Earth Sciences
2022; 11(3): 89-95
http://www.sciencepublishinggroup.com/j/earth
doi: 10.11648/j.earth.20221103.15
ISSN: 2328-5974 (Print); ISSN: 2328-5982 (Online)

Development of a Reservoir Simulator to Model Single-Phase Flow in Porous Media

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To cite this article:
Babilas Hountondji, François de Paule Codo, Paul Maurille Lanmandjèkpogni. Development of a Reservoir Simulator to Model Single-Phase Flow in Porous Media. *Earth Sciences*. Vol. 11, No. 3, 2022, pp. 89-95. doi: 10.11648/j.earth.20221103.15

Received: April 7, 2022; **Accepted**: April 28, 2022; **Published**: June 8, 2022

Abstract: Any groundwater reservoir requires tools to predict future performance as well as to optimize its operation. It is then necessary to simulate groundwater flow in porous media because of the uncertainty and heterogeneity associated with reservoirs. This study developed a reservoir simulator for modeling a single-phase flow in a porous medium. The development of the simulator consists of the physical and mathematical modeling of the reservoir. A MATLAB code was developed to describe groundwater flow in order to appreciate reservoir hydrodynamic pressure distributions from hydraulic head as a function of radial distance while varying the production flow. The formulation equation obtained was solved by the direct method. Examples of graphical plots generated from the simulator illustrate that before the coordinate point P (r=33.74m; h=286.65m) for any value of production flow, hydraulic head or hydrodynamic pressure of the reservoir increases equally with radial distance. This reflects the same drop in the static pressure of the reservoir. Beyond point P, there is a further increase in the hydraulic head, i.e., the hydrodynamic pressure of the reservoir as the production flow increases with the increase in population. This results in a drop in the static pressure of the reservoir in proportion to the increase in the production flow. The variations of the production flow carried out show that the static pressure of the reservoir decreases when the production flow increases. Finally, the simulator to predict the hydraulic head distributions i.e., the hydrodynamic pressure of the reservoir in single-phase flow during production periods is a springboard towards the implementation of multi-phase fluid flow formulations.

Keywords: Simulation, Groundwater, Porous Reservoir, Monzoungodo

1. Introduction

Reservoir simulation is a science that uses physics, mathematics, reservoir engineering and computer science to develop an engineering tool to predict the performance of reservoirs in operation [1-3].

Classical methods for reservoir performance prediction include analog experimental mathematical methods before the advent of reservoir simulators [4].

Mathematical methods use model equations to predict reservoir performance [5-7].

Reservoir simulation is a predictive engineering tool in industry used to obtain production well performance predictions for a groundwater reservoir under different operating conditions [8].

A groundwater abstraction project usually involves a huge investment, and the risk associated with production strategies must be assessed and minimized. These risks include such important factors as the complexity of the reservoir and the fluid that fills it, the complexity of groundwater abstraction mechanisms and the applicability of predictive methods. These complexities can be accounted for in reservoir simulation through input of data into the model, and this applicability can be estimated through good engineering practices and accurate reservoir simulation. It is therefore rightly that the present paper is interested in the development of a simulator to predict the distribution of the hydraulic head, i.e., the pressure in an underground water reservoir. The objectives of this paper are the following:

1) Physically model the flow of groundwater (single-phase fluid) in the reservoir,
2) Derive a mathematical model,

3) Develop Matlab programming codes to effectively solve the equation system,
4) Develop a simulator to predict the drop in the static pressure of the reservoir.

2. Methodologies

2.1. Development of the Flow Simulator in the Reservoir

The development of the groundwater reservoir flow simulator includes the following steps:

1) Formulate the partial differential equations (PDEs) of the model based on the characteristics of the groundwater reservoir, using the three fundamental laws governing the flow of fluids in porous media.
2) Write codes for the system of equations using the Matlab programming environment.

2.2. Mathematical Model

The characteristics of the reservoir are expressed from the partial differential equations (PDEs) as well as the initial and boundary conditions of the reservoir. There are essentially two main laws that govern reservoir simulation [9]:

1) The Law of Conservation of Mass,
2) Darcy's law (transport equation).

2.3. Description of the Mathematical Model (Basic Hypothesis)

The mathematical model was developed under the following assumptions:

1) The reservoir is homogeneous,
2) The fluid is isothermal, monophasic and incompressible,
3) Viscosity is effective at steady state,
4) Fluid flow is linear in the production well,
5) The fluid is Newtonian and its temperature is constant.

2.4. Basic Fluid Flow Equation

2.4.1. Law of Conservation of Mass

Considering the mass balance for a control volume, the mass conservation is given by [10]:

(Total mass entering during Δt) − (Total mass leaving during Δt) = (Net mass change during Δt)

Since the equation constituting the mathematical model of the reservoir is complex to be solved by an analytical method, it is likely to be solved numerically by a computer.

The general partial differential equation for a three-dimensional single-phase flow through a porous medium can be written in Cartesian coordinates by the equation. (1)

$$-\frac{\partial}{\partial t}(\rho_w.\phi) = \frac{\partial}{\partial x}(\rho_w.v_x) + \frac{\partial}{\partial y}(\rho_w.v_y) + \frac{\partial}{\partial z}(\rho_w.v_z) \quad (1)$$

where ϕ is the porosity of the medium, ρ_w the density of water and $\vec{v}$ Darcy's speed.

The fluid being incompressible, i.e ρ_w= cste we have:

$$\frac{\partial}{\partial t}(\phi) + div(\vec{v}) = 0 \quad (2)$$

The porosity being a dimensionless quantity therefore it is constant and we have:

$$\frac{\partial}{\partial t}(\phi) = 0 \quad (3)$$

Thus equation (2) becomes:

$$div(\vec{v}) = 0 \quad (4)$$

2.4.2. Darcy's Law

Darcy's law comes from the experiment of Henri-Darcy [11]. It expresses that the rate of filtration $\vec{v}$ is proportional to the pressure gradient p. This law is only valid at the macroscopic scale and its most common expression in hydrogeology is in the form:

$$\vec{v} = -\frac{k}{\mu}\left(\vec{\nabla}p + \rho_w\, g\vec{\nabla}z_h\right) \quad (5)$$

For an incompressible fluid, this law can also be expressed as a function of the piezometric head by the relationship:

$$\vec{v} = -K\,\vec{\nabla}h \quad (6)$$

with $K = \frac{k\rho_w g}{\mu}$ with $h = z_h + \frac{p}{\rho_w\, g}$

μ: dynamic viscosity of water,
k: intrinsic permeability of the medium,
z_h: dimension defined along the vertical axis,
ρ_w: density of water,
g: gravity acceleration,
h: hydraulic head.

Darcy's law [11] in circular radial flow is expressed by:

$$v = -\frac{\rho g}{\mu}k.\frac{\partial h}{\partial r} \quad (7)$$

With r the radial distance.

2.4.3. General Diffusivity Equation

The continuity equation and that of Darcy's law previously established make it possible to represent the incompressible flows for an idealized porous medium. Their combination leads to the following diffusivity equation:

$$div\left(-K\vec{\nabla}h\right) = 0 \quad (8)$$

Since the flow is radial, then we work along the r-axis and the diffusivity equation is written:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = 0 \quad (9)$$

2.4.4. Initial Condition

Before commissioning (t=0), the head is uniform throughout the reservoir. So, the initial condition is written as:

$$h(x,y,z,o) = h_R\ \forall\ (x,y,z) \quad (10)$$

2.4.5. Boundary Condition of the Study Domain

The outer limit (Σ) of the drainage zone, corresponds to the lateral surface of a cylinder of porous medium of radius R with respect to the axis of the well and corresponding to zero drawdown. Thus, on this face the load is maintained equal to h_R.

At the inner limit ($\sum^1$) the drainage zone, consisting of the lateral surface of the well exploiting the reservoir with a flow rate Q, we have the load on this surface ($\sum^1$) which is noted h_w.

The lower and upper limits ($\sum'$) of the drainage zone formed by the wall rocks of the layer, which are impermeable because, we are dealing with a captive tablecloth. On these surfaces ($\sum'$) so we will have:

$$\frac{\partial h}{\partial n} = 0 \quad (11)$$

2.4.6. Differential Equation of the Flow in the Reservoir

The system of equations to be solved is then the following:

$$\begin{cases} \frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = 0 \\ h = h_R \ on\ \Sigma \\ h = h_w \ on\ \Sigma^1 \\ \frac{\partial h}{\partial n} = 0 \ on\ \Sigma' \end{cases} \quad (12)$$

With:

h_R denoting the hydraulic head at the limit ($\sum$),

h_W designating the hydraulic head at the well ($\sum^1$).

The hydraulic head at the production well h_W and the static pressure p_{st} in the reservoir are [12]

$$h_w = \frac{p_{st}}{\rho_w g} - \frac{Q}{e\,K} \quad (13)$$

$$p_{st} = p_2 + \rho_w g H + \frac{\mu Q}{e\,k} + 0{,}06642\rho_w \frac{H}{D^{4,8}}\ Q^{1,8} v^{0,2} \quad (14)$$

With:

p_2: wellhead pressure,

Q: production flow,

H: the depth of the well,

D: the diameter of the well,

μ: the dynamic viscosity of the fluid,

k: intrinsic permeability,

e: the thickness of the producing layer.

2.4.7. Solving the Differential Flow Equation

The differential equation to be solved is the following:

$$\frac{d^2 h}{dr^2} + \frac{1}{r}\frac{dh}{dr} = 0 \quad (15)$$

That is:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dh}{dr}\right) = 0 \quad (16)$$

After integrations and making use of the initial and boundary conditions of the reservoir, we obtain

$$h = \frac{Q}{2\pi K e}\left[ln\left(\frac{r}{a}\right) - 2\pi\right] + \frac{p_{st}}{\rho_w g} \quad (17)$$

with a: the radius of the production well.

2.5. The Data Needed to Model the Porous Reservoir of Monzoungoudo

2.5.1. Reservoir Hydrodynamic Parameters

Based on test pumping carried out on the Monzoungoudo aquifer by the DGEau from Benin, we obtained some values of hydrodynamic parameters. The confined aquifer is quite transmissive and permeable (permeability of the order of $2.28.10^{-4}$ m/s). The confined aquifer is rich in sand with a porosity of 30% [13].

2.5.2. Characteristics of the Monzoungoudo Aquifer

Based on test pumping carried out on the Monzoungoudo aquifer by the DGEau from Benin, we obtained some values of the characteristics of the aquifer. The fluid which is water has a density $\rho_w = 1000\ kg/m^3$, dynamic viscosity $\mu = 10^{-3} Pa.s$ and a kinematic viscosity $\vartheta = 10^{-6}\ m/s$, [13].

2.5.3. Geometry and Hydrodynamics of the Artesian Well of Monzoungoudo

The data concerning the geometry and hydrodynamics of the borehole are grouped together in Table 1 and mainly concern the technical data of the hydraulic borehole carried out in the village of Monzoungoudo and which are provided by the General Directorate of Water of Benin (DGEau).

Table 1. *Geometric characteristics and hydrodynamic parameters of the artesian spouting well of Monzoungoudo [12].*

Geometric characteristics and hydrodynamic parameters from the artesian well of Monzoungoudo	
Settings	Values
Well depth H (m)	244.18
Well diameter D (m)	0.126
Flow Q (cm^3 /s)	2000
Gravity acceleration g (m/s^2 or N/kg)	9.81
Wellhead pressure p_2 (bars)	4.16
Absolute pipe roughness ε (mm)	0.12

2.5.4. Lithological Section of the Artesian Well of Monzoungoudo

The artesian well gushing from Monzoungoudo crosses all the geological formations in the study area. It is a rotary drilling, which produced cuttings. The interpretation of the cuttings made it possible to recognize the geology of Mouzoungoudo and to carry out the log and the litho-stratigraphic section. The lithological section reveals a surface sedimentary level between 0 and 1m thick, formed of topsoil. Then, between 1 and 3 m there is a formation of lateritic clay. Between 3 and 15 m yellow clay spreads out. Between 15 and 57 m there is plastic clay. Between 57 and 75 m we find shell limestone.

Between 75 and 135 m we find clayey limestone. Between 135 and 184 m we find more or less calcareous pyritic clay. Between 184 and 201 m we find an alternation of clay and limestone. Between 201 and 214 m we find sandy clay. Between 214 and 244.18 m we find fine and medium-grained white quartz sand. Figure 1 shows the cross section of the Monzoungoudo well.

This section confirmed that the Monzoungoudo aquifer is made up of quartz sand and that this aquifer is captive because the roof is made up of a layer of clay that is therefore impermeable. We also note that the static level of the well is +0.95 m, this positive value shows that the Monzoungoudo well is artesian flowing. Similarly, this section allowed us to determine the average thickness of the Monzoungoudo reservoir. The roof and the wall of the sheet being respectively located at 201 m and 244.18 m depth, groundwater inflows into the well take place in the quartz sand between 214 and 244.18 m depth (area in which the strainers are located), we then deduce that the thickness of the reservoir is e = 43.18 m.

3. Results and Discussion

Matlab is a high-level computer language for scientific computing and data visualization built around an interactive programming environment [14]. The codes developed in this article were programmed in the environment Matlab [15].

3.1. Formulation Calculation Method

The Matlab code was written for Eq. (17). An interface that accepts user input data was designed with Matlab for the simulator in this study. The input data required from the user are the reservoir and production well parameters:

1. Reservoir description data such as overall geometry, permeability and porosity,
2. Fluid properties, such as viscosity,
3. Specification of production well location and production rate. Figure 2 shows the interface that was created for the simulator.

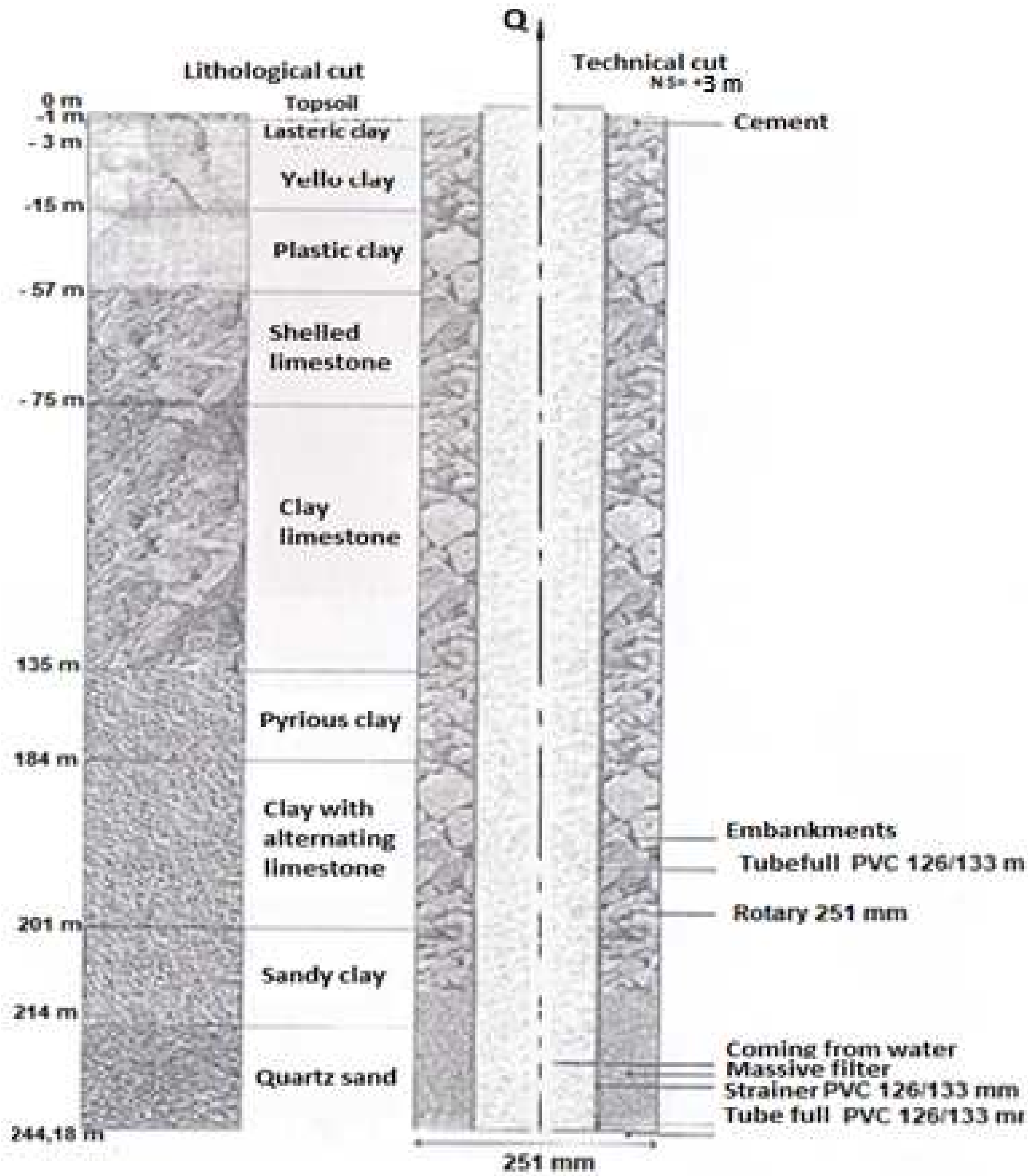


Figure 1. *Lithological section of the artesian well of Monzoungoudo [12].*

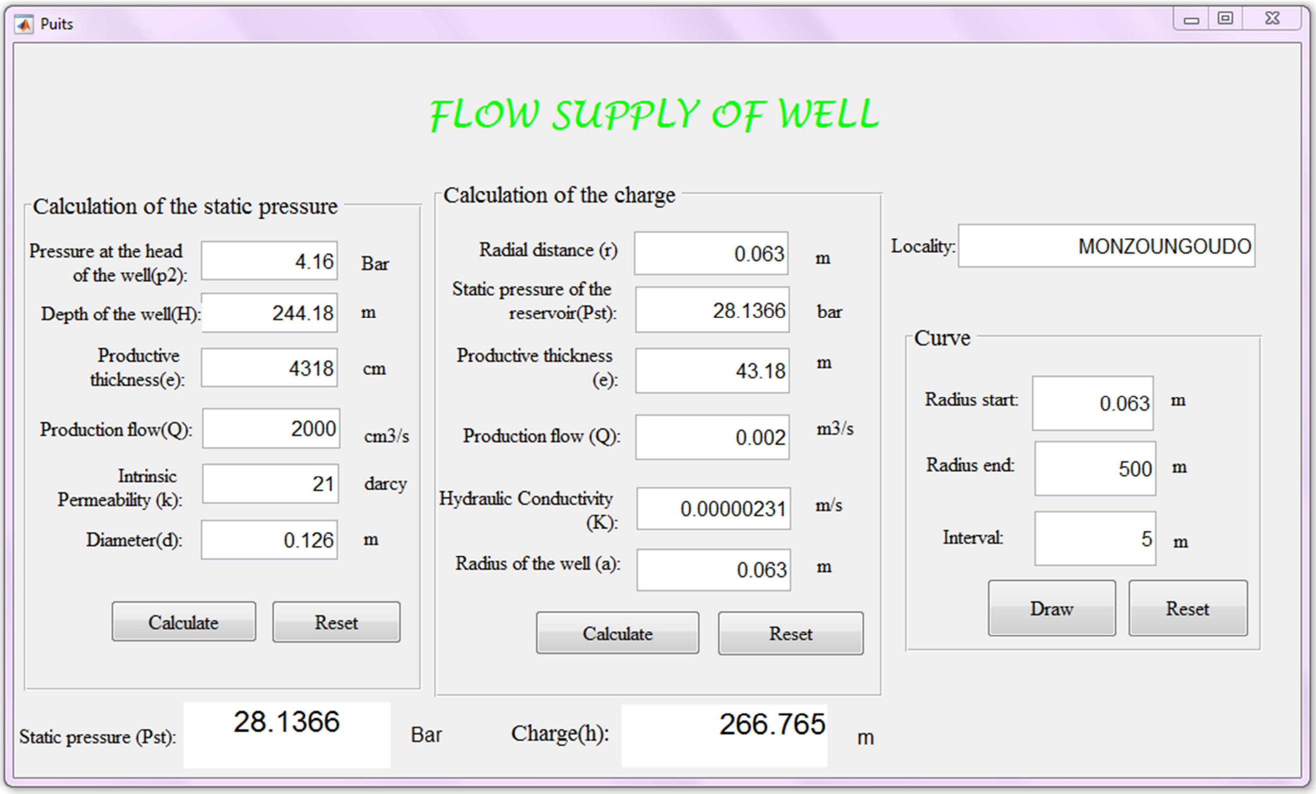


Figure 2. *Interface Matlab for the simulator.*

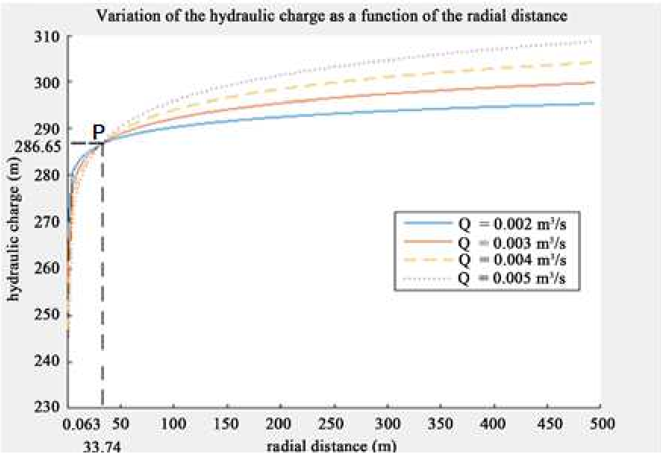
3.2. Simulation Results

Table 2 and the graphs in Figure 3 shows the results generated at each radial distance step after running the simulator. Using Eq. (17), the results obtained for Q = 0,002 m^3/s are recorded in the Table 2.

Figure 3 show the shape of the curves of the hydraulic head in the zone of influence of the well $(a \leq r \leq R)$ for different values of production flow (0.002; 0.003; 0.004; 0.005; 0.006; 0.007; 0.008; 0.009; 0.01m^3/s).

Table 2. *Calculation results of the hydraulic head according to the radial distance [12].*

CALCULATION RESULTS OF THE HYDRAULIC LOAD h(r) AS A FUNCTION OF THE RADIAL DISTANCE (r) FOR ($Q = 0,002\ m^3/s$)											
r (m)	**0.063**	**5**	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**
h(m)	266.89	280.79	282.99	284.28	285.20	285.91	286.49	286.98	287.40	287.78	288.11
r (m	55	60	65	70	75	80	85	90	95	100	105
h(m)	288.41	288.69	288.94	289.18	289.4	289.6	289.80	289.98	290.15	290.32	290.47
r (m	110	115	120	125	130	135	140	145	150	155	160
h(m)	290.62	290.76	290.89	291.02	291.15	291.27	291.38	291.50	291.60	291.71	291.81
r (m	165	170	175	180	185	190	195	200	205	210	215
h(m)	291.91	292	292.09	292.27	292.27	292.35	292.44	292.52	292.60	292.67	292.75
r (m	220	225	230	235	240	245	250	255	260	265	270
h(m)	292.82	292.89	292.96	293.03	293.1	293.16	293.23	293.29	293.35	293.41	293.47
r (m	275	280	285	290	295	300	305	310	315	320	325
h(m)	293.53	293.59	293.64	293.70	293.75	293.81	293.86	293.91	293.96	294.01	294.06
r (m	330	335	340	345	350	355	360	365	370	375	380
h(m)	294.11	294.16	294.20	294.25	294.30	294.34	294.39	294.43	294.47	294.52	294.56
r (m)	385	390	395	400	405	410	415	420	425	430	435
h(m)	294.6	294.64	294.68	294.72	294.76	294.8	294.84	294.88	294.92	294.95	294.99
r (m)	440	445	450	455	460	465	470	475	480	485	490
h(m)	295.02	295.06	295.10	295.13	295.17	295.20	295.20	295.27	295.30	295.33	295.37
r (m)	495	500									
h(m)	295.4	295.43									



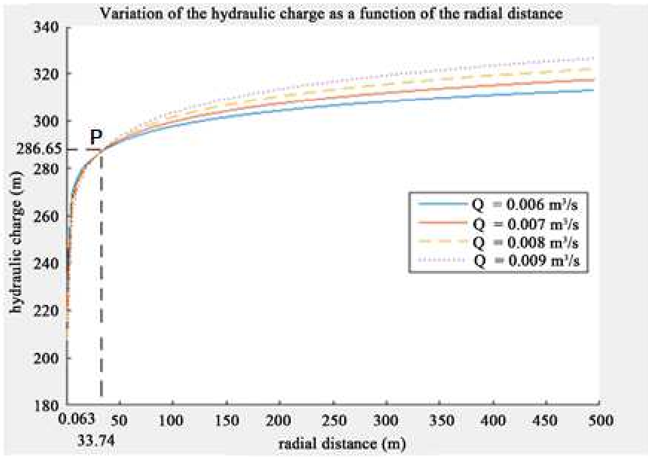


Figure 3. *Hydraulic load depending on the Radial distance [12].*

3.3. Discussion of Results

The objective is to design a tool which makes it possible to estimate the drops in hydrodynamic pressure at the level of the production well in order to appreciate the behavior of the groundwater reservoir as a whole. The hydrodynamic pressure drop is determined in the drainage area of the well. A simulation was made to observe the effect of the production flow on the static pressure of the reservoir through the hydraulic head, i.e., the hydrodynamic pressure, as illustrated by Figure 3. The analysis carried out to study the effect of the production rate on the static pressure of the reservoir shows that the latter decreases as the production rate decreases by 0.009 m^3/s to 0.002 m^3/s.

4. Conclusion

Matlab code is a specific tool in digital reservoir simulators. The mass balance method was used to validate the developed reservoir simulator.

In addition, the 1D numerical simulator developed to predict single-phase pressure distributions in a reservoir during production is a stepping stone to the implementation of two-or three-phase multiphase flow.

Numerical methods such as finite element method, finite difference, integral volume, finite volume method and variational method could be used to discretize the partial differential equation governing the fluid flow process and compared to the method used in this study. This will accommodate regular and irregular reservoir geometry.

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