.74	0.80				
11	**δ**	000	100	010	001	110	101	011	111
	p	0.26	0.99	0.17	0.18	0.13	0.27	0.26	0.51
12	**δ**	0	1						
	p	0.17	0.21						
13	**δ**	0	1						
	p	0.17	0.48						
14	**δ**	0	1						
	p	0.31	0.79						
15	**δ**	0	1						
	p	0.29	0.99						
16	**δ**	00	10	01	11				
	p	0.41	0.85	0.22	0.91				

Table 6 shows G-DINA parameter estimates are different than DINA's. G-DINA does not produce a single parameter per items as it calculates different probabilities for each attributes mastered. For example, 1st item is related to only one attribute and therefore different probabilities of possessing this attribute "1" and not possessing this attribute for this item were calculated individually. Similarly as 3rd item is associated with 2 attributes, students having none of attributes were assigned "00" and those who had only first attributes were assigned "10" and those who had only second attributes were assigned "01" and those who had both attributes were assigned accordingly and probabilities for each separately were estimated.

Fit indices of two models were compared using analysis results calculated for fit statistics and residuals. OX software produces a file containing item fit with analysis results for DINA model and G-DINA model. Item fit file contains residual values for item correlations and correct guessing probabilities and log-odds values. Residual values calculated for DINA and G-DINA models are shown in Table 7 and Table 8 respectively,

Table 7. *DINA Residuals*

	Items															
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	**0.01**	0.06	0.03	0.09	0.06	0.06	0.10	0.06	0.04	0.05	0.09	0.05	0.05	0.06	0.02	0.06
2	0.26	**0.01**	0.06	0.02	0.07	0.04	0.06	0.04	0.02	0.02	0.01	0.01	0.05	0.04	0.04	0.04
3	0.21	0.24	**0.02**	0.01	0.16	0.02	0.05	0.01	0.07	0.06	0.02	0.02	0.03	0.06	0.16	0.01
4	0.38	0.09	0.03	**0.00**	0.06	0.10	0.07	0.02	0.02	0.04	0.08	0.05	0.11	0.03	0.02	0.04
5	0.45	0.33	0.82	0.42	**0.00**	0.02	0.06	0.04	0.12	0.16	0.01	0.01	0.05	0.19	0.19	0.02
6	0.26	0.17	0.10	0.44	0.14	**0.01**	0.05	0.05	0.02	0.03	0.10	0.03	0.08	0.03	0.00	0.05
7	0.45	0.26	0.21	0.32	0.28	0.22	**0.00**	0.07	0.00	0.05	0.08	0.04	0.05	0.01	0.04	0.05
8	0.25	0.18	0.08	0.10	0.27	0.24	0.31	**0.01**	0.01	0.02	0.02	0.02	0.02	0.02	0.01	0.04
9	0.23	0.08	0.28	0.09	0.59	0.11	0.00	0.06	**0.00**	0.07	0.02	0.01	0.01	0.00	0.08	0.00
10	0.29	0.08	0.25	0.16	0.78	0.15	0.22	0.10	0.28	**0.00**	0.02	0.02	0.00	0.07	0.13	0.02
11	0.43	0.03	0.08	0.40	0.06	0.47	0.36	0.08	0.09	0.11	**0.01**	0.02	0.09	0.01	0.01	0.04
12	0.29	0.03	0.08	0.23	0.07	0.17	0.19	0.11	0.07	0.09	0.11	**0.00**	0.01	0.01	0.01	0.00
13	0.20	0.20	0.10	0.46	0.26	0.32	0.21	0.11	0.04	0.02	0.40	0.08	**0.00**	0.03	0.00	0.07
14	0.30	0.15	0.23	0.14	0.87	0.13	0.05	0.07	0.01	0.28	0.07	0.03	0.15	**0.00**	0.13	0.03
15	0.10	0.14	0.58	0.02	0.79	0.08	0.23	0.01	0.24	0.49	0.07	0.03	0.12	0.49	**0.08**	0.07
16	0.25	0.16	0.07	0.18	0.10	0.22	0.20	0.21	0.01	0.06	0.17	0.02	0.28	0.10	0.24	**0.01**

Diagonal=Rate of correct answers; Lower triangular: log-odds rate; Upper triangular: Correlations

Table 8.*G-DINA Residuals*

	Items															
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
1	**0.00**	0.03	0.03	0.02	0.02	0.01	0.05	0.04	0.06	0.02	0.05	0.05	0.11	0.01	0.02	0.02
2	0.12	**0.01**	0.01	0.03	0.04	0.00	0.03	0.02	0.04	0.01	0.01	0.00	0.08	0.02	0.04	0.02
3	0.15	0.06	**0.01**	0.01	0.06	0.01	0.01	0.02	0.02	0.01	0.00	0.01	0.02	0.01	0.06	0.02
4	0.07	0.13	0.07	**0.00**	0.07	0.04	0.02	0.01	0.07	0.01	0.03	0.07	0.14	0.06	0.01	0.00
5	0.14	0.21	0.33	0.47	**0.00**	0.01	0.01	0.02	0.02	0.06	0.03	0.01	0.01	0.10	0.00	0.01
6	0.05	0.01	0.05	0.15	0.03	**0.00**	0.03	0.03	0.04	0.02	0.05	0.04	0.14	0.02	0.01	0.01
7	0.25	0.13	0.06	0.10	0.03	0.11	**0.00**	0.02	0.06	0.00	0.01	0.04	0.10	0.03	0.01	0.05
8	0.19	0.10	0.09	0.04	0.12	0.14	0.08	**0.01**	0.07	0.01	0.00	0.03	0.08	0.00	0.00	0.05
9	0.29	0.18	0.08	0.33	0.12	0.19	0.26	0.31	**0.00**	0.00	0.05	0.03	0.03	0.02	0.03	0.07
10	0.09	0.03	0.02	0.04	0.31	0.09	0.01	0.08	0.01	**0.00**	0.04	0.02	0.00	0.00	0.02	0.01
11	0.23	0.04	0.01	0.11	0.15	0.24	0.05	0.00	0.25	0.20	**0.01**	0.01	0.10	0.04	0.01	0.01
12	0.27	0.02	0.04	0.35	0.03	0.19	0.23	0.19	0.15	0.10	0.03	**0.00**	0.04	0.02	0.00	0.01
13	0.51	0.34	0.11	0.60	0.07	0.62	0.41	0.34	0.12	0.02	0.45	0.22	**0.00**	0.04	0.04	0.13
14	0.07	0.07	0.02	0.26	0.47	0.07	0.13	0.02	0.08	0.00	0.17	0.09	0.20	**0.00**	0.02	0.01
15	0.15	0.19	0.27	0.09	0.01	0.07	0.04	0.03	0.16	0.09	0.04	0.01	0.21	0.08	**0.01**	0.01
16	0.11	0.07	0.08	0.00	0.07	0.05	0.20	0.23	0.28	0.04	0.02	0.06	0.53	0.05	0.03	**0.01**

Diagonal=rate of correct answers; Lower triangular: log-odds rates; Upper triangular: Correlations

Higher residuals indicate mis-fit for model (Henson,Roussos, Templin, 2004). An examination of two models reveals that residuals values pertaining to correct guessing rate represented in diagonals of Tables are higher for DINA model comparing to G-DINA model. In DINA model, these values vary between 0.00 and 0.08 whereas in G-DINA model they vary between 0.00 and 0.01. When residual means pertaining to correct guessing in two models were compared, they are calculated as 0.011 and 0.003 for DINA model and G-DINA model respectively. Table 9 shows indices calculated for residuals, test fit, item fit for DINA and G-DINA models.

Table 9. *Residual, Test and Items fit statistics for DINA and G-DINA models*

		DINA	G-DINA
Test Fit statistics	-2LL	86481.82	85225.0142
	AIC	86575.82	85337.0142
	BIC	86878.99	85698.2373
Item fit statistics	Rate	0.0114	0.0039
	Z(Correlation)	0.0461	0.0312
	Log	0.2109	0.1459
Mean of Residuals	Rate	0.0114	0.0039
	Log- Odds	1.69	1.17
	Correlation	0.37	0.25

-2LL, AIC and BIC indices for Model data fit do not show level of fit between single model and data. These indices rather provide information about data fit of two different models in comparison as to which model had better fit. In this respect, both changes applied to Q Matrix and changes made to model are suitable to be used in CDM applications from data fit point of view.

Table 9 illustrates that for all statistics calculated for DINA and G-DINA models, Model fit level of G-DINA model is higher. For model data fit statistics, calculated values of -2LL (log likelihood), AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) indicated that G-DINA model had better fit. AIC is essentially a powerful model selection criteria used in comparison of two different sized models (Bandolos, 1993; Akıncı, 2007). On the other hand, BIC is a criteria intended for selected model problems in regression.(Ucal, 2006). In both methods, the models with lowest fit coefficients are assumed as better fit model (Cavanaugh, 2009).

Item fit statistics (rate, Z, Log) are intended rather to determine the relation between items in test and data. Those indices should be taken into consideration at the stage of item selection in the test. Nevertheless, it can be told that the model having lower values in item fit statistics had better data fit. It leads to a similar conclusion when item fit indeces were examined. When rates, Z values pertaining to correlations and log values are considered, better fit is obtained in all for G-DINA comparing to DINA model on the basis of items.

Another method to compare model data fit is to analyze residuals. In analysis of residuals, lower values indicate better model data fitness. When statistics for residuals are considered, in correct guessing rates, log-odds value and residual values for correlations among items calculated for each item it is found that G-DINA model produced better fit indices than DINA model.

4. Conclusion

Results of this study showed that G-DINA model had better fit than DINA model for the data analyzed. G-DINA model is a modified form of DINA model. Therefore it may be expected to obtain better fit level for new model than the old one. On the other hand, an important point in studying CDM models is consistency in logical basis of relations prescribed between items and attributes. In this respect, it would not be a correct conclusion to assume each new model would give better fit than previous models. DINA model, by its construction, assumes only an individual mastered all attributes associated with an item had higher probability to answer correctly. In this respect, an individual not mastered only one of attributes would have the same probability to answer correctly as the one who mastered none of attributes. This shows a difficult to attain relation for multiple choice items. When this aspect is considered, DINA model could give rather better results to estimate student profile in tests where higher level and staged behaviors are used together.

On the other hand, G-DINA model weights required attributes for an item when estimating student profile. Therefore in items requiring more than one attributes, contributions of attributes to probability that a student gets the item correct are not equal. This describes a more convenient structure for multiple choice tests.

In developing tests and Question banks, appealing to CDM models furnishes testers with very detailed information regarding test and item attributes in determination of psychometric attributes of items and test.

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