



Development of Mathematical Models for Predicting Customers Satisfaction in the Banking System with a Queuing Model Using Regression Method

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Abstract: The study evaluated the performance of customers in the banking system, using First Bank Nigeria Plc, Owo branch, Ondo state, as a case study, through the aid of queuing theory. The collected data for the analysis was basically for three consecutive days that comprises the arrival and service rate of customers at the bank. The optimal time of the overall server that determines how quick a customer is been attended to after arrival at the bank is also determined. Mathematical models were also developed from the queuing theory parameters that can predict customers' satisfaction in the bank using linear and non-linear model. This was achieved through the aid of a software package called Statistical Package for Social Science (SPSS), version 17. The findings showed that the queuing parameters (Ls, Lq, Ws, and Wq) used to measure customer satisfaction decreases with time as the number of lines increases. The optimal time of the overall server of the bank shows a significant effect because a customer will not be delay unnecessary after arrival to the bank. The developed models suggested the non-linear model as the best model to be recommended to the bank based on this research study because of it significant high value of coefficient of determination (R^2) and calculated F-statistic at 5% level of significance. Possible recommendation is made to the bank management to pay serious attention to this problem. Also, they should put it as a consideration that a line is better than more lines if the principle of fairness and technicality is embraced.

Keywords: Mathematical Models, Customers Satisfaction, Banking System, Queuing Model, Regression Method

1. Introduction

The commercial services have been a predominant sector of the economy where queuing system cannot be over emphasized with respect to the various service customers gets from a business establishment. Many commercial banks in Nigeria have done great effort to increase the service efficiency and customer satisfaction but most of them are facing a serious problem called customers waiting line. In bank, the waiting line of customers appears due to low efficiency of the queuing system, it reflects the lacking of the business philosophy of customer centric, low service rate of the system. The waiting queues of the customer develop because the service to a customer may not be delivered immediately as the customer reaches the service facility [1].

In a formal study of waiting in line and it entire discipline in operations management in Nigeria, [2] used regression

model to examine how queuing theory has been used in offering satisfactory service to customers in the banking industry using both primary and secondary data respectively. The findings of the study showed that most customers in banks are not satisfied based on the queue they experience before been attended to.

The increasing number of queuing theory which has been important tool in modelling a system that ensures efficient service delivery at the service counters and satisfaction of the service offered, in Kenya at JKUAT student finance office, [3] examined how effective the queuing model used when serving students at the finance office was related to customer satisfaction. The study compared the single server model against multi-server model. The findings of the study revealed that M/M/1 model was not the best for the finance department and that almost all customers are not satisfied about the nature of waiting lines which eventually turned

away some students at regular occasions due to the long queues.

The queuing system in Nigerian banks that involves lining up of customers in bank hall in order to be served by bank personnel at each terminal, made [4] to develop a web based application that can assigns each customer queue number on arrival based on touching the screen and the queue number are stored electronically. The aim was to minimize waiting time in queue by proper queue management and better service efficiency.

The formation of waiting lines as a common phenomenon that occurs whenever the current demand for a service exceed the current capacity provided to that service, made [5] to investigate the application of queuing theory to petrol stations in Benin-city, Nigeria. The results showed that queues exist in each of the considered petrol stations. The findings also revealed that the waiting time in the queue and service time at petrol filling stations decrease with increase in the number of servers.

In some province of Pakistan, [6] examined the behaviour and patterns of arrival of students in a university through observation method, considering student affairs department of different universities as the case study. the results showed that more than 70% of the students in universities are unhappy and dissatisfied with the service of the student affairs.

By applying a pure quantitative analysis using five big Nigerian banks as a case study within a framework called queuing technique, [7] investigated the impact of various elements of customer services adopted by the banks to improve the bank profitability in the banking industry. The findings showed that poor customer service management in the banks may reduce banks profitability and this may cause the bank financial distress.

In area of optimization, [8] proposed an optimized model to improve the banking queuing system based on queuing theory. The proposed method can optimize the number of server and improve the service efficiency that could effectively cut down service costs and customer's waiting time.

In the aspect of designing a proper queuing system, [9] analyzed the design and implementation of continuous process improvement in the banking system using queuing model, so as to build processing bank system effectively. The findings showed that the application of continuous process improvement theory to service industries has a good effect, but it should seriously consider the applicability and feasibility.

In organizational performance improvement, [10] sought to establish queuing models that can help organisations to improve on their customers' service within and outside their establishment. The study created new models using non-linear regression analysis which is more convenient for the organisations to assess. The findings showed that the coefficient of determination (R^2) value equal 1 and that the degree of correlation is 100 percent which indicates that 100 percent of the original uncertainty has been explained by the

model.

In Nigeria, [11] highlighted the imperativeness of maintaining an effective customer service delivery by attacking the notorious problem of long queue in service industry especially in Union Bank Plc. The study adopted a survey method with the aid of a structured questionnaire and oral interviews as an instrument of getting the data. The collected data was analysed using descriptive statistics. The findings showed that the long queue is caused by poor supervision of tellers and facilities which had consequently caused a high rate of drop of their customers to other banks.

However, the banking halls in Nigeria have been categorized by large population of customers queuing at various terminals without been prompt attended to. A lot of waiting line at the banks has generated complains by customers despite improvement by the bank management to control the kiosk at the banking hall. Research studies had revealed that a lot has not been extensively done in exploring on mathematical models that can minimize over crowdedness on waiting line (i.e. at various terminals) and increase customers' satisfaction. It is against this background that the study seeks to investigate customers' satisfaction in the banking system using predictive regression models.

2. Queuing Model Formulation

a. M/M/1 ∞ Model:

This queuing system is the simplest to analyse. The system consists of only one server. The arrivals follow Poisson distribution with a mean arrival rate (λ) and the service time has exponential distribution with the average service rate of μ . $P_n = P(N=n)$, ($n=0, 1, 2, \dots$) is the probability distribution of the queue length.

Utilization factor, which is the fraction of time servers are busy:

$$\rho = \frac{\lambda}{\mu} \quad (1)$$

Expected number of customers in the system:

$$L_s = \frac{\rho}{1-\rho} \quad (2)$$

Expected number of customers waiting on the queue:

$$L_q = \frac{\rho^2}{1-\rho} \quad (3)$$

Expected waiting time of customers in the queue:

$$W_q = \frac{\rho}{\mu-\lambda} \quad (4)$$

Expected waiting time of customers in the system:

$$W_s = \frac{1}{\mu-\lambda} \quad (5)$$

b. M/M/Z/ ∞ Model:

This model treats the condition in which there are several service stations in parallel and each customer in the waiting queue can be served by more than one station channel [8].

Consider an M/M/z queue with arrival rate λ , service rate μ and Z servers. The traffic intensity is defined usual by the ratio:

$$\rho_Z = \frac{\lambda}{Z\mu} \quad (6)$$

Xiao *et al.*, (2003) explained the steady distribution of a queuing system as follows:

$P_n = P(N=n)$, ($n=0,1,2,\dots$) is the probability distribution of the queue length N, as the system is in steady state, when the number of system servers is Z, then we have $\lambda_n = \lambda$, $n=0,1,2,\dots$

If there are n customers in the queuing system at any point in time, then the following two cases may arise:

- 1) If $n < Z$, (number of customers in the system is less than the number of servers), then there will be no queue. However, $(Z-n)$ number of servers will not be busy. The combined service rate will then be $\mu_n = n\mu$; $n < Z$.
- 2) If $n \geq Z$, (number of customers in the system is more than or equal to the number of servers) then all servers will be busy and the maximum number of customers in the queue will be $(n - Z)$. The combined service rate will be $\mu_n = Z\mu$; $n \geq Z$.

From the model the probability of having n customers in the system is given by:

$$P_0 = \left[\sum_{n=0}^{Z-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{Z!} \left(\frac{\lambda}{\mu} \right)^Z \frac{Z\mu}{Z\mu - \lambda} \right]^{-1} \quad (7)$$

$$P_n = \begin{cases} (\rho^n / n!) P_0 & n \leq Z \\ \rho^n / (Z! Z^{n-Z}) P_0 & n > Z \end{cases} \quad (8)$$

When $n \geq Z$, it is that the number of customers in the system is not smaller than the number of servers, the next customers must wait, that is,

$$C(Z, \rho) = \sum_{n=Z}^{\infty} P_n = \frac{\rho^Z}{Z!(1-\rho_Z)} P_0 \quad (9)$$

Expected number of customers waiting on the queue:

$$L_q = \left[\frac{1}{(Z-1)!} \left(\frac{\lambda}{\mu} \right)^Z \frac{\mu\lambda}{(Z\mu - \lambda)^2} \right] P_0 \quad (10)$$

Expected number of customers in the system:

$$L_s = L_q + \frac{\lambda}{\mu} \quad (11)$$

Expecting waiting time of customers in the queue:

$$W_q = \frac{L_q}{\lambda} \quad (12)$$

Expecting waiting time of customers in the system:

$$W_s = \frac{L_s}{\lambda} \quad (13)$$

3. Model Specification

The following mathematical model will be used to predict the expected number of customers in the system (L_s), number

of customers waiting in the queue (L_q), waiting time of customers in the queue (W_q), and waiting time of customers in the system (W_s). The models are based on linear and non-linear (quadratic) regression model. The best model will be recommended to be used in this research study.

$$\widehat{L}_s = \beta_0 + \beta_1 \hat{\lambda} \quad (14)$$

$$\widehat{L}_s = \beta_1 + \beta_2 \hat{\lambda} + \beta_3 \hat{\lambda}^2 \quad (15)$$

$$\widehat{L}_q = \beta_4 + \beta_5 \hat{\lambda} \quad (16)$$

$$\widehat{L}_q = \beta_6 + \beta_7 \hat{\lambda} + \beta_8 \hat{\lambda}^2 \quad (17)$$

$$\widehat{W}_s = \beta_9 + \beta_{10} \hat{\lambda} \quad (18)$$

$$\widehat{W}_s = \beta_{11} + \beta_{12} \hat{\lambda} + \beta_{13} \hat{\lambda}^2 \quad (19)$$

$$\widehat{W}_q = \beta_{14} + \beta_{15} \hat{\lambda} \quad (20)$$

$$\widehat{W}_q = \beta_{16} + \beta_{17} \hat{\lambda} + \beta_{18} \hat{\lambda}^2 \quad (21)$$

4. Methodology

4.1. Optimization in the Bank Queuing System

The research study made use of First Bank Nigeria Plc, in Owo Local Government Area of Ondo State, Nigeria. Based on the queuing theory, the queuing challenge of the bank is studied under the following aspects with regards to the available data gotten from the field work:

- a) One line or more; and
- b) Optimal service station.

4.2. Data Analysis

- a) One line or more

The service station in the bank has a queue according to their schedule. For every arriving customer joining the queue, the arrival rate of each service station becomes $\lambda = \frac{\lambda}{2}$; this is also known as two scheduled queuing system. In other words, when there are two lines, the system can be considered as two isolated M/M/2 systems. If there is a line, the system will be M/M/1 and L_s , L_q , W_s and W_q will be calculated respectively and compared to know which one is more efficient from a technical point of view. The analysis is as follows:

The research study made use of observed data that was collected at the bank premises. The data was collected for three (3) consecutive days based on the arrival and service rate of customers to the bank. They are of the value of 173, 308, & 142, and 79, 142, & 91, respectively for arrival and service rate.

The arrival rate for the overall system for the three days is:

$$\lambda_T = \lambda = \lambda_1 + \lambda_2 + \lambda_3 \quad (22)$$

$$\lambda = 173 + 208 + 142 = 523$$

The service rate for the overall system for the three days is:

$$\mu_T = \mu = \mu_1 + \mu_2 + \mu_3 \quad (23)$$

$$\mu = 79 + 142 + 91 = 312$$

The number of servers (Z) or service station in the bank is 8. The waiting lines of the customers in the bank do not disturb the number of servers. A customer (s) might decide to change from a particular line to another, keeping the number of servers constant. The servers is constant all through in the bank system under review as long as it is not been altered. The number of servers varies from one bank to another with respect to their banking system. The number of servers in this researcher study is strictly for First Bank Nigeria Plc, Owo, Ondo State.

Assuming eight (8) waiting lines, the number of servers will be constant all through the lines irrespective of interchangeability.

When there is a line, $Z = 8, \lambda = 523, \mu = 312, \rho = \frac{523}{312}, n = 0, 1, 2, \dots, 7$.

$$P_0 = \left[\sum_{n=0}^{Z-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n + \frac{1}{Z!} \left(\frac{\lambda}{\mu} \right)^Z \frac{Z\mu}{Z\mu - \lambda} \right]^{-1}$$

Where

$$\sum_{n=0}^{Z-1} \frac{1}{n!} \left(\frac{\lambda}{\mu} \right)^n = 5.3438$$

$$P_0 = \left[5.3438 + \frac{1}{8!} \left(\frac{523}{312} \right)^8 \frac{8 \times 312}{(8 \times 312) - 523} \right]^{-1} = 0.1871000$$

$$L_q = \left[\frac{1}{(8-1)!} \left(\frac{523}{312} \right)^8 \frac{312 \times 523}{[(8 \times 312) - 523]^2} \right] \times 0.1870000 = 0.0000973$$

$$L_s = L_q + \frac{\lambda}{\mu} = 0.0000973 + \frac{523}{312} = 1.6763791$$

$$W_s = \frac{L_s}{\lambda} = 0.0032053$$

$$W_q = \frac{L_q}{\lambda} = \frac{0.0000973}{523} = 0.0000002$$

When there are two lines, $\frac{\lambda}{2} = 261.5, \mu = 312, \rho = 0.8381000$

$$L_s = \frac{\rho}{1-\rho} = 5.176652$$

$$L_q = \frac{\rho^2}{1-\rho} = 4.3385523$$

$$W_s = \frac{1}{\mu-\lambda} = 0.0198000$$

$$W_q = \frac{\rho}{\mu-\lambda} = 0.016596$$

Similarly, the analysis is same for, when the lines are 3, 4, 5, 6, 7, & 8 respectively.

Table 1. gives the details of the summary of the parameters of the bank queuing system.

Table 1. Parameters of the bank queuing system.

Number	λ	μ	L_s	L_q	W_s	W_q
1	523	312	1.6763791	0.0000970	0.0032053	0.0000002
2	261	312	5.1766520	4.3385523	0.0198000	0.0165960
3	174	312	1.2665000	0.7077000	0.0073000	0.0041000
4	130	312	0.7215000	0.3024000	0.0055000	0.0023000
5	104	312	0.5044000	0.1691000	0.0048000	0.0016000
6	87	312	0.3877000	0.1083000	0.0044000	0.0012000
7	74	312	0.3149000	0.0754000	0.00421	0.0010000
8	65	312	0.2650000	0.0555000	0.0041	0.0008000

b). Optimal Service Station

The service station is also known as the number of server. An optimal service station is the probability that a customer will arrive at the bank premises and wait for service either more or less based on different purpose. The optimal service station of the bank queuing system is given as:

$$C(Z, \rho) = \sum_{n=Z}^{\infty} P_n = \frac{\rho^Z}{Z!(1-\rho_Z)} P_0 \text{ Or } \left(\frac{\lambda}{\mu} \right)^Z \frac{P_0}{Z!(1-\frac{\lambda}{Z\mu})}$$

Where: $Z = 8, \rho = \frac{523}{312} = 1.6762821, P_0 = 0.1871000$

$$C(Z, \rho) = \sum_{n=Z}^{\infty} P_n = \frac{\rho^Z}{Z!(1-\rho_Z)} P_0 = 0.0003660 \text{ or } 0.03660\%$$

4.3. Estimated Regression Model

The following are the estimated regression models based on the queuing system of the bank. The models are based on linear and non-linear (quadratic) regression model.

Expected number of customers in the system:

$$L_s = 0.389 + 0.005\lambda \quad (24)$$

$$R^2 = 0.225; F_{cal} = 1.737; Prob. = 0.236$$

$$L_s = -2.856 + 0.042\lambda - 6.291E - 5\lambda^2 \quad (25)$$

$$R^2 = 0.742; F_{cal} = 7.190; Prob. = 0.034$$

Expected number of customers in the queue:

$$L_q = 0.387 + 0.002\lambda \quad (26)$$

$$R^2 = 0.038; F_{cal} = 0.237; Prob. = 0.643$$

$$L_q = -2.857 + 0.039\lambda - 6.290E - 5\lambda^2 \quad (27)$$

$$R^2 = 0.680; F_{cal} = 5.309; Prob. = 0.058$$

Expected waiting time of customers in the system:

$$W_s = 0.006 + 5.069E - 6\lambda \quad (28)$$

$$R^2 = 0.021; F_{cal} = 0.126; Prob. = 0.735$$

$$W_s = -0.007 - 2.431E - 7\lambda^2 \quad (29)$$

$$R^2 = 0.729; F_{cal} = 6.719; Prob. = 0.038$$

Expected waiting time of customers in the queue:

$$W_q = 0.003 + 5.130E - 6\lambda \quad (30)$$

$$R^2 = 0.021; F_{cal} = 0.128; Prob. = 0.128$$

$$W_q = -0.010 - 2.439E - 7\lambda^2 \quad (31)$$

$$R^2 = 0.732; F_{cal} = 6.827; Prob. = 0.037$$

Table 2. shows the F-statistics values of the models against the test of significant at 5% level.

Table 2. F-statistics values of the models.

S/N	Model (s)	F-statistic	P-value	Test of sig.	Decision
1	$L_s = 0.389 + (0.005*\lambda)$	1.737	0.236	0.05	Not Significant
2	$L_s = -2.856 + (0.042*\lambda) - (0.00006291*\lambda^2)$	7.190	0.034		Significant
3	$L_q = 0.387 + (0.002*\lambda)$	0.237	0.643		Not Significant
4	$L_q = -2.857 + (0.039*\lambda) - (0.00006290*\lambda^2)$	5.309	0.058		Significant
5	$W_s = 0.006 + (0.000005069*\lambda)$	0.126	0.735		Not Significant
6	$W_s = -0.007 - (0.000002431*\lambda^2)$	6.719	0.038		Significant
7	$W_q = 0.003 + (0.000005130*\lambda)$	0.128	0.128		Not Significant
8	$W_q = -0.010 - (0.000002439*\lambda^2)$	6.827	0.037		Significant

Source: Author computation from SPSS 17.

Decision: test of significant is taken as 0.05 (5%). If the p-value of the F-statistic of the formulated model is lesser or equal to the test of significant (0.05), model is therefore significant and if otherwise, model is not significant.

4.4. Discussion of Findings

Table 1 shows the parameters of the bank queuing system in this research study. The L_s , L_q , W_s , and W_q in the bank decreases as the number of lines increases. This values shows that in any banking services, in term of “First Come, First Serve”, based on the principle of fairness and technicality, a line is better than more lines in the banking structure. Also, to avoid unnecessary delay of customers in the bank for a particular transaction purpose or waiting lines based on the collected data; some mathematical models were formulated based on regression analysis that can determine the expected number or waiting time of customers in the queue while been attended to in the bank. The model was based on linear and non-linear models. The reason is to check which of the model will be recommended best for the bank. The findings show that the non-linear model will be recommended to the bank because of it high effect of coefficient of determination (R^2) and the p-values of the calculated F-statistic that is lesser than the test of significant at 5% level as shown in Table 2. The value of the optimal servers (0.03660%) shows a significant effect because there is practically no existence of queue in the bank because a customer arriving to the bank will not have to wait more before been attended to.

5. Conclusion

The principle of fairness and technicality should be emphasize more on the term “First Come, First Serve

(FCFS)” because a line is better than more lines. Hence, managers of the bank should pay serious attention on this banking system problem. The queuing theory parameters (L_s , L_q , W_s , and W_q) determines the effectiveness and efficiency of the bank as investigated in this research study. Also, we can see from the case study that the waiting time of customer is reduced, customer satisfaction is increased, and the estimated queuing model is feasible. Finally, the most important aspect of the models created in this work is that it shows the management of the bank that despite the limited number of staff they have, they can distribute the available staff among the units effectively and efficiently.

Nomenclature

ρ = Utilization factor

L_s = Expected number of customers in the system

L_q = Expected number of the customers waiting on the queue

W_q = Expected waiting time of customers in the queue

W_s = Expected waiting time of customers in the system

λ = Arrival rate

μ = Service rate

Z = Number of servers

P_n = Probability of exactly n customers in the system

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