Control Science and Engineering
2017; 1(1): 1-3
http://www.sciencepublishinggroup.com/j/cse
doi: 10.11648/j.cse.20170101.11

On One Class of Undirected Graph

Kochkarev Bagram, Sibgatullovich

Department of Mathematics and Mathematical Modelling, Institute of Mathematics and Mechanics Named After N. I. Lobachevsky, Kazan (Volga Region) Federal University, Kazan, Russia

Email address:
bkochkarev@rambler.ru (K. Bagram)

To cite this article:
Kochkarev Bagram Sibgatullovich. On One Class of Undirected Graph. *Control Science and Engineering*. Vol. 1, No. 1, 2017, pp. 1-3. doi: 10.11648/j.cse.20170101.11

Received: April 1, 2017; **Accepted**: April 18, 2017; **Published**: May 31, 2017

Abstract: Introduced the concept of polynomial combinatorial sets in enumerative combinatorics and formulates the problem of finding some element with an easily recognized symptom among elements of a combinatorial set. We build an efficient algorithm to solve this problem. We prove that this algorithm does not fit into the formal definition of an algorithm (e.g. “Turing machine”). It is proved that all NP-complete problems are not polynomial. We consider a countable class of undirected Hamiltonian graphs with an odd number of vertices without loops and multiple edges. We prove one typical feature of such graphs: almost every simple path containing all the vertices of the graph is not Hamiltonian cycle. In other words, in the langage of probability theory, the probability that a randomly selected a simple path in this graph containing all vertices, is Hamiltonian cycle tends to zero with growth of number vertices.

Keywords: An Undirected Graph, A Simple Path, Hamiltonian Cycle, Polynomial Combinatorial Set, Non-polynomial Combinatorial Set

1. Introduction

In The Clay Mathematical Institute named seven outstanding mathematical problems –“Millennium Prize Problems” for each of which will be paid 1 million USD. To be considered decision published in the well-known mathematical journal, and not earlier than 2 years after publication. It should be noted that one of the named issues, namely, “the Problem of Poincare” was recognized by the Mathematical Institute resolved, although the stated conditions of publication had not been implemented (the publication was only in the archive).

Other from called the Institute of problems is the so called “Cook’s Problem”, which the author of this work has investigated for nearly 50 years, but in a different wording (see the work of the author [1]) and decided back in 2008 and reported at the International Conference “Malmeet” November 2008, Novosibirsk [2]. Then there was the reports on this issue in March 2009 International Conference Science in the Univesities Moscow [3] and published in the archive [4-6], report at the International Conference “Computing Claster” CC2013 (Ukraine, Lviv, June 3-5 2013) [7], the publications [8-9] in the Russian langage and, finally, publications in International journals [10-11].

The impression is that this attitude of the Clay Institute only to scholars who are not citizens of the United States. For example, in the Internet strenuously advocated one work of A. Wiles [12], published in 1995 in the Annals of mathematics, dedicated to the well-known in number theory the problem of P. Fermat and for which in the past 2016, he received the Abel prize. Unfortunately, the title of this work of A. Wiles does not match the content, since an elliptic curve Frey does not exist in nature [13-15], and therefore Fermat’s Last theorem, for alleged proof where he got 21 years after the publication of his wrong work, was not proved. One can only wonder how the 6 independent experts are unable to give a correct estimate of the work and allowed her to publish in such a prestigious journal like the Annals of mathematics. Meanwhile by us published [13-15] work that finally put a point on the issue of Fermat and moreover provide a proof of a proposition is a refinement of the original statement of Fermat: Diophantine equation of Fermat $u^n + v^n + w^n, n > 2, uvw \neq 0$, has no solutions in the field of rational numbers.

Since the work of [5] was submitted to the archive, we decided in paragraph 2 to give just one fragment of this work.

2. Polynomial Combinatorial Set

In enumerative combinatorics [16] is usually given an infinite class of finite sets S_i, where i runs over some set of indices I (such as the set of non-negative integers N), and we want to count combinatorial number $|S_i|$ of elements in each S_i "simultaneously". Set S_i we call combinatorial sets. However, for our purposes we need to know a definite answer to the question "S_i polynomial or not from i?" In the case of an affirmative answer to the question it is enough to prove that $|S_i| < O(i^k)$, where k is some constant. In enumerative combinatorics, the combinatorial elements of the set S_i have a simple combinatorial definition and some additional structure. Revealed that the set S_i contains many elements and the main question is to determine (estimate their combinatorial number), not search, for example, some special item. The problem that we're going to outline in this work is, namely, the construction of an efficient algorithm to find the element among the elements of some combinatorial set.

Definition 1. Combinatorial set S_i is called polynomial if $|S_i| < O(i^k)$, where k is some constant, otherwise it is called non-polynomial.

Problem statement: Let S_i be combinatorial set and x some element of S_i easy to check (in polynomial time the length of the input $|x|$) characteristic (property) α. It is necessary to build an algorithm that for $\geq |S_i|$ the number of steps find the element x. We propose the following algorithm for solving the problem: we assume that a finite set S_i of combinatorial objects is in some capacity Ω. Of capacity Ω we derive successively without returning all objects to the last inclusive. For the latter object we check that the properties of α. If the last fetched object has the property α, then problem solved. If this object does not have property α, then all Ω extracted from the objects we returned in Ω renewable and the extraction process of Ω objects, but with each subsequent extraction of the object to check whether the extracted object property α. Obviously, for some $|S_i| + m, 1 \leq m \leq |S_i|$ step, the object will have the property α and the problem will be solved. Of the constructed algorithm follows the theorem.

Theorem 1. If a combinatorial set S_i is polynomial, then the formulated problem is solved in polynomial number of steps, namely, the number of steps t of the constructed algorithm satisfies the inequality $t < O(|S_i|)$. If a combinatorial set S_i is not polynomial, then the problem is not polynomial.

Obviously, if $|S_i| \geq k^i, k \geq 2$, then combinatorial set S_i is not polynomial.

The algorithm, which implies the above theorem clearly satisfies all the requirements of the intuitive notion of algorithm, but does not fit into the formal definition of an algorithm (e.g., "Turing mashine"), because x is sought, generally speaking, for an indefinite number of steps t that satisfies the inequality $t \geq |S_i|$, and a Turing mashine on any input gives a valid response within a certain number of steps. Therefore, the algorithm cancels famous Turing's thesis: "any algorithm we can construct a Turing mashine that is equivalent to his algorithm". Because of the equivalence of the Turing mashine other well-known formal concepts of algorithm (recursive functions, normal algorithm) relevant abstracts are also voided.

Corollary 1. If a combinatorial set S_i is not polynomial, the algorithm is non-polynomial.

From [5] we know that in complexity class NP there exist combinatorial set S_i that are not polynomial. Hence the result.

Corollary 2. NPC-problems are not polynomial.

Proof. We can build one non-polynomial combinatorial set S_i. Let $S = \{1, 2, ..., n\}$ be the set of natural numbers, where $n \geq 5$ is odd number. Let j be a number of S. We form the family of all subsets $F \subset S$ such that $j \notin F$ and $|F| = \frac{n-1}{2}$. Next, we'll add to some subset F number j and denote this subset $\widehat{F} = F \cup \{j\}$ using. As a combinatorial set S_i, consider the family formed subsets of F, including a subset of $\widehat{F}$. We consider the following problem: it is necessary from an urn containing cards with the recorded subsets of F and $\widehat{F}$ to get a card with a $\widehat{F}$ after the $\begin{pmatrix} n-1 \\ \frac{n-1}{2} \end{pmatrix} - 1$ in the number of retrievals. If $\begin{pmatrix} n-1 \\ \frac{n-1}{2} \end{pmatrix}$ step card with a $\widehat{F}$ will not be all removed cards are returned to the urn and the extraction processes that use renewable cards until a card with a $\widehat{F}$. Obviously, since $\begin{pmatrix} n-1 \\ \frac{n-1}{2} \end{pmatrix}$ has exponent [17], our problem can be solved in exponential number of steps. Thus, the formulated problem belongs to the NPC and, therefore, is not polynomial. To verify the solution you need to count the number of items on the map that is extracted from the urn. If this number equals $\frac{n-1}{2} + 1$, then the problem is solved correctly.

In [20], it was found that the problem of recognition of a Hamiltonian graph is NP-complete problem. In [10] we proved that this problem is not polynomial and thus the question of the complexity of NP-complete problems were finally solved and the hypothesis of Jacques Edmonds $P \neq NP$ was proved. In the present paragraph we have repeated the publication of one piece of work [5], which also proves the above hypothesis.

3. Class Hamiltonian Graphs with Odd Number of Vertices

Let $i \geq 5$ is an odd natural number and $G = (V = V_1 \cup V_2, E)$ is the class of bipartite graph such that $|V_1| = \left\lfloor \frac{i}{2} \right\rfloor, |V_2| = \left\lceil \frac{i}{2} \right\rceil, |E| = \left\lfloor \frac{i}{2} \right\rfloor \cdot \left\lceil \frac{i}{2} \right\rceil$. Thus in graph G every vertex of V_1 is connected by an edge to each vertex of V_2. It is known [18, 846] that the graph G is not Hamiltonian. Further, let $G' = (V, E')$ is the graph obtained from graph G by adding one edge u, connecting the two vertices $v \in V_2, v' \in V_2$. As combinatorial set S_i we consider the set of all simple paths from vertex v in the graph G which contain every vertex in V. Obviously, all simple paths from vertex v to vertex v' in graph G that contain each vertex in V adding edges u in graph G' is converted into Hamiltonian cycles. [10] reported that the number of Hamiltonian cycles in the graph G' is equal to $\left(\left\lfloor \frac{i}{2} \right\rfloor!\right)^2$. Evidently, the combinatorial number $|S_i|$ for the graph G and G' is the same and $|S_i| = \left\lceil \frac{i}{2} \right\rceil \left(\left\lfloor \frac{i}{2} \right\rfloor!\right)^2$.

Theorem 2. The probability that a randomly selected simple path in a graph G', containing all vertices will be Hamiltonian cycle tends to zero with increasing number of vertices i.

Proof. In fact, from the above the probability of this happening is

$$\frac{\left(\left\lfloor \frac{i}{2} \right\rfloor!\right)^2}{\left\lceil \frac{i}{2} \right\rceil \cdot \left(\left\lfloor \frac{i}{2} \right\rfloor!\right)^2} = \frac{1}{\left\lceil \frac{i}{2} \right\rceil}$$

4. Conclusion

Although the hypothesis of Jacques Edmonds $P \neq NP$, was confirmed and proved in the works [1-5], [7-10], but this result is still The Clay Mathematics Institute not recognized and therefore probably in [19] this issue is considered unresolved.

References

[1] Kochkarev B. S. Gipoteza J. Edmondsa I problema S. A. Kuka //Vestnik TGGPU, 2011 2 (24) s. 23-24 (in Russian).

[2] Kochkarev B. S. On Cook's problem. http://www.math.nsc.ru/conference/.

[3] Kochkarev B. S. Prilogenie monotonnykh funktsii algebry logiki k probleme Kuka, Nauka v Vuzakh: matematika, fizika, informatika. Tezisy dokladov Mejdunarodnoj nauchno-obrazovatelnoj konferentsii, 2009, pp274-275 (in Russian).

[4] Kochkarev B. S. About one class polynomial problems with not polynomial certificates //arXiv: 1210. 7591v1 [math. CO] 29 Oct. 2012.

[5] Kochkarev B. S. Proof of the hypothesis Edmonds's, not polynomial of NPC problems and classification of the problems with polynomial certificates. //arXiv: 1303. 2580v1 [cs. CC] Mar 2013.

[6] Kochkarev B. S. Typical property of one class of combinatory objects and estimatin from above corresponding combinatory numbers //arXiv: 1304. 4363v1 [cs. DM] 16Apr 2013.

[7] Kochkarev B. S. About one class polynomial problems with not polynomial certificates //Second International Conference "Claster Computing" CC 2013 (Ukraine, Lviv, June 3-5, 2013) P. 99-100.

[8] Kochkarev B. S. Dokazatelstvo gipotezy Edmondsa I reshenie problem Kuka. Nauka, Tekhnika I Obrazovanie, 2014, 2 (2) s. 6-9 (in Russian).

[9] Kochkarev B. S. Vzaimootnosheniya mejdu slojnostnymi klassami P, NPi NPC Problems of modern science and education. 2015, 11(41). S. 6-8 (in Russian).

[10] Kochkarev B. S. Problem of Recognition of Hamiltonian Graph //International Journal of Wirless Communication and Mobile Computing 2016; 4 (2): 52-55.

[11] Kochkarev B. S. Typical Properties of Maximal Sperner Families of Type (k, k+1) and Upper Esnimate. //IOSR Journal of Mathematics (IOSR-JM) Volum 13, Issue 2 Ver II (Mar-Apr. 2017) PP. 16-23.

[12] Wiles A. Modular elliptic curves and Fermat's Last Theorem Annals of Mathematics. V. 141 Second series 3 May 1995 pp. 445-551.

[13] Kochkarev B. S. Ob odnom klasse algebraicheskikh uravnenij, ne imejutshikh ratsionalnykh reshenij //Problem of moden science and education. 2014. 4(22) s. 9-11.

[14] Kochkarev B. S. Algorithm of search of Large Prime Numbers //International Journal of Discrete Mathematics, 2016; 1 (1): 30-32.

[15] Kochkarev B. S. About One Binary Problem in a class of algebraic equations and Her Communication with The Great Hypothesis of Fermat //International Journal of Current Multidisciplinary Studies Vol. 2, Issue, 10, pp. 457-459, October, 2016.

[16] Stanley R. P. Enumerative Combinatorics v. 1, 1986.

[17] Yablonsky S. V. Vvedenie v diskretnuyu matematiku Moskow, 1986, P. 384.

[18] Cormen T. H., Ch. E. Leiserson, Rivest R. L. Algoritmy, postroenie I analiz, 2002, MTSNMO, Moskow P. 960.

[19] Ravenstvo klassov P i NP https://ru.wikipedia.org/wiki

[20] Karp R. M. Reducibility among combinatorial problems. //In Raymond E, Miller and James W. Thatcher, editors, Complexity of computer Computations, P. 85-103, Plenum Press, 1972.