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Approximate Solutions for Mathematical Model of Carcinogenesis Using Adomian Decomposition Method

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Abstract: In this paper, the Adomian decomposition method (ADM) is applied to obtain the approximate solution of a mathematical model of carcinogenesis which is a Riccati differential equation derived by Moolgavkar and Venzon (see [9]). The numerical solution obtained by this way have been compared with the exact solution which obtained by Moolgavkar and Venzon (see [11]). This comparison show that the (ADM) is a powerful method for solving this differential equations. The method does not need weak nonlinearity assumptions or perturbation theory, the decomposition procedure of Adomian will be obtained easily without linearization the problem by implementing the decomposition method rather than the standard methods for the exact solutions.

Keywords: Adomian Decomposition Method, Riccati Differential Equation, Carcinogenesis, Error Absolute

1. Introduction

Over the years, many mathematical models of carcinogenesis have appeared under various biological conditions. In most cases, these models do not admit analytical solution. So these equations should be solved using some particular numerical techniques such as Adomian method. The accuracy of the (ADM) was studied extensively by Cherrault[6]. In this paper we apply (ADM) to a Riccati differential equation:

$\frac{d}{dt}\phi(t) = b_2\phi^2(t) + [\alpha_2 x_3 - (b_2 + d_2 + \alpha_2)]\phi(t) + d_2$ (see [9]) and find an approximate solution. then, by comparing the analytical solution with the exact solution we will find a small absolute Error [using Matlab 7.0].

2. Adomian Decomposition Method

In the 1980's, George Adomian introduced a new method to solve nonlinear functional equations (see [14]). The ADM involves separating the equation under investigation into linear and nonlinear portions. The linear operator representing the linear portion of the equation is inverted and the inverse operator is then applied to the equation. Any given conditions are taken into consideration. The non- linear portion is decomposed into a series of Adomian polynomials. This method generates a solution in the form of a series whose terms are determined by a recursive relationship using these Adomian polynomials. A brief outline of the method follows.

Let the general form of a differential equation be

$$Fy = g \tag{1}$$

Where F is a non- linear differential operator with linear and non linear terms, y and g are functions of t.

The linear term is decomposed as

$$F = L + R + N \tag{2}$$

Where L is an operator representing the linear portion of F which is easily invertible, R is a linear operator for the remainder of the linear porion, and N is a nonlinear operator

representing the nonlinear terms in F

Begin by rewriting the equation in operator form

$$Ly + Ry + Ny = g \tag{3}$$

For convenience L is taken as the highest order derivative. Solving L_y from (3) we have

$$L_y = g - R_y - N_y$$

Because L is invertible, the equivalent expression is

$$L^{-1}(L_y) = L^{-1}(g) - L^{-1}(R_y) - L^{-1}(N_y) \tag{4}$$

Since F was taken to be a differential operator and L is linear, L^{-1} would represent an integration and with any given initial or boundary conditions. If L is a second order operator, then L^{-1} is a two fold integration operator.

$$L^{-1}(L_y) = y(t) - y(0) - ty'(0)$$

Then equation (4) for y yields

$$y(t) = y(0) + ty'(0) + L^{-1}(g) - L^{-1}(R_y) - L^{-1}(N_y) \tag{5}$$

Therefore, y can be presented as a series

$$y(t) = \sum_{n=0}^{\infty} y_n(t). \tag{6}$$

Where y_0 identified as $y(t_0) + ty'(t_0) + L^{-1}(g)$ and y_n ($n \succ 0$) is to be determined. The non-linear term N(y) will be decomposed by the infinite series of Adomian polynomials A_n which is written as

$$N(y) = \sum_{n=0}^{\infty} A_n \tag{7}$$

In order to determine the Adomian polynomials, a grouping parameter λ is introduced.

$$y(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \tag{8}$$

$$N[y(\lambda)] = \sum_{n=0}^{\infty} \lambda^n A_n \tag{9}$$

From (8) and (9) we deduce

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N(\sum_{n=0}^{\infty} \lambda^n y_n) \right]_{\lambda=0} \tag{10}$$

Now, substituting (6) and (7) into (5), we obtain

$$\sum_{n=0}^{\infty} y_n = y_0 - L^{-1}[R(\sum_{n=0}^{\infty} y_n)] - L^{-1}(\sum_{n=0}^{\infty} A_n)$$

Consequently we can write

$$\begin{aligned} y_0 &= y(t_0) + ty'(t_0) + L^{-1}(g) \\ y_1 &= L^{-1}R(y_0) + L^{-1}(A_0) \\ y_2 &= -L^{-1}R(y_1) - L^{-1}(A_1) \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ y_{n+1} &= -L^{-1}R(y_n) - L^{-1}(A_n) \end{aligned} \tag{11}$$

Based on the Adomian decomposition method, we considered the solution $y(t)$ as

$$y = \lim_{n\to\infty} \Phi_n$$

Where the (n+1) term approximation of the solution is defined in the following form

$$\Phi_n = \sum_{k=0}^{n} y_k(t), \quad n \succ 0$$

As stated above, the ADM produces a convergence series solution. The issue of convergence is addressed by several researchers (see [15], [16] and [17]). According to Cherrault (see [6]), the series produced by the decomposition method is absolutely convergent as well as uniformly convergent. This is the case because the series "rearranges a strongly convergent Taylor series of the analytic functions u and f(u). The series converges uniformly (and absolutely and in norm), henc e the sum is not changed by rearrangement of the terms". Babolian and Biazar (see [15]) provide a definition from which the order of convergence for the method could be determined. Of course, having a higher order of convergence is desirable since then the series will converge more rapidly. Then, the particular solution Φ_n the n-term approximation is converging and accurate for low values of n.

3. Mathematical Model of Carcinogenesis

The two-stage model of carcinogenesis as proposed by Moolgavkar and Knudson (see [9]) assumes that a cancer tumor develops from a single normal stem cell by clonal expansion and views carcinogenesis as the end result of two-discrete events in normal stem cells.

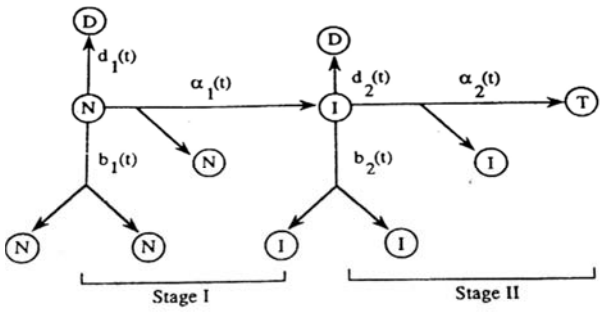

Figure 1. *Two stage model of carcinogenesis.*

N, I, T stand for normal stem cell, intermediate cell and tumor cell, respectively, D stands for death.

As shown by Wai –tan (see [3]), we can find the age specific incidence function of tumors and hence the probability distribution of time required for a normal stem cell to develop into a tumor.

Let the transition probabilities be denoted by

$$P_1(s,t;\ u_1,\ u_2\ ,u_3\ \ ;i_1\ ,i_2\ ,i_3) =$$
$$\mathrm{P}_r\{N(t)=i_1, I(t)=i_2, T(t)=i_3 \,|\, N(s)=u_1, I(s)=u_2, T(s)=u_3\}$$

and $P_2(s,t;v_1,v_2;j_1,j_2) = \mathrm{P}_1\{I(t)=j_1, T(t)=j_2 \,|\, I(s)=v_1, T(s)=v_2\}$

Let $\phi(x_2,x_3;s,t)$ be the PGF of [I(t), T(t)] given [I(s)=1,N(s)=T(s)=0] that is $\phi(s,t)=\sum_{j_1}\sum_{j_2} x^{j_1}x^{j_2}p(j_1,j_2;s,t)$

In this model, if $\alpha_i(t)=\alpha_i$, $b_2(t)=b_2$, $d_2(t)=d_2$ and N(t) are deterministic growth curve functions. In this case, $\phi(t_0,t)=\phi(t_0-t)$ is a function of t_0-t. Putting t_0= 0, then the calenderer time may be taken as a Person's age.(see [7]).

In Wai tan (see [3]), If $\alpha_2(t)=\alpha_2$, $b_2(t)=b_2$ and $d_2(t)=d_2$ then $\phi(x_2,x_3;0,t)=\phi(t)$ satisfies the following Riccati differential equation with initial condition $\phi(0)=x_2$

$$\frac{d}{dt}\phi(t)=b_2\phi^2(t)+\left[\alpha_2x_3-(b_2+d_2+\alpha_2)\right]\phi(t)+d_2 \quad (12)$$

Now we will solve the Riccati differential equation (12) using (ADM).

4. Analysis of General Riccati Differential Equation

Before we give the formal definition of Riccati equations, a little introduction may be helpful. Indeed, consider the first order differential equation

$$\frac{dy}{dx}=f(x,y).$$

if we stop at y, we will get a linear equation. Riccati looked at the approximation to the second degree: he considered equations of the type

$$\frac{dy}{dt}=Q(t)y+R(t)y^2+P(t)\ \ (13)\ \ y(0)=G(t)$$

Where $Q(t),R(t),P(t)$ and $G(t)$ are scalar functions. These equations bear his name, Riccati equations (see [18]). They are nonlinear and do not fall under the category of any of the classical equations. In order to solve a Riccati equation, one will need a particular solution. Without knowing at least one solution, there is absolutely no chance to find any solutions to such an equation. Indeed, let y_1 be a particular solution of the previous equation. Consider the new function z defined by $z=\frac{1}{y-y_1}$, Then easy calculations give

$$\frac{dz}{dx}=-(Q(x)y+2y_1R(x))z-R(x).$$

which is a linear equation satisfied by the new function z. Once it is solved, we go back to y via the relation

$$y=y_1+\frac{1}{z}.$$

Keep in mind that it may be harder to remember the above equation satisfied by z. Instead, try to do the calculations whenever you can.

In this paper, to solve (12) by means of (ADM), we construct a Adomian polynomials A_n.

5. Numerical Solution for the Given Model Using (ADM)

From equations (12), (13) we get

$$Q(t)=\ a_2\ ,$$

where $a_2=\left[\alpha_2x_3-(b_2+d_2+\alpha_2)\right]$,

$$R(t)=b_2\ ,\ P(t)=d_2\ ,\ G(t)=x_2$$

Applying the Adomian Decomposition method to the problem (12), we have

$$\phi(t)=\phi(0)+L^{-1}\left[R(t)\phi^2\right]-L^{-1}\left[Q(t)\phi\right]+L^{-1}\left[P(t)\right] \quad (14)$$

Where;

$$L^{-1}=\int_0^t (.)dt \quad (15)$$

$$\phi(t)=\phi(0)+b_2L^{-1}\left[\phi^2\right]-a_2L^{-1}\left[\phi\right]+L^{-1}\left[d_2\right]$$

$$Let\ \phi(\mathrm{t})=\sum_{n=0}^{\infty}\phi_n\ \ and\ \ \ \phi^2=\sum_{n=0}^{\infty}A_n$$

From (11), the recurrent scheme of (ADM) is written as

$$\phi_0=\phi(0)+t\phi'(t_0)+L^{-1}\left[\phi\right]+L^{-1}\left[P(t)\right]$$

Then

$$\phi_0=x_2+td_2 \quad (16)$$

$$\text{And } \phi_{n+1}=L^{-1}\left(R(t)A_n\right)-L^{-1}\left(Q(t)\phi_n\right) \quad (17)$$

Let $n=0$ and using equations (8) and (9) we get

$$N(\mu(\lambda))=N(\phi_0)=A_0$$

So $A_0=\phi_0{}^2=(x_2+td_2)^2$

From (17) we have

$$\phi_1 = b_2 L^{-1}(A_0) - a_2 L^{-1}(\phi_0)$$

$$= b_2 L^{-1}(x_2 + td_2)^2 - a_2 L^{-1}(x_2 + td_2)$$

$$= b_2\left[tx_2^2 + \frac{1}{3}d_2^2 t^2 + x_2 d_2 t^2\right] - a_2\left[tx_2 + \frac{1}{2}d_2 t^2\right]$$

$$\phi_1 = \left[b_2 x_2{}^2 - a_2 x_2\right]t + \left[\frac{1}{3}b_2 d_2^2 - \frac{1}{2}a_2 d_2\right]t^2. \quad (18)$$

Let $n = 1$ and using equations (8) and (9) we get

$$N(\mu(\lambda)) = N(\phi_0 + \lambda\phi_1) = (\phi_0 + \lambda\phi_1)^2$$

$$A_1 = \frac{d}{d\lambda}(\phi_0{}^2 + 2\lambda\phi_0\phi_1 + \lambda\phi_1{}^2)\Big|_{\lambda=0}$$

$$N(\mu(\lambda)) = N(\phi_0 + \lambda\phi_1) = (\phi_0 + \lambda\phi_1)^2$$

$$A_1 = \frac{d}{d\lambda}(\phi_0{}^2 + 2\lambda\phi_0\phi_1 + \lambda\phi_1{}^2)\Big|_{\lambda=0}$$

So $A_1 = 2\phi_0\phi_1$

From (17) we have

So $\phi_2 = b_2 L^{-1}(A_1) - a_2 L^{-1}(\phi_1)$

$$\phi_2 = 2b_2 L^{-1}[(b_2 x_2{}^3 + d_2 x_2{}^2 - a_2 x_2{}^2)t + (\frac{1}{2}b_2 d_2 x_2 - \frac{1}{2}a_2 x_2 d_2)t^2$$
$$+ (b_2 d_2 x_2{}^2 + d_2{}^2 x_2 - a_2 d_2 x_2)t^2 + (\frac{1}{2}b_2 d_2{}^2 + \frac{1}{2}a_2 d_2{}^2)t^3]$$
$$- a_2 L^{-1}[(b_2 x_2{}^2 - a_2 x_2)t$$
$$+ (\frac{1}{3}b_2 d_2^2 - \frac{1}{2}a_2 d_2)t^2]. \quad (19)$$

We use $\Phi_n(t)$ as the approximating of $\phi(t)$

$$\phi(t) = \lim_{n\to\infty} \Phi_n, \text{ Where } \Phi_n = \sum_{k=0}^{n}\phi_k(t)$$

Put n = 2 and using equations (16, 18 and 19) we have

$$\Phi_n(t) = \phi_0 + \phi_1 + \phi_2 + \ldots$$

Moolgavkar, S.H. and Venzon (see [11]) have shown the exact solution of the given model (12) as:

$$\phi(t) = \{y_2(x_2 - y_1) + y_1(y_2 - x_2)\exp[b_2(y_2 - y_1)t]\}\times$$
$$\{(x_2 - y_2) + (y_2 - x_2)\exp[b_2(y_2 - y_1)t]\}^{-1}$$

Where

$$2b_2 y_2 = b_2 + d_2 + \alpha_2 - \alpha_2 x_3 + h(x_3)$$
$$2b_2 y_1 = b_2 + d_2 + \alpha_2 - \alpha_2 x_3 - h(x_3)$$

With $h(x_3) = [(b_2 + d_2 + \alpha_2 - \alpha_2 x_3)^2 - 4b_2 d_2]^{1/2}$

For both solutions (numerical and exact), let these factors b_2, d_2, α_2, x_3 *equal* 0.01 *and* x_2 *equal* 0.3.

Using Mat lab (version 7.0) we get this computations:

Table 1. *Numerical comparison for the problem.*

t	Exact Solution	Iterate ADM	Absolute Error
0.1	0.30019169	0.300999004	$8.0731 * 10^{-4}$
0.2	0.30038769	0.30458198	$4.19429 * 10^{-3}$
0.3	0.300579033	0.306878955	$6.299625 * 10^{-3}$
0.4	0.300771124	0.30917992	$8.408795 * 10^{-4}$

6. Conclusion

We introduced a technique, Adomian decomposition method to numerically solve the Riccati differential equation of a biological model is presented. All the numerical results obtained by using (ADM) descried earlier show very good agreement with the exact solutions for only a few terms. Comparing the decomposition method with several other methods that have been advanced for solving Riccati equation shows the new technique is reliable, powerful and promissing. The computations associated the problem in this paper have been performed Using (Matlab 7.0).

The ADM does have some disadvantages, however. The first is that the method gives a series solution which must b e truncated for practic al applications. In addition, the rate and region of convergence are potential short comings. According to Jiao (see [19]), "although the series can be rapidly convergent in a very small region, it has very slow convergence rate in the wide r region...and the truncated series solution is an inaccurate solution in that region, which will greatly restrict the application are a of the method." An investigation into this claim would greatly benefit the scientific community. Nonetheless, the ADM is proving to be a very useful tool with wide application. According to Wazwaz (see[20]), "The main advantage of the method is that it can b e applied directly for all types of deferential and integral equations, linear or nonlinear, homogeneous or inhomogeneous, with constant coefficients or with variable coefficients.

Another important advantages that the method is capable of greatly reducing the siz e of computational work while still maintaining high accuracy of the numerical solution." These advantages are presumably the basis for the wide-ranging applicability of the method.

7. Future Research

The completion of this study has led to an awareness of several topics that require further investigation, such as the homotopy analysis method (HAM) which able to solving a nonlinear ordinary/partial differential equations. The (HAM) employs the concept of the homotopy from topology to generate a convergent series solution for nonlinear systems. This is enabled by utilizing a homotopy-Maclaurin series to deal with the nonlinearities in the system (see [13]). The (HAM) is an analytic approximation method as apposed to discrete computational method. Further, the (HAM) uses the homotopy parameter only on a theoretical level to demonstrate that a nonlinear system may be split into an infinite set of linear systems which are solved analytically,

while the continuation methods require solving a discrete linear system as the homotopy parameter is varied to solve the nonlinear system. So, from Medhi Dehghan (see [4]), showing that solving a Ricatti differential equation is very difficult exactly

$$\frac{d}{dt}\psi_1(t) = b_1\psi_1^{\,2}(t) + \left[\alpha_1\phi(t) - (b_1 + d_1 + \alpha_1)\right]\psi_1(t) + d_1 \quad \text{with}$$

initial condition $\psi_1(0) = x_1$

Although Moolgavkar and Venzon (see [9]) used a numerical solutions of the previous equation to compute some incidence curves due presumably to the slow convergence of the series. In general, Applying (HAM) require further examination. To begin with, an understanding of the convergence should be developed. This would hopefully lead to an understanding of limited effectiveness of Padé after treatment. In addition, the degree to which the strength and type of non linearity affect the quality of the solution should also be investigated. (Comp. [21 – 29]).

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