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A Univariate Time Series Autoregressive Integrated Moving Average Model for the Exchange Rate Between Nigerian Naira and US Dollar

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Abstract: This research fit a univariate time series ARIMA model to the Monthly data of exchange rate between Nigerian Naira and US Dollar from January 1980 to December 2015. The Box-Jenkins Autoregressive Integrated Moving Average (ARIMA) model was estimated and the best fitted ARIMA model is used to obtain the post-sample forecasts for three years (January 2016 to December 2018). The data was analyzed with the aid of R statistical package and the best model was selected using Auto. ARIMA. The fitted model is ARIMA (0,1,1) with Akaike Information Criteria (AIC) of 2313.19, Normalized Bayesian Information Criteria (BIC) of 2325.39. This model was further validated by Ljung-Box test with no significant Autocorrelation between the residuals at different lag times and subsequently by white noise of residuals from the diagnostic check performed which clearly portray randomness of the standard error of the residuals, no significant spike in the residual plots of ACF and PACF. The forecasts value indicates clearly that Naira will continue to depreciate against the US Dollar between the periodsunderstudy.

Keywords: Arima, Time Series, Box- Jenkins, Ljung-Box, Stationarity, Unit Root, Naira, US Dollar

1. Introduction

Exchange rates are quoted as foreign currency per unit of domestic currency or domestic currency per unit of foreign currency. Exchange rate can also be seen as the price of one country's currency in relation to another country. It is the required amount of units of currency that can buy another amount of units of currency. It is the price in which one currency is exchange for another. It measures the domestic worth of an economy; especially in terms of the currencies of most industrialized countries such as United States of America Dollars, British Pound Sterling, German Duetsche Mark, Japanese Yen, French Frank, Italian Lira and the Canadian Dollar, [1]. According to [7], Exchange rate policy has been identified as one of the endogenous factors that can affect the economic performance of a nation.

In Nigeria, the management of the exchange rate is carried out by the central bank of Nigeria, following the adoption of the structural adjustment program policy in 1986, the country has moved from a pegged or rigid exchange rate regime to a more flexible regime; [3]. In practice, no exchange rate is "clean or pure float", that is a situation where the exchange rate is left completely to be determined by the market forces of demand and supply but rather the prevailing system is the managed float whereby the monetary authorities intervene periodically in the foreign exchange market of a country in order to attain some strategic objectives- [9]. Monetary policy has always been seen as a fundamental instrument over the years for the attainment of macroeconomic stability which is often seen as a prerequisite to achieving sustainable growth of output.

In the recent years, there has been considerable interest in modelling and forecasting exchange rate using the ARIMA model. The necessity for such an investigation arises from the fact that, the ARIMA model has come to play a very important role in the modelling of non-stationary time series data and can take into account the serial correlation found in time series dataset.

Investigation of the behaviour of daily exchange rates of the Indian Rupee (INR) against the United States Dollar (USD), British Pound (GBP), Euro (EUR) and Japanese Yen (JPY) from January 2010-April 2015 was carried by [4], They examine the predictability of these exchange rates using classical time series method (ARIMA), and complex nonlinear methods such as Neural Network and Fuzzy Regression Neurons. They concluded that, in predicting exchange rate market in India, ARIMA model does better than those of the complex nonlinear models.

The exchange rate between the Chana Cedi and the US Dollar from January 1994 to December 2010 was modeled by [2]. In their work, they developed ARIMA model using Box-Jenkins method of time series analysis and found out that ARIMA (1, 1, 1), model is the most suitable model for the data, and use the model to make two years forecast from January 2011 to December 2012 and found a depreciation of Chana Cedi's against the US Dollar.

The exchange rates of European Union Currency vs Romanian Leu (EURRON), United State dollar/Romanian Leu (USDRON), British Pound/Romanian Leu (GBPRON), Japanese Yen/Romanian Leu (JPYRON), Chinese Yuan/Romanian Leu (CNYRON), Russian ruble/ Romanian Leu (RUBRON) was forecasted by [8], using 80 daily observations taken from 3 January 2011 and 22 April 2011 with ARIMA method and EST. They find the appropriate models as ARIMA (1,0,0) for nEURRON, ARIMA (1,0,0) for USDRON, ARIMA (1,1,1) for GBPRON, ARIMA (4,0,6) for JPYRON, ARIMA (1,0,0) for CNYRON, ARIMA (1,1,3) for RUBRON. However, the authors conclude that EST gives more significant results than ARIMA.

The exchange rate between Naira and US Dollar taken Monthly data from January 1994 to December 2011 using ARIMA model was forecasted by [10]. Their result reveals that there is an upward trend and the second difference of the series was stationary (I(2)). Based on the selection criteria AIC and BIC, and the best model that explains the series was found to be ARIMA (1,2,1). They used the fitted model to forecast for the period of 12 Months terms which indicates that the Naira will continue to depreciate against the US Dollar.

The success of ARIMA model against Monetary Model, fitting the United State Dollar and Turkish Lira rate with the monthly observations taken from the dates between January 1980 and July 2001 was compared by [12],. They found out that ARIMA (3,1,2) is the most appropriate model for the series and concluded that ARIMA is more efficient in fitting United State Dollar and Turkish Lira rate compared to Monetary Model.

This research will contribute to the literature by estimating and forecasting the exchange rate between Naira (NGN) and US dollar (USD) using a univariate time series ARIMA model between the periods 1980 to 2015. The specific objectives are:

i. To evaluate the trend and changes between Nigeria Naira and United State Dollar from 1980 – 2015
ii. To fit a univariate time series ARIMA model to the data, and select the best model for the data
iii. To use the fitted model to make three years forecast.

2. Methodology

2.1. Model Specification

The model used in this study is the ARIMA model proposed by [5]. The preliminary test for stationarity and seasonality of the data was conducted in which differences (d) as well as transformation were taken. After the stationarity of the series was attained, the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF) of the stationary series were employed to select the order p and q of the ARIMA model. At this stage, different candidates' model manifested and their parameters were estimated using the maximum likelihood method. Based on the model diagnostic tests and parsimony, the best fitting ARIMA model is obtained. The Mathematical model for Auto Regressive of order p as well as that of Moving Average of order q is given respectively as

$$y_t = \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \cdots + \Phi_p y_{t-p} + \epsilon_t \quad (1)$$

And $y_t = \in_t - \theta_1 \in_{t-1} - \theta_2 \in_{t-2} - \cdots - \theta_q \in_{t-q}$ (2)

The ARMA process of order (p,q) is written as

$$y_t - \Phi_1 y_{t-1} - \Phi_2 y_{t-2} - \cdots - \Phi_p y_{t-p} = \in_t - \Phi_1 \in_{t-1} - \Phi_2 \in_{t-2} - \cdots - \Phi_q \in_{t-q} \quad (3)$$

2.2. Method of Estimation - ARIMA Methodology

The Box-Jenkins model building techniques consists of the following four steps:

Step 1: Preliminary Transformation: If the data display characteristics violating the stationarity assumption, then it may be necessary to make a transformation so as to produce a series compatible with the assumption of stationarity. After appropriate transformation, if the sample autocorrelation function appears to be nonstationary, differencing may be carried out.

Step 2: Identification: If y_t is the stationary series obtained in step 1, the problem at the identification stage is to find the most satisfactory ARMA (p,q) model to represent y_t.

[5] determined the integer parameters (p,q) that govern the underlying process y_t by examining the autocorrelations function (ACF) and partial autocorrelations (PACF) of the stationary series. [11] explained that it is better to entertain more than one structure for further analysis because the evidence examined at this stage does not point clearly in the direction of a single model [11] stated that this decision can be justified on the ground that the objective of the identification phase is not to rigidly select a single correct model but to narrow down the choice of possible models that will then be subjected to further examination.

Step 3: Estimation of the model

This deals with estimation of the tentative ARIMA model identified in step 2. The estimation of the model parameters can be done by the conditional least squares and maximum likelihood.

Step 4: Diagnostic checking: Having chosen a particular ARIMA model, and having estimated its parameters, the adequacy of the model is checked by analyzing the residuals. If the residuals are white noise; accept the model, else go to step 1 again and start over.

3. Analysis and Results

TIME SERIES GRAPH OF THE RAW DATA

Time series plots which display observations on the y-axis against equally spaced time intervals on the x-axis used to evaluate patterns and behaviours in data over time is displayed in Figure1 below. The data used for this research was sourced from Central Bank of Nigeria Statistical Bulletin. The data was analyzed with the aid of R statistical package.

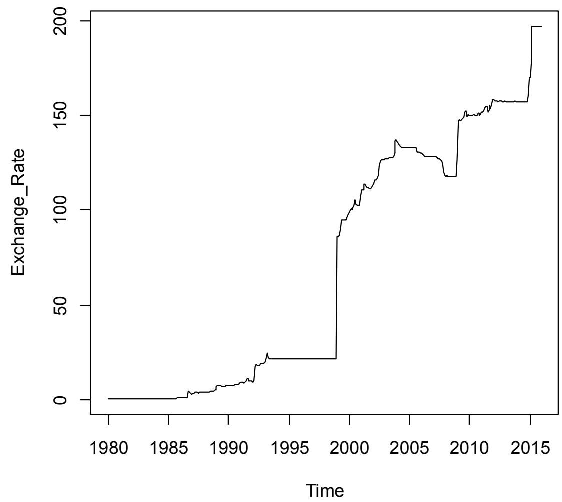

Figure 1. *Time Series plot of Naira-Dollar exchange rate from 1980 to 2015.*

Figure1 display the Time Series graph of Exchange Rate between Naira and US Dollar from January 1980 to December 2015. The researchers observed that while the exchange rate data maintained stability with the US Dollar from 1980 to 1986, there occur gradual changes in the exchange rate which continue to depreciate against the US Dollar at different stages over time. From 2005 to 2010 the Naira appreciated against the US Dollar, and started depreciating from 2010 till 2015. This behaviour of non-monotonous indicates that non-stationarity is inherent in the data. The formal test for stationarity was conducted to augment the graphical analysis. This test, conducted at 5% significance level is displayed in table 1.

Table 1. *Unit Root Test for Naira-Dollar Exchange Rate.*

Test type	Test Statistic	Lag order	p-value
“tseries” ADF	-2.2528	7	0.4707

The Augmented Dickey-Fuller (ADF) test for stationarity is shown in Table 1. The test shows the presence of a unit root in the data ($p>0.05$). This pattern indicates clearly that the series has to be transformed or differenced to stabilize or stationarize the data before its capability is assessed or before improvements are initiated. The stationarity of the data was however achieved at first difference.

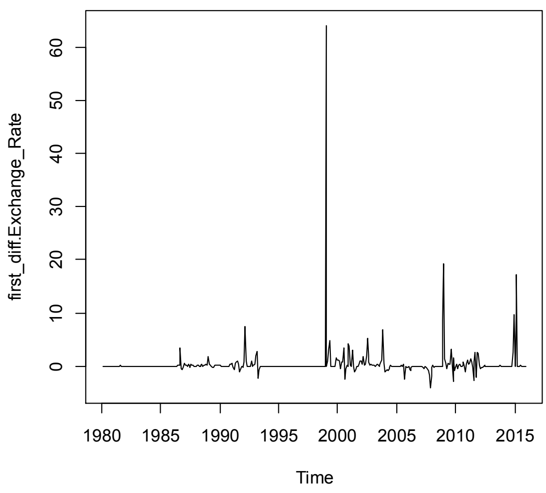

Figure 2. *Time Series Plot of the First Difference of Naira-Dollar exchange rate.*

Figure 2 shows the first difference of the data, the pattern of the data in Figure 2 indicates that the mean and variance of the series were stable over time. This pattern confers stationarity of the data at first differenced. The ADF test of staionarity in Table 2 also corroborates the graphical analysis that the series is stationary at first difference ($p<0.05$)

Table 2. *Unit Root Test for First Difference.*

Test type	Test Statistic	Lag order	p-value
ADF	-7.0722	7	0.01

Table 2 above depicts the unit root test for the first differenced of the data. The ADF test of staionarity in Table 2 also corroborate the graphical analysis that the series is stationary at first difference ($p<0.05$)

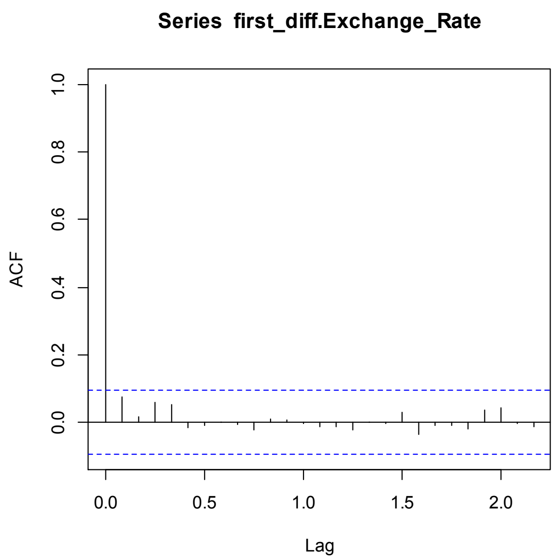

Figure 3. *ACF Plot of the First Difference.*

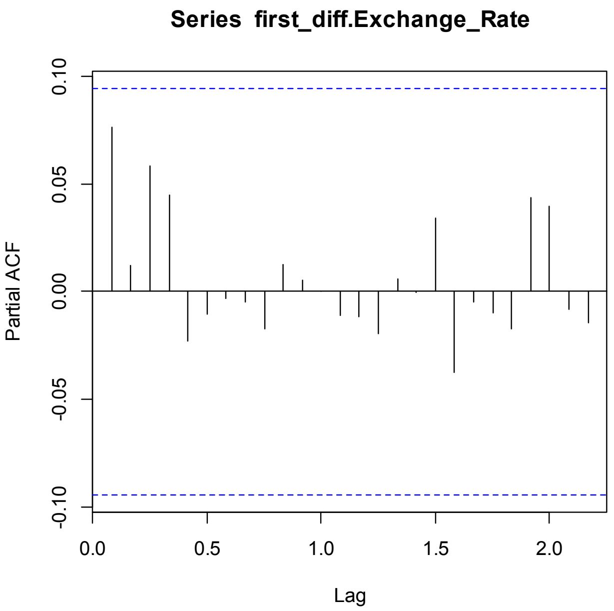

Figure 4. *PACF Plot of the First Difference.*

Figure 3 and figure 4 comprises the plots of Autocorrelation function (ACF) and Partial Autocorrelation function (PACF) of the series. If the PACF display a sharp cut-off while the ACF decay more slowly (i.e., has significant spikes at higher lags), we say that the series display an Autoregressive (AR) signature, however, if the ACF display a sharp cut-off while the PACF decay more slowly, we say that the series display a Moving Average(MA) signature. The lags at which the ACF cut off is the indicated number of MA order, while the lags at which the PACF cut off is the indicated number of AR order. From the graphs of this research, the ACF has a cut off at the first lag while there is no cut-off at PACF; this pattern is typical of ARIMA (0, 1, 1) model. But, [11] explained that it is better to entertain more than one structure for further analysis because the evidence examined at this stage does not point clearly in the direction of a single model, [11] stated that this decision can be justified on the ground that the objective of the identification phase is not to rigidly select a single correct model but to narrow down the choice of possible models that will then be subjected to further examination. As a result of this, therefore, the higher spikes of ACF and PACF in this research were considered for further examination. These models are: ARIMA (0,1,1) ARIMA (0,1,2) ARIMA (0,1,4) ARIMA (1,1,1) ARIMA (1,1,4) ARIMA (3,1,1) ARIMA (3,1,2) ARIMA (3,1,4) ARIMA (4,1,1) ARIMA (4,1,2). The estimates of these models were summarized in Table 3 below.

Table 3. *Summary Results And The Parameter Estimates of Possible ARIMA Models.*

ARIMA Structures	Parameter Estimates	p-value	S.E	AIC	BIC
ARIMA(0,1,1)	0.0746 0.4557	0.0123	0.0471	2313.19	2325.39
ARIMA(0,1,2)	*MA1= 0.0868 MA2=0.0199*	*0.07196956 0.65884860*	*0.0483 0.0451*	*2319.14*	*2331.345*
ARIMA(0,1,4)	*MA1=0.0836MA2=0.0261MA3=0.0685MA4=0.0664*	*0.08153814 0.58724254 0.15127288 0.16239120*	*0.04800.04800.04770.0475*	*2319.4*	*2339.739*
ARIMA(1,1,1)	*AR1= 0.7766MA1= -0.7030*	*9.785901e-05 1.848499e-03*	*0.19930.2258*	*2316.88*	*2329.09*
ARIMA(1,1,4)	*AR1=0.0404MA1=0.0435MA2=0.0228MA3=0.0675MA4=0.0638*	*0.9581216 0.9548292 0.7702054 0.1894196 0.3567822*	*0.76870.76740.07820.0514 0.0693*	*2321.4*	*2345.808*
ARIMA(3,1,1)	*AR1=0.5246AR2=-0.0196AR3=0.0639MA4=-0.4410*	*0.2111095 0.7620363 0.2497358 0.2931621*	*0.41950.06470.05550.4195*	*2319.82*	*2340.16*
ARIMA(3,1,2)	*AR1=0.6905AR2=-0.2854AR3=0.0842MA1=-0.6070MA2=0.2520*	*0.2558280 0.6176831 0.1962669 0.3188717 0.6394577*	*0.60760.57190.06520.6089 0.5378*	*2321.62*	*2346.026*
ARIMA(3,1,4)	*AR1=0.2644AR2=-0.3447AR3=0.3874MA1=-0.1808MA2=0.3492MA3=-0.297MA4=0.0244*	*0.7122448 0.6380213 0.6453716 0.8011092 0.6235849 0.7172157 0.8206213*	*0.71670.73270.84170.7177 0.71140.8200.1075*	*2325.21*	*2357.759*
ARIMA(4,1,1)	*AR1=-0.1111AR2=0.0354AR3=0.0695AR4=0.0695MA1=0.1947*	*0.8839246 0.6652221 0.1690954 0.2899031 0.7984512*	*0.76110.08180.05060.0657 0.7627*	*2321.3*	*2345.707*
ARIMA(4,1,2)	*AR1=-0.0334AR2=-0.0657AR3=0.0764AR4=0.0660MA1=0.1171MA2=0.0951*	*0.9703598 0.9284391 0.2884749 0.3713142 0.896497.2 0.8916525*	*0.89950.73160.07200.0738 0.90020.6983*	*2323.29*	*2351.77*

Table 3 contained the summary results and the parameters estimate of the possible ARIMA models. Comparing theNormalized Bayesian Information Criteria (BIC), and the Akaike Information Criteria (AIC) of the models, clearly prefers ARIMA (0,1,1) model as the best since it has the smallest AIC and BIC. In addition, the estimate of all the AR models were found to be statistically insignificant ($p>0.05$). Therefore the null hypothesis (H_o) of parameter is or equal zero is not rejected resulting in their removal from the model. The estimates of the MA model on the other hand, was found to be statistically significant ($p<0.05$). These attributes clearly prefers ARIMA (0, 1, 1) to other models.

Table 4 depicts the summary of the parameter estimates of ARIMA (0, 1, 1). The model is thus given as:

$$\nabla' Y_t = 0.4557 - 0.0746\varepsilon_{t-1} - \varepsilon_t \Rightarrow Y_t - Y_{t-1} = 0.4557 - 0.0746\varepsilon_{t-1} - \varepsilon_t \quad (4)$$

***Table 4.** Parameter Estimate of ARIMA (0, 1, 1) Model.*

R^2	BIC	AIC	
.997	*2325.39*	*2313.19*	
Coefficient	Estimate	S.E	*p-value*
MA 1	*0.0746*	*0.0471*	*0.0123*
Constant	*0.4557*		

This model is a special case of ARIMA model, which is called an Integrated Moving Average (IMA) Model.

***Table 5.** Ljung-Box Test of ARIMA (0,1,1).*

Test Type	Q-statistic	p-value
Ljung-Box	7.886	0.859

The fitted model was diagnosed by Ljung-Box test (Table 5), with (p>0.05), and therefore accepts the null hypothesis, thus the residuals appears to be uncorrelated. This indicates that the residuals of the fitted ARIMA (0,1,1) model is a white noise, and for that reason, the model fit the series quietly well, the parameter of the model is significant and the residuals are uncorrelated. Hence, the model is good for forecast.

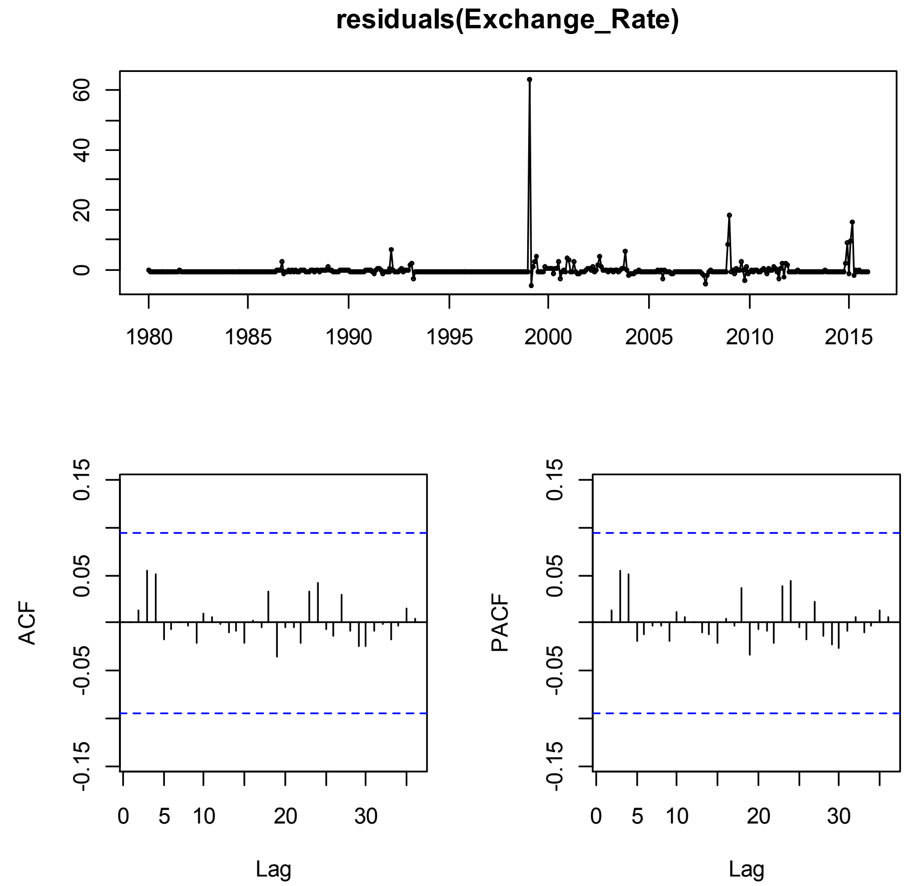

***Figure 5.** ACFAND PACF RESIDUAL (Z) PLOT FOR ARIMA (0,1,1).*

Figure 5 comprises the ACF and the PACF plots of the residuals, these plots shows no evidence of a significant spike (the spikes are within the confidence limits) indicating that the residuals seems to be uncorrelated. Therefore, the ARIMA (0,1,1) model appears to fit well so, the model is good to make forecasts.. This also shows that the residuals of ARIMA (0,1,1) model is white noise.

***Table 6.** Forecast of exchange rate between Nigerian Naira to United State Dollar from January 2016 to December2018.*

YEAR	POINT FORECAST	LCL	UCL
Jan 2016	197.4140	190.5048	204.3233
Feb 2016	197.8697	187.7276	208.0118
Mar 2016	198.3254	185.7562	210.8946
Apr 2016	198.7810	184.1828	213.3793
May 2016	199.2367	182.8589	215.6145
Jun 2016	199.6924	181.7102	217.6745

YEAR	POINT FORECAST	LCL	UCL
Jul 2016	200.1480	180.6935	219.6026
Aug 2016	200.6037	179.7805	221.4269
Sep 2016	201.0594	178.9522	223.1666
Oct 2016	201.5150	178.1944	224.8357
Nov 2016	201.9707	177.4967	226.4447
Dec 2016	202.4264	176.8510	228.0018
Jan 2017	202.8821	176.2508	229.5133
Feb 2017	203.3377	175.6909	230.9846
Mar 2017	203.7934	175.1670	232.4198
Apr 2017	204.2491	174.6755	233.8226
May 2017	204.7047	174.2134	235.1960
Jun 2017	205.1604	173.7782	236.5426
Jul 2017	205.6161	173.3675	237.8646
Aug 2017	206.0717	172.9795	239.1639
Sep 2017	206.5274	172.6126	240.4422
Oct 2017	206.9831	172.2651	241.7011
Nov 2017	207.4387	171.9357	242.9417
Dec 2017	207.8944	171.6234	244.1654
Jan 2018	208.3501	171.3269	245.3732
Feb 2018	208.8057	171.0455	246.5660
Mar 2018	209.2614	170.7782	247.7446
Apr 2018	209.7171	170.5242	248.9100
May 2018	210.1727	170.2828	250.0627
Jun 2018	210.6284	170.0534	251.2034
Jul 2018	211.0841	169.8354	252.3328
Aug 2018	211.5397	169.6282	253.4513
Sep 2018	211.9954	169.4313	254.5595
Oct 2018	212.4511	169.2443	255.6579
Nov 2018	212.9067	169.0667	256.7468
Dec 2018	213.3624	168.8981	257.8267

Table 6 depicts the forecast values between Naira and US Dollar for three years. We computed one-step ahead forecast with the fitted ARIMA (0, 1, 1), with 95% confidence limit and with minimum error as possible. The result shows an increase in trend between Naira and US Dollar. This clearly indicates that the Naira will continue to depreciate against the US Dollar within the three years period. The forecast plot in Figure 6 corroborates the increase in trend forecasts presented in Table 6.

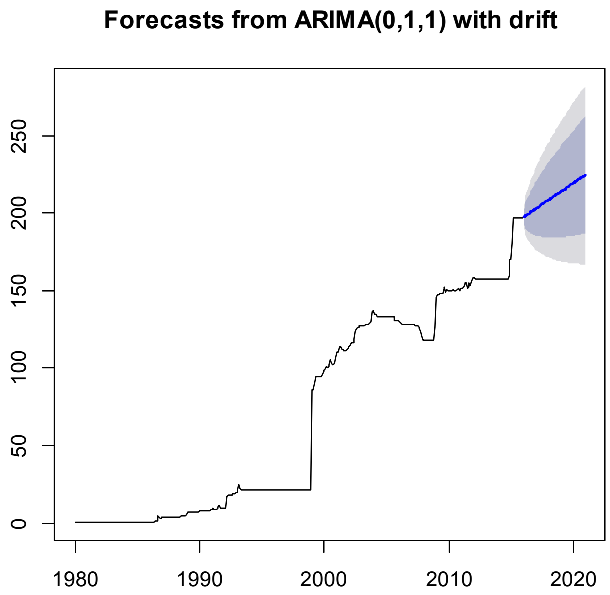

Figure 6. *Exchange rate of Naira-Dollar forecast plot.*

The forecast plot in Figure 6 corroborates the increase in trend forecast as presented in Table 6.

4. Conclusion

This research fit a univariate time series Autoregressive Integrated Moving Average (ARIMA) model to the exchange rate between Naira and the US Dollar using monthly data from January 1980 to December 2015. The evaluation of pattern shows that, while the exchange rate data maintained stability with the US Dollar from 1980 to 1986, there occur gradual changes in the exchange rate which continue to depreciate against the US Dollar at different stages over time. From 2005 to 2010 the Naira appreciated against the US Dollar, and started depreciating from 2010 till 2015.

The Box-Jenkins Autoregressive Integrated Moving Average (ARIMA) model was estimated and the best fitted ARIMA model is ARIMA (0, 1, 1). with Normalized Bayesian Information Criteria (BIC) of 2325.39, and Akaike Information Criteria (AIC) of 2313.19. This model was further validated by Ljung-Box test with no significant Autocorrelation between the residuals at different lag times and subsequently by white noise of residuals from the diagnostic checks performed which clearly portray randomness of the standard error of the residuals, no significant spike in the residual plots of ACF and PACF.

The fitted model was used to obtain the post-sample forecast for three years. The forecasting performance of Box-Jenkins models is accessed. The one-step ahead forecasts is computed with the fitted mode ARIMA (0,1,1). These forecasts and their 95% confidence interval i.e. lower confidence limit (LCL) and upper confident limit (ULC) for three years (i.e. 2016 to 2018) indicates that, Naira will continue to depreciate against the US Dollar within the forecasted period.

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