

# Modeling the Residuals of Financial Time Series with Missing Values for Risk Measures Using R

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**Abstract:** This paper is to fit an appropriate model on the returns of daily stock price and determine the appropriate model for the residuals in order to compute some risk measures. The daily stock price of First Bank Nigeria, Plc was collected from Nigerian Stock Exchange Market. The methods of weekly average, regression imputation and repetition were used in computing the missing values. Also, adopted was deleting days with missing values. The method of transformation was determined in each of the series and log transformation was adopted for the four series. In the model selection, the ARMA+GARCH model of the repetition had the minimum AIC as compared to other methods of dealing with missing values. The distribution of the residuals was found to be suitable to the Generalized Parato Distribution (GPD). The parameters of this distribution were used in computation of risk measures. The computed Value at Risk (VaR) has a value of 49438.79 and that of the Expected Shortfall (ES) as 49291.24 with position of 1,000,000. This is an indication that the risk of investing in the stock of the First Bank Nigeria, Plc is eminent.

**Keywords:** Missing Value, Volatility, Stock Price, Residuals, Transformation, Value at Risk, Expected Shortfall

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## 1. Introduction

The series of stock price is of financial time series and according to Tsay, [1] both financial theory and its empirical time series contain an element of uncertainty. The choice of the model is of paramount important to the study because of the volatile nature of financial time series.

The residuals in the study are from model based on conditional variance structure and conditional mean structure, that is, the combination of Autoregressive moving Average (ARMA) and Generalized Autoregressive Conditional Heteroscedastic (GARCH) models. The residuals will be without Autoregressive Conditional Heteroscedastic (ARCH) effect which can now be used to get a model for the purpose of computing Value at Risk (VaR) and Expected Shortfall (ES).

The series in this research is a financial time series, that is, the daily stock price of First Bank of Nigeria Plc (FBN). In the series, there are some days without recorded price due to non-operation by the Stock Exchange Market which become a concern. The Nigerian Stock Exchange Market does not

open its floor on week days of public holidays and during civil disturbances or crisis, thus causing the missing values. But in forecasting such days will also be forecasted without contributing fully in the series.

In most of the researches, some fundamental issues were ignored, like using a statistical tool to determine the type of transformation of the series, estimation of the missing values in time series and outright use of GARCH models without the determination of the presence of ARCH in the series. Some went as far as using ARMA on purely financial time series. These areas will be addressed in this study in order to have efficient and reliable results.

This study is carried out in order to fit an appropriate model on the returns of daily stock price, to determine the appropriate model of the residuals and to determine risk measures associated to investment in stock.

## 2. Literature Review

Some researchers go for outright omission of the periods with missing values and mostly silent on that aspect as seen

in Hamadu and Mojekwu, [2], in an investigation of Nigerian insurance stock option prices. Contrary to the outright omission, Honaker and King, [3] adopted the use of modern methods of computing missing value because deleting the days with missing values can bring about biased and inefficiency. Three methods were adopted and they are multiple imputation method, analysis to incorporate knowledge from area experts via priors on individual missing cells values and developing of an algorithm that substantially expands the range of computationally feasible data types and sizes.

In other to see the extent of the effect of outright deletion, the series from the outright deletion need to be compared with series of estimated missing values. Shukur and Lee, [4] in the study of wind speed in Iraq and Malaysia made a comparison of the classical and AR-ANN methods of imputation. The classical methods include linear, nearest neighbor, and state space. In the analysis using ARIMA, the AR-ANN performed better than linear, nearest neighbor and state space. They had already considered AR, even though with deleting approach, before the use of ARIMA to test the effectiveness.

In some instances the days without dealings in stock market is overtaken by considering weekly or monthly stock price averages instead of daily prices. Chow and Lawler, [5] used weekly data in studying the time series of Shanghai and New York stock price indices. Ajie and Nenbee, [6] used yearly data of stock price in analyzing the monetary policy and stock price in Nigeria. But the effect of compressing the data which may lead to loss of information cannot be ruled out.

This research work will estimate the missing values of the daily stock price of First Bank Nigeria, Plc from 1998 to 2007 before modeling. This is a diversion of the least square principle of Ferreiro, [7] which the model is identified before using the fitted model to estimate the missing values.

Kabaila and Mainzer, [8] in studying estimation risk used the error from GARCH (1, 1) for approximate value at risk (VaR) and estimated shortfall (ES) that differ from exact VaR and ES. In an effort to get a reliable estimate of VaR and ES because of its important in managing financial risk discovered from the study that the approximate VaR and ES are close to being an unbiased estimator of the exact value of VaR and ES.

Stavroyiannis, [9] used GARCH modelling followed by Filtered Historical Simulation in determining the VaR and ES of digital currencies found out that digital currencies are subject to higher risk. Abad, Benito and Lopez, [10] in the review of value at risk methodologies stated that of all the approaches of forecasting VaR the approaches based on the Filtered Historical Simulation and Extreme Value Theory are the best methods. In line with their submission, this study will make use of the extreme value theory in the estimation of value at risk and expected short fall.

### 3. Missing Value and Transformation of a Financial Time Series

The missing values are recorded on week days without trading on the floor of Nigeria Stock Exchange Market. For instance, the Nigerian independent day that is not on same day of the week for all the years. The weekend of Saturdays and Sundays are not considered. Four methods of dealing with the missing values are used. These are; the average imputation by Velicer and Colby, [11], regression imputation, Cameron and Trivedi [12], the eliminating or deleting (days) method, Soley – Bori [13] and the repetition of preceding values.

The transformation of the series is determined by Taylor's Power Law, [14], see Table 1 below.

**Table 1.** Slope and Transformation of Time Series.

b	1-b/2	Transformation	Y <sup>1</sup>
0	+1.0	No transformation	Y <sup>1</sup>
1	+0.5	Logarithmic	Y <sup>0.5</sup>
2	0.0	Reciprocal Square	Log(Y)
3	-0.5	Root	Y <sup>-0.5</sup>
4	-1	Reciprocal	Y <sup>-1</sup>

The logarithm transformation is required for the four series and the differencing of the logarithm transformation gives the returns.

### 4. Combination of ARMA and GARCH Model

One of the extensions to GARCH (p, q) is ARMA(u, v)+GARCH(p, q). Let  $\{r_t\}$  be a time series of the stock returns and  $\tilde{r}_t = r_t - \mu$ , then ARMA(u, v)+GARCH(p, q) is given as

$$\tilde{r}_t = \phi_1 \tilde{r}_{t-1} + \dots + \phi_u \tilde{r}_{t-u} + e_t - \theta_1 e_{t-1} - \dots - \theta_v e_{t-v}$$

$$e_t = \sigma_{t/t-1} \varepsilon_t \quad (1)$$

$\sigma_{t/t-1}^2 = w + \alpha_1 e_{t-1}^2 + \dots + \alpha_q e_{t-q}^2 + \beta_1 \sigma_{t-1/t-2}^2 + \dots + \beta_p \sigma_{t-p/t-p-1}^2$   
 $\{\tilde{r}_t\}$  is a return series which is often a serially uncorrelated sequence with zero mean.

$\varepsilon_t$  is independent and identically distributed (iid) random variables with mean zero and variance 1, otherwise known as innovations or standardized residuals.

$\sigma_{t/t-1}^2$  is the conditional variance of  $\tilde{r}_t$ .

Cryer and Chan [15] and Tsay [1].

### 5. Generalized Pareto Distribution (GPD)

The GPD is an approximation of General Extreme Value Distribution (GEVD). McNeil et al, [16] gives the cumulative density function (CDF) of GDP as two parameter distribution

function below.

$$G_{\xi, \beta}(x) = \begin{cases} 1 - (1 + \frac{\varepsilon_t \xi}{\beta})^{-\frac{1}{\xi}}, \xi \neq 0 \\ 1 - \exp(-\frac{\varepsilon_t}{\beta}), \xi = 0 \end{cases} \quad (2)$$

Where

$\xi$  is the tail index which is a measure of the shape of the tail

$\beta > 0$  is the scaling parameter and

$\varepsilon_t \geq 0$  when  $\xi \geq 0$  and

$0 \leq \varepsilon_t \leq -\frac{\beta}{\xi}$  when  $\xi < 0$

## 6. Risk Measures

The type of risk measures that will be considered in this research is that of market risk which involves the computation of Value at Risk (VaR) and Expected Shortfall (ES).

### 6.1. Value at Risk (VaR)

This is the estimate of an amount by which an institution's position in a risk category could fall gradually due to general market movements during a given holding period. It can also be referred to as a measure of minimum loss of a financial

$$p = [DV(l) \geq VaR] = 1 - [DV(l) \leq VaR] = 1 - F_l(VaR) \quad (4)$$

The VaR is concerned with tail behavior of the  $F_l(x)$ . The left tail of  $F_l(x)$  is important for a long position (adopted for this study) while the right tail of  $F_l(x)$  is for a short position.

If  $F_l(x)$  is known, then VaR is the same as  $p$ th quantile  $x_p$ .

$$VaR = x_p$$

$$x_p = \inf\{x / F_l(x) \geq p\} \text{ for } 0 < p < 1 \quad (5)$$

Where  $\inf$  denotes the smallest real number satisfying  $F_l(x) \geq 0$

Tsay, [1].

### 6.2. Peak Over Threshold (POT) Approach to VaR Calculation

There are two approaches; Homogeneous case (parameters are fixed over time) and Non-homogeneous case (parameters are time-varying, according to some explanatory variables). This approach was proposed by Davison and Smith, [17] and Smith, [18].

Case I (Homogeneous).

$$VaR = \begin{cases} \beta + \frac{\alpha}{k} \{1 - [-D(\ln(1-p))^k]\} \text{ if } k \neq 0 \\ \beta + \alpha \{-D(\ln(1-p))\} \text{ if } k = 0 \end{cases} \quad (6)$$

position during a particular period of time for a given (small) probability.

Considering the following;

i. Let  $t$  be the time index

ii. Let  $l$  be the periods of the financial position

iii. Let  $DV(l)$  be the change in value of the assets in the financial position from time

$t$  to  $t+l$

iv. Denote the CDF of  $DV(l)$  by  $F_l(x)$

The VaR of a long position over the time horizon  $l$  with probability  $p$  under a probabilistic approach is given as

$$p = [DV(l) \leq VaR] = F_l(VaR) \quad (3)$$

For  $DV(l) < 0$  (the holder of a long financial position suffers a loss), the VaR above typically assumes a negative value when  $p$  is small signifying a loss. From the definition, the probability that the holder would encounter a loss greater than or equal to VaR over the time horizon  $l$  is  $p$ . Therefore, the probability  $(1 - p)$  is the potential loss encountered by the holder of the financial position over the time horizon  $l$  is less than or equal to VaR.

For  $DV(l) < 0$  (the holder of a short position suffers a loss when the value of the asset increases), hence VaR is defined as

where  $D$  is the baseline time interval, mostly the number of days trading in a year, used in estimation. In the United States, one typically uses  $D = 252$ .

Case II (Non-Homogeneous).

$$VaR_q = u + \frac{\beta}{\xi} \{1 - [\frac{T}{N_u(1-q)}]^\xi\} \dots \dots \dots (7)$$

Where

$u$  is the threshold

$T$  is the sample size

$N_u$  is the number of exceedances and

$\beta$  and  $\xi$  are the scale and shape parameters of the GPD

Tsay, [1].

This work will consider a two parameter case of non-homogeneous.

### 6.3. Expected Shortfall (ES)

This is expected loss given that the VaR is exceeded otherwise referred to as Expected Tail Loss (ETL). The expected shortfall

(ES) is defined as

$$ES_q = E(L / L > VaR_q) \quad (8)$$

### 6.4. Generalised Pareto Distribution GPD Approach to ES Calculation

Using the properties of GPD the ES is defined as

$$ES_q = \frac{VaR_q}{1+\xi} + \frac{\beta + \xi u}{1+\xi} \quad (9)$$

Where  $\xi$  and  $\beta$  are the parameters of GPD.  
Tsay, [1].

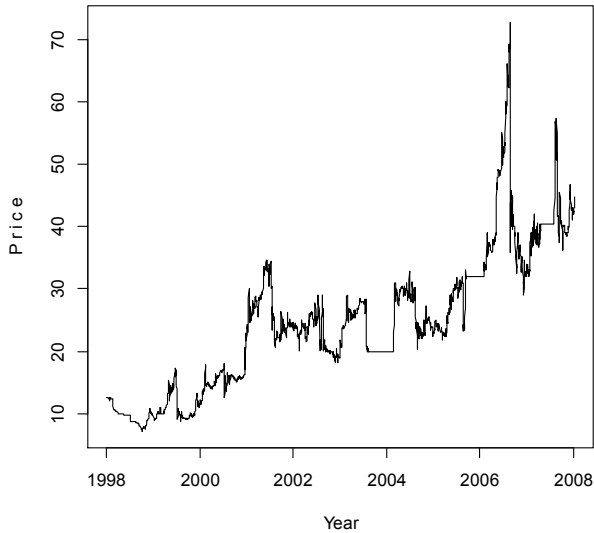
## 7. Modeling

The modeling will be done with help of R software for statistical computing, R-3.2.2, R Core Team [19]. The packages and codes used are displayed in the Appendix.

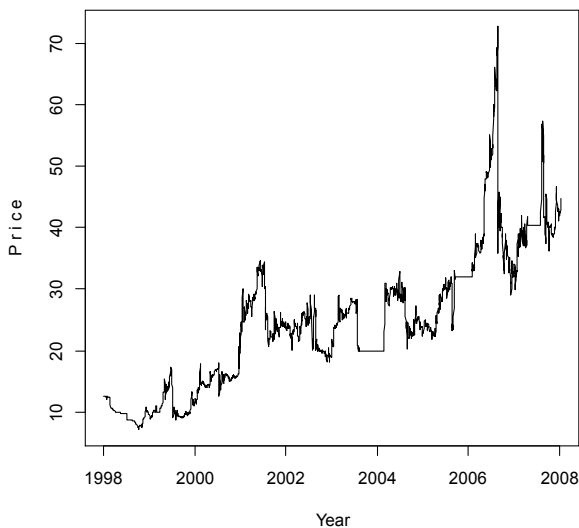
### 7.1. Time Series Plot

The four series are based on the estimation of missing values from the daily stock price of First Bank Nigeria (FBN); FBN1- weekly average, FBN2-weekly regression estimates, FBN3-repetition of preceding value and FBN4-deletion of missing values.

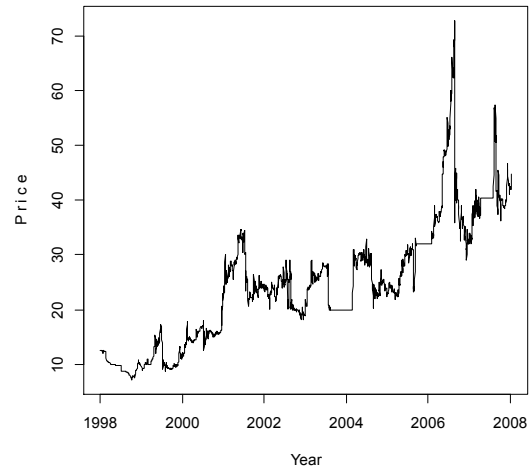
The time plot in Figure 1 to 4 indicates an upward trend.



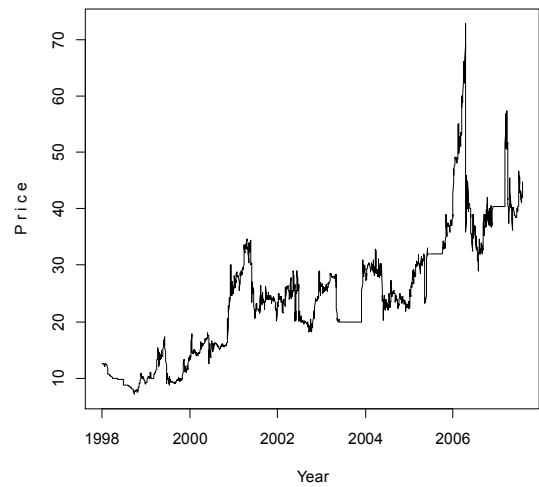
**Figure 1.** Time plot of Daily Stock Price of FBN1.



**Figure 2.** Time plot of Daily Stock Price of FBN2.



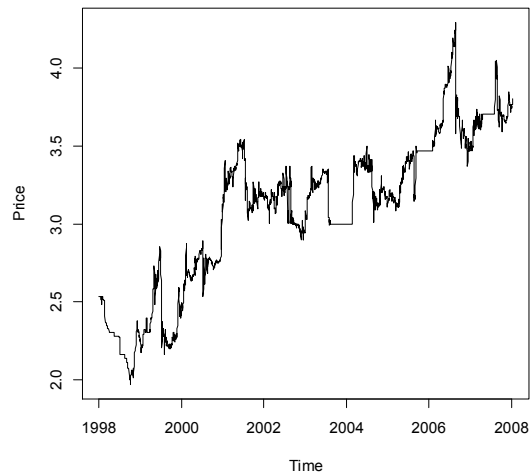
**Figure 3.** Time plot of Daily Stock Price of FBN3.



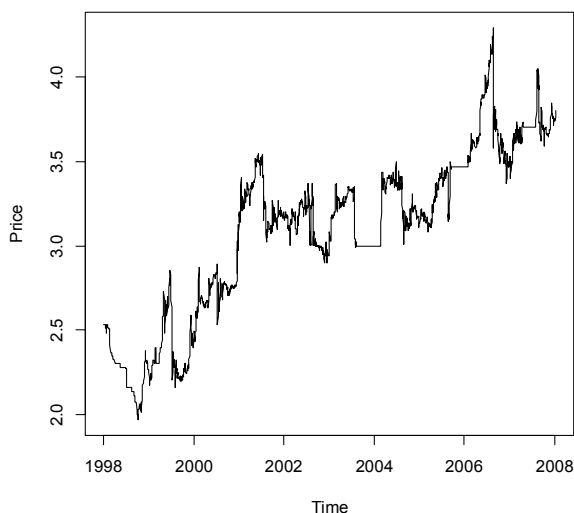
**Figure 4.** Time plot of Daily Stock Price of FBN4.

The behavior of the logarithm transformation of the four series is shown in the plots of the log of the four series from Figure 5 to 8.

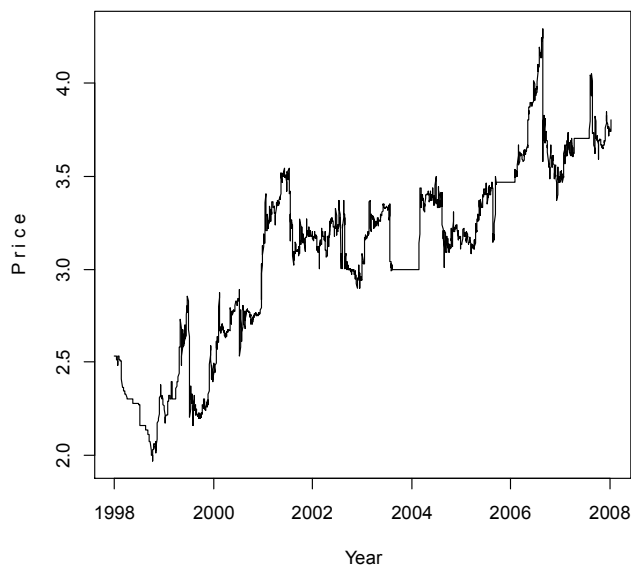
The logarithm transformation plot from figure 5 to 8 exhibited trend in all the four series which implies that the series are still non-stationary.



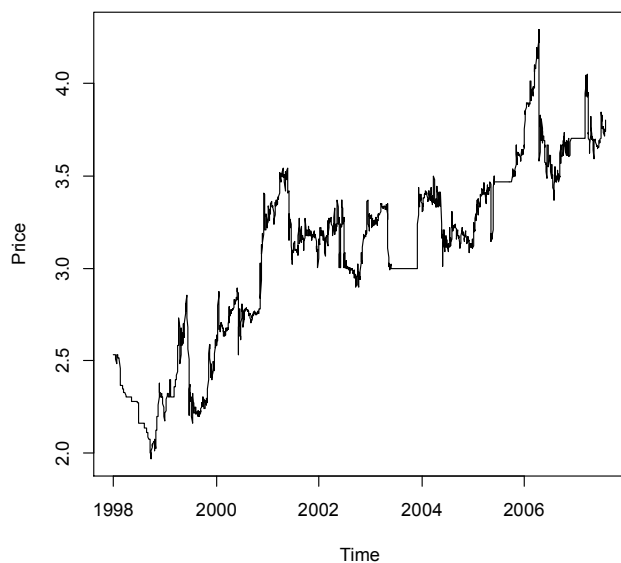
**Figure 5.** Time plot of Log Daily Stock Price of FBN1.



**Figure 6.** Time plot of Log Daily Stock Price of FBN2.



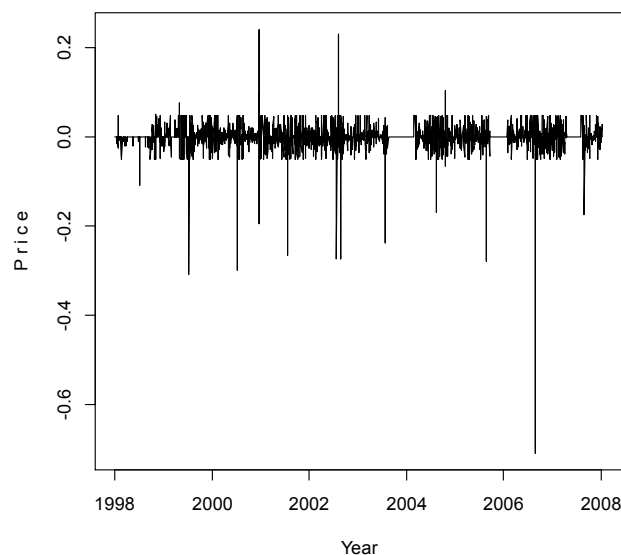
**Figure 7.** Time plot of Log Daily Stock Price of FBN3.



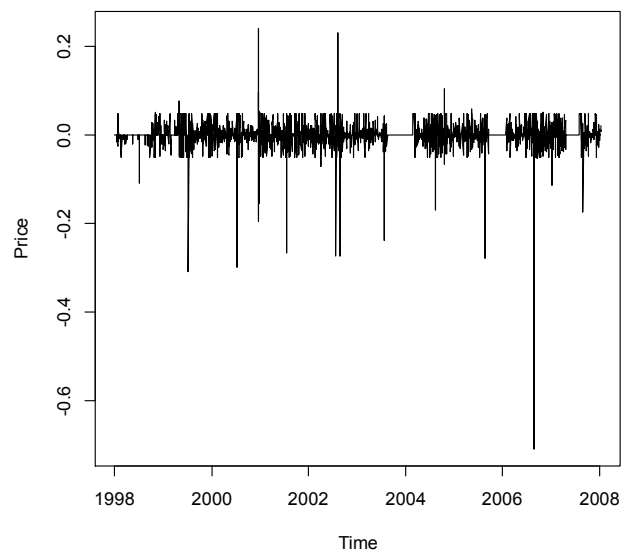
**Figure 8.** Time plot of Log Daily Stock Price of FBN4.

In the plot of the returns  $(r_t) = \log(y_t) - \log(y_{t-1})$  of the four series in figure 9 to 12 shows the removal of trend, making it stationary.

This satisfies use of the returns of an asset which Campbell, Lo, and MacKinlay, [20] stated that it is a complete and scale-free summary of the investment opportunity and easier to handle than price series. A series of this nature (volatile) is assumed to be serially uncorrelated with zero mean as noted by Cryer and Chan, [15].



**Figure 9.** Time plot of Returns of FBN1.



**Figure 10.** Time plot of Returns of FBN2.

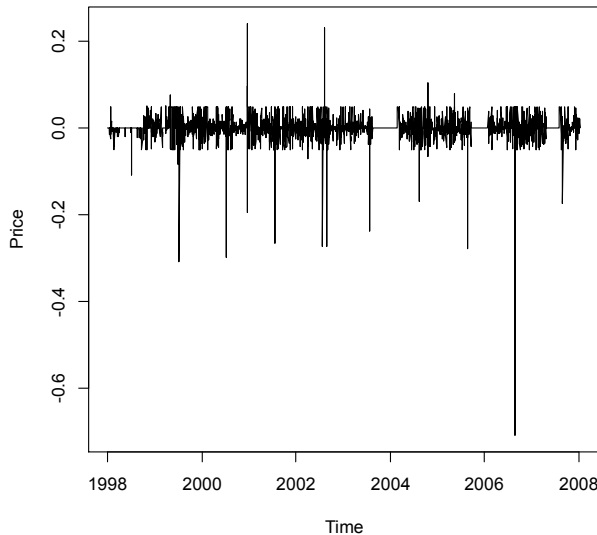


Figure 11. Time plot of Returns of FBN3.

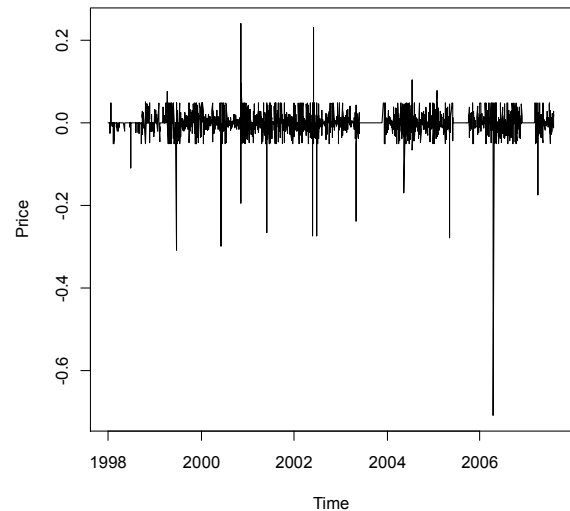


Figure 12. Time plot of Returns of FBN4.

Table 2. Models and AIC.

Series	ARMA(1,1)+GARCH(1,1) <sub>t</sub> AIC	ARMA(2,1)+GARCH(2,1) <sub>t</sub> AIC	ARMA(1,1)+GARCH(1,1)Norm AIC	ARMA(2,1)+GARCH(2,1)Norm AIC
FBN1	-6.642874	-6.773584	-4.556857	-4.554653
FBN2	-6.637693	-6.772759	-4.545477	-4.547152
FBN3	-6.709260	-6.837350	-4.576206	-4.577212
FBN4	-6.573305	-6.705362	-4.506181	-4.506617

## 7.2. Model Selection

The tentative model is to be selected based on the AIC. The best of the models from Table 2 below is ARMA(2,1)+GARCH(2,1)<sub>t</sub> of FBN3 and its AIC is - 6.837350 which is the minimum.

### 7.2.1. Estimate of the Model Parameters

The values of the estimated parameters from the tentative model can be seen in Table 3 below. Even though there is not enough evidence to accept the normality assumption from the Jarque-Bera Test and Shapiro-Wilk Test. But the Ljung-Box Test shows that there is no intercorrelation.

Table 3. Estimated Values of the Parameters with p-values.

ARMA(2,1)+GARCH(2,1) <sub>t</sub>		p-value
Parameter	Value of Parameter/Statistic	
ar1	0.5279	0.00175
ar2	-0.040353	0.01585
ma1	-0.49403	0.00501
W	0.00000000095237	0.98031
alpha1	1	0
alpha2	1	0
beta1	0.29796	0
Jarque-Bera Test	43195034	0
Shapiro-Wilk Test	0.03741205	0
Ljung-Box Test R Q(15)	3.705121	0.9985541
Ljung-Box Test R <sup>2</sup> Q(15)	0.1326831	1

### 7.2.2. Symmetric Test

The Kurtosis (K) of the residuals is 128.2811, which means  $K > 0$ , hence K is heavy tailed as described by Cryer and Chan, [15]. Therefore, the generalized extreme value distribution of a heavy tail behavior is that of Frechet type of distribution, having the shape parameter  $\xi > 0$ . This according to Tsay, [1] is good for risk management and Bystrom, [21] stated that it is for financial time series which is usually fat tailed. This is approximated to Generalized Pareto Distribution (GPD).

## 8. Estimates of the Generalized Pareto Distribution (GPD) Parameters

The threshold (u) of choice at  $u \geq 0$  is 0.05. This gives 50 values above the threshold (exceedances). The tail index is 2.175344675 and the scaling parameter is 0.001394252.

### 8.1. The GPD Model Diagnosing

The model will be diagnosed through the following plots (figure 13 and figure 14). The residual plot in Figure 13 does not show any form of trend but a rectangular scatter points. The model is adequate with Figure 14 showing an iid standard exponential distribution.

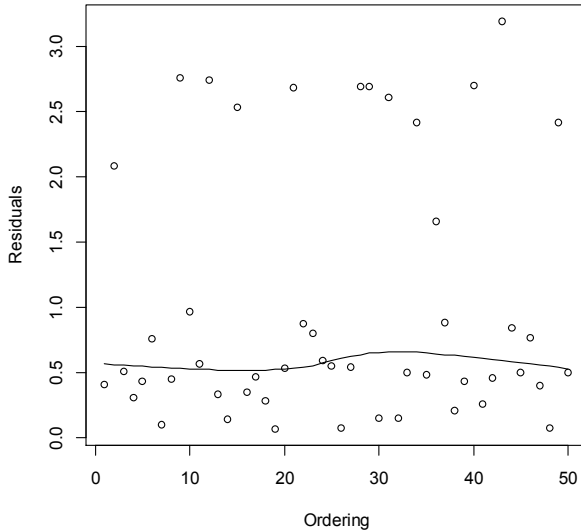


Figure 13. Scatter plot of Residuals.

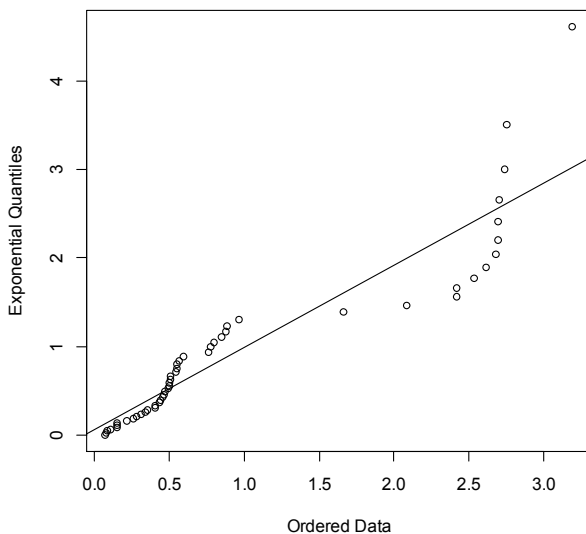


Figure 14. QQ plot of Residuals.

### 8.2. Computation of VaR and ES from GPD Parameters

The quantile of the two measures is stated below.

$$VaR_q = 0.04943879$$

$$ES_q = 0.04929124$$

Therefore,

$$VaR = 0.04943879 \times 1000000 = 49438.79$$

$$ES = 0.04929124 \times 1000000 = 49291.24$$

The value at risk (VaR) is 49438.79 while that of expected shortfall is 49291.24 at a position of 1,000,000.00.

## 9. Discussion

The daily stock price exhibited the same features of other financial series. It is non-stationary in the observed prices but with the returns on the daily stock price, the trend seems to be removed. The series, daily stock price of FBN, has both conditional mean and conditional variance not significantly zero, meaning that ARMA model alone or GARCH model alone would not have fit the data appropriately.

In this work, it would have been faster approach to use the return series in the Generalized Pareto Distribution (GPD) model but it would have amounted to ARCH effect in subsequent computations. Therefore, the study adopted the use of the second stage approach; finding the residuals and using the series in GPD. In the computation of VaR the position of N1000000 was also considered because most single individual investment were of about a million naira. This is because most of them were from the middle income earners, mostly civil servant, who can assess such loans from their salary account.

The loss of information about the series by deleting the days with missing values was exposed. This is seen in the fact the method of repetition of preceding value gave the best model having the least AIC. Also, in all the models in table 2 the highest AIC is at FBN4 (deleting of the days with missing values). This is a clear indication that an outright deletion will give the worse result than any other method used in this study.

## 10. Conclusions

The various approaches adopted in handling the missing values contributed immensely to the selection of the appropriate model used in estimating the residuals. In the computation of VaR and ES the position of N1000000 determines the risk of investing N1000000 in the stock of First Bank Nigeria, Plc at long and short position.

## Appendix

```
R Code
library(TSA)
library(fGarch)
FBN1=read.csv('FBN1.csv',header=T)
FBN2=read.csv('FBN2.csv',header=T)
FBN3=read.csv('FBN3.csv',header=T)
FBN4=read.csv('FBN4.csv',header=T)
FBN1=ts(FBN1, start=c(1998,1), frequency=260)
FBN2=ts(FBN2, start=c(1998,1), frequency=260)
FBN3=ts(FBN3, start=c(1998,1), frequency=260)
FBN4=ts(FBN4, start=c(1998,1), frequency=260)
plot(FBN1)
plot(FBN2)
```

```

plot(FBN3)
plot(FBN4)
FBN1=read.csv('FBN1.csv',header=T)
FBN2=read.csv('FBN2.csv',header=T)
FBN3=read.csv('FBN3.csv',header=T)
FBN4=read.csv('FBN4.csv',header=T)
lm1 <- lm(log.variance.~log.mean., data=FBN1)
lm1
lm2 <- lm(log.variance. ~ log.mean., data=FBN2)
lm2
lm3 <- lm(log.variance. ~ log.mean., data=FBN3)
lm3
lm4 <- lm(log.variance.~log.mean., data=FBN4)
lm4
plot(log(FBN1))
plot(log(FBN2))
plot(log(FBN3))
plot(log(FBN4))
r.FBN1=diff(log(FBN1))
r.FBN2=diff(log(FBN2))
r.FBN3=diff(log(FBN3))
r.FBN4=diff(log(FBN4))
plot(r.FBN1)
plot(r.FBN2)
plot(r.FBN3)
plot(r.FBN4)
m1=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN
1,include.mean=F,trace=F,cond.dist="std")
m2=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN
2,include.mean=F,trace=F,cond.dist="std")
m3=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN
3,include.mean=F,trace=F,cond.dist="std")
m4=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN
4,include.mean=F,trace=F,cond.dist="std")
m5=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN
1,include.mean=F,trace=F,cond.dist="std")
m6=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN
2,include.mean=F,trace=F,cond.dist="std")
m7=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN
3,include.mean=F,trace=F,cond.dist="std")
m8=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FBN
4,include.mean=F,trace=F,cond.dist="std")
m9=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FBN
1,include.mean=F,trace=F)
m10=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FB
N2,include.mean=F,trace=F)
m11=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FB
N3,include.mean=F,trace=F)
m12=garchFit(formula=~arma(1,1)+garch(1,1),data=r.FB
N4,include.mean=F,trace=F)
m13=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FB
N1,include.mean=F,trace=F)
m14=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FB
N2,include.mean=F,trace=F)
m15=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FB
N3,include.mean=F,trace=F)
m16=garchFit(formula=~arma(2,1)+garch(2,1),data=r.FB
N4,include.mean=F,trace=F)
summary(m1)
summary(m2)
summary(m3)
summary(m4)
summary(m5)
summary(m6)
summary(m7)
summary(m8)
summary(m9)
summary(m10)
summary(m11)
summary(m12)
summary(m13)
summary(m14)
summary(m15)
summary(m16)
residuals(m7)
kurtosis(residuals(m7))
library(evir)
y=residuals(m7)
ny=-y
meplot(ny)
m17=gpd(ny,threshold=0.05)
m17
plot(m17)
Selection:3
Selection:4
riskmeasures(m17,0.95)

```

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