

Two Body IsoElectronium Model of the Heliumic Systems

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Abstract: In preceding works, one of us (R. M. Santilli) has shown that, according to quantum chemistry, identical electrons cannot create the strong bond occurring in molecular structures due to their strongly repulsive Coulomb interaction; has constructed hadronic chemistry as a non-unitary covering of quantum chemistry solely valid at mutual distances of 10^{-13} cm; has introduced contact non-Hamiltonian interactions in the deep penetration of the wavepackets of valence electrons that overcomes said Coulomb repulsion, resulting in a strongly attractive bond of valence electron pairs in singlet called 'isoelectronium' and shown that the new valence bond allows an exact and time invariant representation of the binding energy of the hydrogen and water molecules. By using these advances and our inference that (from the fact that an atomic lone pair of electrons form a coordinate covalent bond identified by G. N. Lewis) the lone pairs of electrons are indeed isoelectronium, in this paper we present, apparently for the first time, a new structure model of the Helium atom under the name of Iso-Helium, in which the two electrons of a given orbital are strongly coupled into the isoelectronium that provided a quantitative description of Pauli exclusion principle. In particular, as a result of the strongly bound state of Santilli isoelectronium, the iso-Helium reduces to be a two-body system, thus admitting exact analytic solution. The presented analytic solution is applicable to all Helium-like systems. Using it we have calculated effective charge on the nuclei of Helium-like systems that are in excellent agreement with the literature values.

Keywords: Covalence, Isoelectronium, Helium, Hadronic Chemistry

1. Introduction

In quantum mechanics, the Helium atom (a two electron system) is cited as a first case wherein no analytic solution of the Schrödinger equation is possible. Therefore, in conventional quantum mechanics one has to go for approximate methods, such as perturbation and variation methods [1-4]. Indeed, each one of these two methods is a cumbersome mathematical technique obviously yielding merely approximate results. On the other hand, the solution of Schrödinger equation of Hydrogenic systems yields the concept of orbitals whose energy depends on the principal quantum number. As a result the energy gap between $1s$ and $2s$ orbitals is obtained as being quite large. Hence, as we go from a hydrogen atom to a Helium atom the second electron either should go to the $1s$ or to the $2s$ orbital. If it goes to the $1s$ orbital, then very strong electrostatic repulsion would make it an unstable atom. Whereas, if the second electron goes to the $2s$ orbital, then

most probably they would remain spin unpaired (Ortho-helium). In the latter event again it would be a chemically very reactive atom being a diradical. Even if we assume that the second electron of $2s$ orbital remains paired with $1s$ electron (Para-helium) still they being separated electrons would be vulnerable to chemical reactivity besides this state has been experimentally found as a higher excited state than the triplet first excited state. However, Helium is a noble atomic gas having practically no chemical reactivity. Hence, on both the counts of energetics and chemical reactivity there is only one choice that is of housing both the electrons in the $1s$ orbital. In quantum mechanics, this state is prescribed as allowed by the Pauli exclusion principle requiring the two electrons in the $1s$ orbital to be coupled in the singlet state (that is with spins antiparallel). But how this spin pairing imparts Helium atom the inertness towards chemical reactivity has not been so far addressed to. Unless the two electrons in the same orbital present themselves as one entity, say as a quasi-particle, they are no different than when they occupy two different orbitals with

opposite spins as far as chemical reactivity is concerned. However, the quantum mechanics remains silent on how two electrons of a given orbital remain stable in spite of very strong electrostatic repulsion between them. In fact, two electrons at a mutual distance of 62 pm will experience a *repulsive* force of about 60 nN¹.

Another fact that we need to consider is the case of lone pairs of electrons in the valence shell of the central atom of molecules. For example, in the case of NH₃ molecule there we have one lone pair of electrons on N atom. The electrons constituting the pair have never been observed to form two separate covalent bonds with other atoms. On the contrary the said lone pair forms another type of covalent bond named by G. N. Lewis as the directed or coordinate covalent bond (1913-1919). This led him to classify acids and bases as Lewis acids and Lewis bases [5]. The Lewis bases are the donor of a pair of electrons. Some of the examples of Lewis acids are BF₃, AlCl₃, SO₃, etc. and those of Lewis bases are NH₃, HO⁻, etc. In inorganic chemistry the Lewis bases are also termed as ligands that form complexes with transition metal ions, which are known as coordination compounds.

From the above mentioned facts we assert that the coordinate covalent bond formation can take place only if the lone pair of electrons exist as a quasiparticle. Otherwise there should have been at least a few examples of formation of two separate covalent bonds from a lone pair of electrons. This then implies that every spin paired electrons in a given atomic orbital must be existing as a quasiparticle.

We recall that the strong electrostatic repulsion between two electrons in an orbital was, for the first time, pointed out by Santilli (see monograph [6-9] and original papers quoted therein) in connection with his pioneering research on chemical bonding of hydrogen and water molecules. In essence, Santilli showed that, according to quantum chemistry, two identical electrons in the singlet coupling cannot create a valence bond because of the above indicated, very strong, repulsive Coulomb force. Therefore, Santilli constructed an invariant non-unitary covering of quantum chemistry under the name of *hadronic chemistry* and showed that such a covering theory does indeed permit the achievement for the first time of an *attractive* force between valence electron pairs in singlet coupling at mutual distances of the order of 10⁻¹³ cm. Santilli then proved that such an attractive force does indeed permit an essentially exact representation of the binding energies of the Hydrogen and water molecules. The central mechanism is provided by the first consistent representation of the contact, non-linear, non-local and non-potential interactions caused by the deep overlapping of the wave packets of the identical valence electrons which are outside the representational capabilities of quantum chemistry. The evident non-Hamiltonian character of the new interaction then mandates a *non-unitary* covering of quantum chemistry. When the said non-Hamiltonian interaction is treated with the suitable covering of the mathematics underlying quantum

chemistry, known as *isomathematics*, the deep mutual overlapping of the wave packets of the valence electrons in singlet coupling creates a force representable with the Hulthén potential which in the model adopted by Santilli is so strongly *attractive* to “absorb” the repulsive Coulomb potential, resulting in the first known *attractive* force between identical valence electron in singlet coupling², resulting in a new state nowadays known as *Santilli Isoelectronium* (IE). Santilli also indicated that, electrons are predicted to experience a “mutation” of their conventional characteristics when they are the constituents of the isoelectronium that he named *isoelectrons* technically due to the fact that they are no longer characterized by the Poincaré symmetry but by its non-unitary covering known as the *Poincaré-Santilli isosymmetry*. It should be stressed to avoid misinterpretations that *hadronic chemistry* solely applies at mutual distances of 10⁻¹³ cm called the “hadronic horizon” and less; it recovers uniquely and identically quantum chemistry for all its molecular studies for distances larger than 10⁻¹³ cm; and the basic axioms of hadronic chemistry are exactly those of quantum chemistry, solely subjected to a broader representation permitted by the novel isomathematics (see Ref. [6, 7] for details).

As stated above to achieve the chemical inertness it is not sufficient that two electrons be housed in the same atomic orbital with opposite spin but also they should exist as a one entity. This strongly points out that each pair of two electrons with opposite spin housed in an orbital necessarily exist as a single entity, which in all probability is none else but the isoelectronium proposed by Santilli in describing the covalent bonds of water and hydrogen molecules [10, 11]. For example, in the case of ammonia molecule there we have one lone pair of electrons in the valence shell of nitrogen which is chemically inert in the sense of conventional covalent bonding because it must be an isoelectronium (IE).

Hence, with the coining of the concept of isoelectronium by Santilli as described above we hereby assert that all spin paired electrons of atomic orbitals are individually isoelectronium (IE).

Thus our task is to incorporate the concept of IE in Helium and Helium-like systems. To avoid any ambiguity to persist, we have taken this opportunity to first take a stock of the interactions that are supposed to operate in these systems. We recall Santilli’s proposal and assertions that at atomic scales an IE may be treated as a quasi-particle hence Helium-like systems turns out as a two body problem namely the nucleus consisting of neutrons and protons as a point mass and the IE an another point mass. Both the nucleus and IE individually are of hadronic dimensions. On the lines of the conventional quantum mechanical approach we would be tempted to identify only two interactions, namely:

- 1) the electrostatic attraction between the IE, the proposed quasi-particle, and the nuclear protons and;
- 2) the electrostatic repulsion between the two isoelectrons of the IE.

But on little deeper pondering over the physical state of the

¹ The distance of 62 pm is the diameter of 1s orbital of Helium atom and if the two electrons are assumed to remain farthest away of each other then roughly they would be mutually 62 pm away (c.f. Tables 8 – 10 and also Appendix A).

² It should be recalled that according to Santilli’s model the triplet coupling of two electrons of a given orbital creates strongly *repulsive* hadronic forces.

system one easily realizes that because the size of an IE is of the order of 1 fm or even less there should exist an equally important and strong magnetostatic attraction between the two electrons of IE (so far in none of the atomic systems this interaction has been separately dealt with in determining the total potential energy of the system. Most probably because it is expected that the two electrons of a given orbital would try to lie as far distance apart as possible to minimize electrostatic repulsion but then at such separation distances the magnetostatic interaction would be insignificantly small to reckon with (c.f. Appendix A)). This interaction originates because each spinning electron acts as a tiny magnet (refer also to Appendix A). In addition to this we also realize that the formation of an IE is through a deep overlap of the wave packets of two electrons which, indeed, is a new phenomena. Because of it a new type of interaction comes into play and would obviously contribute in determining the total potential energy of the system. This latter contribution we have quantified using Hulthén potential through the tools of hadronic mechanics and it turns out as a repulsive hadronic contribution (c.f. Section 5). On the face of it, this appears to be in contradiction to the demonstration of Santilli that the corresponding hadronic force is attractive in nature. However, it gets reconciled on realizing that, in fact, Santilli is not treating the above stated magnetostatic interaction separately hence his hadronic contribution is bound to be attractive in nature³.

Thus in this paper, we apply the above advances to Helium and Helium-like systems apparently for the first time. As a result of the above considerations the IE can be considered in first approximation to be a single stable quasi-particle with spin 0, charge $2e$, rest energy of the order of 1 MeV, and an essentially null magnetic moment. Therefore, the structure model of the Helium and Helium-like systems, here proposed under the name of *iso-Helium-like systems*, has been earlier treated in all *quantum chemistry* texts with the sole exception of the short range interactions of the electron pair, which in this presentation has been treated for the first time by recognizing the short range very strong magnetostatic interaction and the covering known as *hadronic chemistry* to deal with the new hadronic force.

The advantages of this conception of the iso-Helium atom and hence iso-Helium-like systems are the following:

1) The hadronic model of the iso-Helium-like systems permits one of the few quantitative representations of Pauli exclusion principle because for the triplet coupling of two identical electrons in the same orbital there would be a strong repulsive magnetostatic interaction that would not allow IE formation hence no hadronic chemistry is warranted, thus only possibility that is left is the strong attraction due to the singlet coupling of two electrons in an orbital in excellent agreement with Pauli's exclusion principle.

2) To the best of our knowledge, the proposed model permits a quantitative representation of the lack of chemical

reactivity of Helium evidently because either the unpaired electrons or the paired but un-united electrons are not statistically available for conventional chemical reactions.

3) The proposed model permits, apparently for the first time, the reduction of the Helium atom and Helium-like systems to a *two-body system*, with ensuing analytic solutions. It should be, however, recalled that the IE is permanently stable only in first approximation in view of Heisenberg's uncertainty principle and other factors [6].

A brief description of isomathematics and hadronic mechanics we have given in our preceding paper on nuclear spins [12] and for the sake of brevity we are not repeating that subject matter herein. For details of hadronic chemistry the reader is advised to refer to the excellent reviews by Santilli [6] and Tange [7-9].

In Section 2 we provide a brief description of the concept of IE. In Section 3 we have presented the conventional quantum mechanical model of Helium and the Helium-like system, in Section 4 we have reformulated the Helium and Helium-like system by incorporating Santilli's IE and studied the corresponding implications both within and outside the hadronic horizon. In Sections 5 and 6 we have quantified the isopotential originating from the deep mutual overlap of the wave packets of the two isoelectrons of IE. Since we are treating IE as a quasi-particle which implies that it contributes either the null potential energy or a constant and presumably very small value of it to the net potential energy of the system⁴. It, therefore, legitimizes the assumption that the isopotential nullifies the algebraic sum of electrostatic and magnetostatic potentials of the two isoelectrons of the IE or results into a very small but constant value of it. As a result of it there ensues the quantum mechanical analytic solution of Helium-like systems. The details of this latter part has been described in Section 6. The wave functions so generated consists of radial and angular parts and consequently provide three quantum numbers namely principal, azimuthal and magnetic ones. The expression of energy so obtained is dependent on the principal quantum number, in exactly the same way as in the case of Hydrogenic system. The expressions so obtained are the same as those we obtain in the case of Hydrogenic system except that, instead of the reduced mass of the electron, there we have the reduced mass of the IE and instead of the nuclear charge, Z , there we have the effective nuclear charge, Z^* .

The results of our studies are tested with the estimation of the effective nuclear charge for the iso-Helium-like systems by using corresponding experimental values for ionization energies, which are presented in Section 6.1. Our calculations match exceedingly well with the effective nuclear charges obtainable from the Slater rule [13] and the more recent ones provided by Clementi et al using SCF theory [14-17]. However, our estimated values of effective nuclear charge are a shade superior over the above two earlier methods. Section

³ In the case of π^0 meson formation from an electron and a positron obviously the magnetostatic interaction would be strongly repulsive and it gets countered by the electrostatic attraction and new hadronic attraction originating from deep overlap of wave packets of these two particles.

⁴ Recall that in hydrogenic atoms the potential energy of nucleons to its net potential energy is assumed to contribute either null or a constant value not dependent on the distance between nucleus and extra-nuclear electron. Still it produces excellently well all the frequencies of emission spectra of such atoms.

6.2 presents our calculations of most probable radius of Helium-like systems that has been compared with the radius of hydrogenic systems. In the final Section 7 we have placed our concluding remarks.

This paper also includes three Appendices. In Appendix A we present the comparative calculations of electrostatic and magnetostatic potentials with varying distance between the two electrons. In Appendix B we illustrate the notion of the "trigger" needed to bring the two electrons inside the hadronic horizon and in Appendix C we present the energy of simple harmonic motion of the IE within the hadronic horizon.

2. The Concept of Isoelectronium

The Pauli exclusion principle states that no two electrons in an atom can have all the four quantum numbers identically same [1-4]. It, therefore, means that in an orbital two electrons can be housed only if their spin quantum numbers are different. However, the spin quantum number can only have two values $+1/2$ and $-1/2$, that we conventionally represent as up and down arrows because basically it is a vectorial quantity originating from the spinning of electrons (the charge particle), hence the dictate of the Pauli principle is that the two electrons of a given orbital are spin paired. Since the spinning electron generates a magnetic field perpendicular to the plane of the spin the net spin magnetic moment of two electrons remains zero in an orbital. The spins of the electrons, the corresponding spin quantum number values ($m_s = 1/2$ and $-1/2$), their magnetic moment vectors and their pairing with opposite spins are shown in Figure 1.

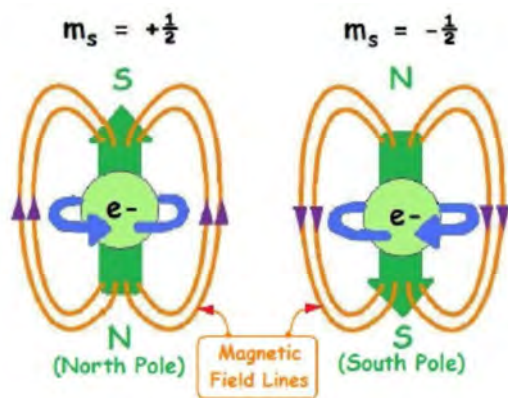


Figure 1. Schematic representation of the spins of two electrons, correspondingly generated magnetic fields and magnetic moment vectors and the spin pairing. The values of the spin quantum number, m_s , are also shown.

Thus the net magnetic moment equals to zero is evident from Figure 1.

Recall that, the strong hold of spin pairing is meticulously maintained in all quantum mechanical descriptions wherever two spinning fundamental particles are described in quantum mechanics but the physics of concomitantly implied strong interaction between the two particles is not at all attended in any quantum mechanical description. From Figure 1 it is evident that when two electrons are spin paired there originates strong magnetostatic attraction and simultaneously

due to their identical charges they must experience very strong electrostatic repulsion. Now for the time being if it is considered that the electrostatic repulsion nullifies the magnetostatic attraction when two electrons are housed in an orbital then it is easy to realize that the nullification of the said attractive and repulsive potentials would take place at a particular distance between two electrons, hence the assumption of their free movement about the nucleus within the extranuclear space cannot be guaranteed. Though, on an average for a collection of atoms the net magnetic moment would be zero. In Appendix A we have presented our calculations of the potential energies of electrostatic repulsion and magnetostatic attraction to illustrate their relative magnitudes.

Of course, on the one hand one would be tempted to assert that to minimize the electrostatic repulsion between the two electrons of 1s orbital they need to be farthest apart from each other, which according to the average diameter of 1s orbital of a Helium atom (that is 62pm) should be 62pm away. In this case to maintain the spin pairing of two electrons situated such a long distance apart would be extremely difficult (How can the Spin entanglement be considered at such a very short distances between the two spin paired electrons is debatable because there also exist very strong electrostatic and magnetostatic interactions.). On the other hand, in Appendix A we have seen that the null potential energy position (without involving Hulthén potential) is at $r_{null} = 0.2731 \text{ pm} = 273.1 \text{ fm}$ (which, indeed, is a very small a distance compared to the average diameter, 62pm, of 1s orbital of He atom). This would imply that in all doubly filled orbitals the two electrons remain situated at identically the same distance away from each other to fulfill the Pauli principle, which indeed is a too demanding a requirement as well as is a non-realistic one. Moreover, by considering the rest mass of an electron is entirely electromagnetic in origin J. J. Thomson in 1881 had calculated the radius of electron equal to $2.82 \times 10^{-13} \text{ cm} = 2.82 \text{ fm}$, which is termed as the classical radius of an electron [18]. Thus we see that the wave packets of two spin paired electrons would not even touch each other but should be about 265 fm apart to have null potential energy. Therefore, as stated above to maintain spin pairing at such a distance apart for two electrons is too stringent a requirement to be followed in reality.

From the above discussion it is clear that the Pauli principle cannot hold if the two electrons of the same atomic orbital are allowed to remain a distance away. Precisely for this reason Santilli proposed that the two electrons come so much close to one another that their wave packets overlap, that is, the centers of two electrons achieve a distance of about 1 fm or less (1fm is less than half of the predicted radius (2.82fm) of the electron by Thomson). Of course, this assertion of Santilli is in connection with two spin paired electrons of the covalent bonds of hydrogen and water molecules⁵. Then he succeeded

⁵ At a first glance one may have apprehension about the physical reality of the overlap of the wave packets of two electrons because of their identical charge that should cause strong repulsion. Hence, the concept of a trigger has been coined by Santilli (c.f. Appendix B). This proposed mutual overlap of wave packets of electrons can be compared with the quantum mechanics given concept of overlap of electronic atomic orbitals that forms a chemical bond. If one considers the electrostatic repulsion between any two electrons then the said overlap cannot be a

in unearthing a new kind of interaction that comes into play, which is of non-potential, non-local and non-Hamiltonian type originating from the physical contact between the two electrons by way of deep overlap of their wave packets. We recall that the small volume of the wave packets of electrons behave as a hyper-dense medium as asserted by Santilli.

This proposed mutual penetration of the wave packets of electrons has been termed as the mutation of electrons. Thus the union of mutated electrons, as stated above, has been given the name of IE and the two electrons are now termed as *isoelectrons*. The prefix iso- originates because the physics of this union can only be described by using Santilli isomathematics [19, 20]. Indeed, the IE formation has already been demonstrated by Santilli in molecular bonding of hydrogen and water molecules [6, 10, 11] and in Cooper pair [21, 22]. The schematic representation of coupling that forms IE is given in Figure 2.

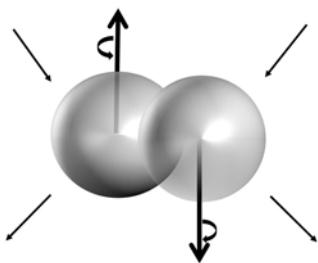


Figure 2. The schematic representation of isoelectronium by way of deep overlap of wave packets of two electrons at a distance of 1 fm or less.

An experimental evidence asserted by Santilli in support of the formation of IE is that in the Helium atom the two electrons are bounded most of the time, to such an extent that they are emitted in such a bonded form during photo-disintegrations, and in other events [6].

For detailed discussion on the concept of formation of IE the reader is directed to read the original sources namely [6, 10, 11]. However, in brief we state that the IE formation provides, for the first time, a quantitative theory of Pauli principle. *Due to the deep mutual overlap of wave packets of two electrons of an IE an entirely new type of interaction comes into play that Santilli identified as being of non-Hamiltonian, non-local and non-potential character.* To quantify it he has formulated hadronic mechanics by using his isomathematics. Consequently, its quantitative expression is obtained as Hulthén type potential, which at short distances behaves as

reality. However, in chemical bonding this apprehension is set aside on the grounds that the electrostatic attraction exerted by two nuclei on the two electrons forces the two electronic orbitals to overlap. In Santilli's language one can now say that it is the required trigger for forming a chemical bond. Moreover, as we have described in Appendix A when two electrons come close within hadronic volume then very strong magnrtostatic force of attraction starts exerting making the said overlap a reality. At the same time now a new hadronic effect comes into play that works to maintain balance between the attractive and repulsive forces within an IE. Another implication of IE when incorporated in the description of chemical bonding would be that the conventional overlap of the orbitals gets described as IE formation by the two valence electrons of the two atoms forming chemical bond. This subject matter we will discuss separately.

constant/r, that is as Coulomb type.

This is the conceptual basis of the Pauli principle applied to the two spin paired electrons in a molecular orbital. Accordingly, Santilli and co-workers have described the hadronic mechanics (a quantitative description) of π^0 meson, the IE of Cooper pairs and the covalent bonds of hydrogen and water molecules.

Now in this paper we are presenting our investigations using an approximation of IE formation (of the two spin paired electrons) in atomic systems of Helium and Helium-like atoms (see for example [23] and the original references cited therein).

3. Helium and Helium-Like Systems According to Quantum Chemistry

A Helium atom consists of two extranuclear electrons and, two protons and 2 neutrons in its nucleus as depicted in Figure 3 without neutrons as they are of no relevance in the present discussion.

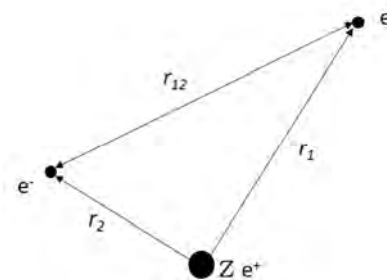


Figure 3. Schematic representation of Helium like atoms (Ions).

Therefore, there we have the following three potential energy terms, namely:

$$-\frac{Ze^2}{r_1}, \quad -\frac{Ze^2}{r_2}, \quad +\frac{e^2}{r_{12}} \quad (1)$$

where as depicted in Figure 3 r_1 and r_2 are the distances of the electron 1 and the electron 2 from the nucleus and r_{12} is the distance between two electrons. Thus in eq.(1) the first two terms are the attractive potentials and the third one is the repulsive one. Also as the distances r_1 , r_2 and r_{12} are of comparable magnitude thereby all the terms of eq.(1) are also of comparable magnitude. Notice that it is indeed a three body problem.

4. IE in Helium and Helium-Like Systems According to Hadronic Chemistry

Recall that, according to Santilli, an IE is formed when two electrons in singlet spin coupling occupy the same molecular orbital. The same proposal we now adopt also in the case of two spin paired electrons of an atomic orbital.

Recall also that the IE, by definition, is formed by the deep mutual overlap of the wave packets of two electrons. In singlet coupling (only), this wave overlap produces a new

contribution of non-potential, non-local and non-Hamiltonian type to the net potential energy of the system.

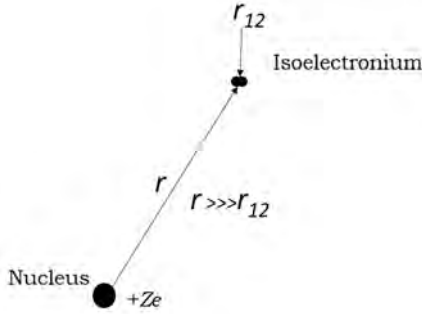


Figure 4. A schematic representation of two body model of Helium atom ($Z = 2$) with isoelectronium.

When we treat two spin paired electrons in an atomic orbital as a tiny quasi-particle its implication is that for all practical considerations we have $r_{12} \lll r_1$ and r_2 . In the conventional quantum mechanical approach the potential energy terms of this system would get represented as,

$$V = -\frac{2Ze^2}{r} + \frac{e^2}{r_{12}} - \frac{\mu_e^2}{r_{12}^3} \quad (2)$$

where we have used $r (= r_1 \equiv r_2)$ which is the distance between the IE and the nucleus and r_{12} is the internal distance of IE and by definition is many fold smaller compared to r . The last term on the right hand side of eq.(2) originates from the magnetostatic interaction that start significantly exerting at short distances between the two spin paired electrons of the IE because they behave as two tiny magnets (due to their spinning) obviously placed at a short distance apart⁶ and the magnitude of the corresponding interaction steeply increases as the distance between the two electrons decreases towards hadronic dimension (c.f. Appendix A).

In this way, we have approximated the three body model of the Helium into a two body model that we call IsoHelium. This gets pictorially represented as shown in Figure 4. Thus we see that, because of $r_{12} \lll r$ the second and third terms on the right hand side of eq.(2) completely overpower the first electrostatic term due to attraction between the IE and the protons of the nucleus and the net effect turns out as a strong attraction. *Even though there emerges a strong magnetostatic attractive potential on bringing two electrons within the hadronic distances the story of interaction still remains incomplete. This is so because the new phenomena of the deep mutual overlap of the wave packets of two electrons, bound to bring in the corresponding entirely new source of potential*

that in essence implies having the non-potential, non-local and non-Hamiltonian character; that imparts the status of a quasi-particle to so situated spin paired two electrons, which Santilli christened as an IE. However, as asserted by Santilli the state of deep mutual overlap of wave packets of two electrons implies their mutation that in his words is the realization of iso-electrons characterized by the Lorentz-Santilli isosymmetry [24, 25].

In order to quantify the said new potential originating within an IE we recall that the situation in Helium and Helium-like atomic systems is somewhat like an IE of hydrogen and water molecules described earlier by Santilli. In the present case the electrostatic attraction between the nucleus and IE serves as the required *trigger* [6, 10, 11] that forces two orbital electrons to form an IE (c.f. Appendix B).

In this model of iso-Helium atom and iso-Helium-like systems we have the following situation. The electrostatic and magnetostatic interactions between the two isoelectrons of IE still remain governed by the corresponding conventional laws. Whereas the attraction between the nucleus and the IE, and the kinetic energy of IE relative to the stationary nucleus involve distances many fold higher than 1 fm. Hence, the latter two aspects are needed to be treated by quantum mechanics and related conventional mathematics. That is they get adequately described by the conventional quantum mechanical tools. However, we have to quantify the new interaction originating from the deep overlap of the wave packets of two electrons of IE by the methods of hadronic mechanics. Thus we have a mixed situation of requiring to handle both the hadronic and the conventional interactions and two distances of non-comparable magnitudes. We recall that the case of π^0 meson is different in which we remain entirely well within the hadronic horizon because it has been postulated by Santilli as the hadronic union of isoelectron and isopositron [26].

However, as IE behaves as a quasiparticle all the interactions between two electrons of it should produce either a null potential or a very small value of it not dependent on the distance between nucleus and extra-nuclear IE (c.f. Section 5).

Accordingly, for the time being if we assume that the attractive, repulsive and hadronic potential energies within IE exactly get nullified, then for the system under consideration there are two inputs to the Hamiltonian, H , namely:

1. the kinetic energy, T , of IE relative to the nucleus,

$$T = \frac{p^2}{2\mu_{IE}} \quad (3)$$

where p is the momentum of the IE and its reduced mass, μ_{IE} , reads as,

$$\mu_{IE} = \frac{m_{IE} \times M_{nucleus}}{m_{IE} + M_{nucleus}} \quad (4)$$

where m_{IE} is the rest mass of IE and $M_{nucleus}$ is the rest mass of the nucleus.

2. the potential energy, V_i , is determined by the attraction between the effective nuclear charge and an electron of the

⁶ This magnetostatic potential has not been incorporated in any of the earlier quantum mechanical descriptions. Perhaps because its magnitude remains insignificantly small compared to the electrostatic repulsion between two free electrons of an orbital as the average distance between them is expected to be of the order of atomic dimension (c.f. Table 11 of Appendix A). Whereas in Santilli's model of a molecular orbital housing an IE it seems that by default this magnetostatic interaction gets covered within the hadronic Hulthén potential that he calculated hence it is no wonder that the result is an attractive hadronic potential.

IE⁷,

$$V_i = -\frac{Z^* e^2}{r} \quad (5)$$

where Z^* is the effective nuclear charge experienced by the IE.

It should be noted that, according to the model under consideration, the IE is seen from the nucleus of nuclear charge, Z^* , as one single entity with effective charge e and is treated quantum mechanically. Hadronic chemistry solely holds for the structure of the IE that necessitates the use of isomathematics. The rudiments of it we have described in the following Section 5.

5. The Isopotential in the Semi-IsoHelium-Like Systems

In order to understand the quantitative treatment of the strong hadronic *contact* force between two electrons in singlet coupling at very small mutual distances, it is essential to recall that such a force simply cannot be formulated via quantum mechanics and its known mathematical structure, e.g., via the conventional Hilbert space. This is due to the fact that the latter formulations can only represent interactions derivable from a potential, while *contact interactions have no potential by their very conception*. Consequently, the sole mathematically and physically consistent representation of an isolated IE is that via isomathematics with particular reference to the use of the Schrödinger-Santilli isoequation on an iso-Hilbert space over Santilli isocomplex isonumbers (see for example [6] and references cited therein). Therefore, we have the following three contributions in iso-Hilbert space, namely:

1. The non-linear, non-local, non-potential and non-Hamiltonian contribution due primarily to the deep mutual overlap of wave packets of two electrons we represent as,

$$\hat{g} = \hat{g}(\hat{r}_{12}) \quad (6)$$

2. The repulsive electrostatic interactions between the identical charge of two isoelectrons;

$$\hat{V}_{electrostatic} = +\hat{V}_e \hat{\times}_e (\hat{e}^2 \hat{\div} \hat{r}_{12}) \quad (7)$$

where $\hat{V}_e = V_e \times \hat{I}_e$ and

3. The attractive magnetostatic potential due to the two identical tiny isomagnets in singlet coupling, namely:

$$\hat{V}_{magnetostatic} = -\hat{V}_\mu \hat{\times}_\mu (\hat{\mu}^2 \hat{\div} \hat{r}_{12}) \quad (8)$$

where $\hat{V}_\mu = V_\mu \times \hat{I}_\mu$ and $\hat{\mu}$ is the iso-magnetic moment of

the isoelectron and μ is the conventional magnetic moment of an electron.

Recall that isoquantities are equal to the quantum quantities multiplied by the isounits \hat{I} , \hat{I}_e , \hat{I}_μ , etc., all isoproducts $\hat{\times}$, $\hat{\times}_e$, $\hat{\times}_\mu$, etc. are given by $\times \hat{I} \times$, $\times \hat{I}_e \times$, $\times \hat{I}_\mu \times$, etc., where \times is the conventional associative product, and the same holds for the isoquotients [19, 20].

Notice that the above three interactions are taking place within the hadronic space of an IE, which we are treating as a quasi-particle. However, this quasi-particle when treated as a part of a heliomic system it is justified to treat it as a point mass because the distance between the nucleus and the IE is many fold larger than the size of an IE (c.f. $r \gg r_{12}$). As stated above to the first approximation the interactions within a quasi-particle by its definition would not contribute to the potential energy of the heliomic system (even if it is a non-null contribution the net contribution would be a very small constant in the sense not dependent on r , the distance between nucleus and IE. Hence, our main result would not get altered in any way). Therefore, we are led to assume,

$$\hat{g}(\hat{r}_{12}) = \hat{V}_\mu \hat{\times}_\mu (\hat{\mu}^2 \hat{\div} \hat{r}_{12}) - \hat{V}_e \hat{\times}_e (\hat{e}^2 \hat{\div} \hat{r}_{12}) \quad (9)$$

However, it is profitable to use the projected version of eq.(9) on the conventional Hilbert space (the space to which the Helium-like system belongs). The said projections of each term of eq.(9) are⁸:

$$\hat{g}(\hat{r}_{12}) \rightarrow g(r_{12} \times \hat{I})$$

$$\hat{V}_\mu \hat{\times}_\mu (\hat{\mu}^2 \hat{\div} \hat{r}_{12}) \rightarrow \frac{\mu_e^2}{r_{12}^3}$$

$$\hat{V}_e \hat{\times}_e (\hat{e}^2 \hat{\div} \hat{r}_{12}) \rightarrow \frac{e^2}{r_{12}}$$

Therefore, the projected version on the conventional Hilbert space of eq.(9) reads as,

$$g(r_{12} \times \hat{I}) = g(\hat{T}(r_{12})) = h \times \hat{T}(r_{12}) = \frac{\mu_e^2}{r_{12}^3} - \frac{e^2}{r_{12}} \quad (10)$$

Notice that we have used $\hat{r}_{12} = r_{12} \times \hat{I}'$ and then absorbed the r_{12} dependence of g entirely in the isotopic element by defining new isotopic element, \hat{T} , equal to $r_{12} \times \hat{I}'$ that allowed us to adopt,

$$g(r_{12} \times \hat{I}') = g(\hat{T}(r_{12})) = h \times \hat{T}(r_{12}) \quad (11)$$

where $g(r_{12} \times \hat{I}')$ is the projected function on the Hilbert

⁷ Though we are considering the attractive electrostatic interaction between the nuclear charge and the two electrons of an IE but the use of the effective nuclear charge, Z^* , in eq.(5) implies that one of the electrons of the IE joins into the act of shielding of nuclear charge hence the factor 2 in the numerator of this equation is not appearing.

⁸ We have chosen isotopic elements, \hat{T}_e and \hat{T}_μ such that when \hat{V}_e and \hat{V}_μ are projected on the conventional Hilbert space the result is unity i.e. $V_e \times \hat{T}_e = V_\mu \times \hat{T}_\mu = 1$. This choice of mathematical handling of electrostatic and magnetostatic interactions between the two electrons of an IE implicitly assigns the entire quantification of hadronic effects to the term $\hat{g}(\hat{r}_{12})$ or equivalently to $g(r_{12} \times \hat{T})$.

space of $\hat{g}(\hat{r}_{12})$. Thus h is obtained as independent of r_{12} . In this way we need to quantify two quantities \hat{T} and h each positive definite.

Although the approximation of eq.(9) or that of eq.(10) seems to be purely ad hoc but its physical reality appears to be as described below. Let us consider the case of a diatomic molecule as an analogy. The separated atoms have three translational degrees of freedom individually. On forming the union, similar to a diatomic molecule, IE possesses only three translational degrees of freedom. Hence the remaining three translational degrees of freedom generate new degrees of freedom. They are two rotational and one vibrational (symmetric). The physical existence of both the new types of degrees of freedom in a molecule have been experimentally and unequivocally proved by IR, UV and microwave spectroscopies. On the same lines the separated electrons in an orbital will have in all six translational degrees of freedom corresponding to their motion within an orbital. However, on forming IE there still remains three translational degrees of freedom for its motion within the volume of an orbital. Hence the remaining three translational degrees of freedom get transformed to one vibrational and two rotational degrees of freedom of the IE. The reason of vibration of an IE can be traced out from the fact that at very short distances of hadronic scales the magnetostatic attraction between two isoelectrons over powers the electrostatic repulsion between them (c.f. Appendix A). In this way ultimately a dielectron would be formed by way of the complete superposition of the wave packets of the two electrons. But so far there is no experimental evidence of such a species. The physical non-realization of such a species lead us to conjecture and believe that there originates a new form of interaction coming into play due to the deep overlap of the wave packets of two isoelectrons of an IE. That starts countering the very strong magnetostatic interaction at such small distances i.e. 1 fm and less. That on the lines of Santilli we have quantified via the Hulthén potential. Because of this mutual countering of attractive and repulsive interactions within an IE it maintains the vibrational mode. Similarly, each IE will have two rotational modes. But we must realize that all the degrees of freedom of IE would be constrained by the very strong electrostatic attraction between the nuclear charge and IE. Perhaps still it would be permissible to use the tools of hadronic mechanics for describing vibrational motion of an IE on the lines of the quantum mechanics of the vibrational motion of a diatomic molecule and that we have described in Appendix C. On the other hand, the rotational motion of IE takes place in the conventional Hilbert space, that is within the entire volume of an orbital. Hence, this motion needs to be computed using quantum mechanical tools on the similar lines of rotational motion of a diatomic molecule. The striking difference would be the drastic reduction of moment of inertia in going from a molecule to an IE. This aspect we have not investigated yet. However, it is clear that an IE in a Helium-like systems possesses (and for that matter all spin paired electrons of atomic and molecular orbitals) vibrational,

rotational and translational energies⁹.

In view of the above approximations the net potential energy of the system is solely determined by,

$$V = -\frac{Z^* e^2}{r} \quad (12)$$

originating from electrostatic attraction between the nuclear charge and one of the electrons of the IE in extra-nuclear region. Of course, under the condition of eq.(10) we need to evaluate $\hat{T}(r_{12})$ and also assign a suitable expression to h .

Since, the IE formation is assumed as triggered by the electrostatic attraction between nuclear charge and the two electrons (c.f. Appendix B) hence to the first approximation we adopt the following expression,

$$h = \frac{Z^* e^2}{r} \quad (13)$$

Of course, h has been assumed as dependent on r and the latter is independent of r_{12} , hence the expression on the right hand side of eq.(13) serves as practically of constant magnitude within the hadronic volume.

The isounit \hat{I}' and its corresponding isotopic element are computed on the similar lines as it has been done in the case of an IE of hydrogen and water molecules [6, 10], that is,

$$\hat{I}' = e^{N \times \psi / \hat{\psi}} \approx 1 + N \times \frac{\psi}{\hat{\psi}}, \quad N = \int \hat{\psi}_{1\downarrow}^\dagger(r_{12}) \times \hat{\psi}_{2\uparrow}^\dagger(r_{12}) d^3 r_{12} \quad (14)$$

that gives,

$$\hat{T}' \approx 1 - N \times \frac{\psi}{\hat{\psi}} \quad (15)$$

where ψ is the conventional wave function of an electron, $\hat{\psi}$ is the iso-wave function of an isoelectron, the subscripts 1 and 2 refer to the two isoelectrons of IE and their spin paired state has been shown by up and down arrows in the subscript.

Moreover, the wave functions ψ , $\hat{\psi}$, $\hat{\psi}_{1\downarrow}^\dagger$ and $\hat{\psi}_{2\uparrow}^\dagger$ are the functions of r_{12} . That is the computation of isounit and the isotopic element is being carried out within the hadronic volume. Therefore, from the preceding equations, we have,

$$\hat{I}' \gg 1, \quad \hat{T}' \ll 1, \quad \lim_{r_{12} \gg 1\text{fm}} \hat{I}' = I \quad (16)$$

As the explicit form of ψ is of Coulomb type, it behaves as,

$$\psi(r_{12}) \approx N \times e^{-(b \times r_{12})} \quad (17)$$

where N is approximately constant at distances termed as the coherence length, r_c , of the IE ($= 1/b$), while $\hat{\psi}$ behaves like,

$$\hat{\psi}(r_{12}) \approx M \times \left(\frac{1 - e^{-(b \times r_{12})}}{r_{12}} \right) \quad (18)$$

⁹ Among them the translational energy appears to overwhelmingly dominate that gets quantified and is contained in the electronic energy of the Helium-like system given by eq.(24).

where M is also approximately constant within the same range of applicability [6].

Therefore, the isotopic element \hat{T}' of eq.(15) on using eqs.(17) and (18) gets expressed as,

$$\hat{T}' = 1 - \frac{N^2}{M} \times r_{12} \times \frac{e^{-(b \times r_{12})}}{1 - e^{-(b \times r_{12})}} = 1 - r_{12} \times V_0 \times \frac{e^{-(b \times r_{12})}}{1 - e^{-(b \times r_{12})}} = 1 - r_{12} \times V_{Hulthén} > 0 \quad (19)$$

where we have the constant $V_0 = N^2 / M$ and the Hulthén potential is given by,

$$V_{Hulthén} = V_0 \times \frac{e^{-(b \times r_{12})}}{1 - e^{-(b \times r_{12})}} \quad (20)$$

Yet another expression of $V_{Hulthén}$ under the condition of eq.(11) in combination with eqs.(10) and (19) is obtained that reads as,

$$V_{Hulthén} = \left(\frac{r}{(r_{12}^\dagger)^3} \times \frac{1}{Z^*} + \frac{1}{r_{12}^\dagger} \right) - \frac{\mu_e^2}{Z^* e^2} \times \left(\frac{r}{(r_{12}^\dagger)^5} \right) \quad (21)$$

where r_{12}^\dagger in eq.(21) is that value of r_{12} at which the net null potential is achieved.

Notice that r represents the most probable distance of IE from the nucleus. Obviously it would be different for different orbitals. That is the fixing of r implies considering IE in a given orbital. Thus we learn from eq.(21) that the magnitude of Hulthén potential varies from orbital to orbital and thereby the hadronic and conventional interactions between the two isoelectrons of IE also change from orbital to orbital implying that r_{12} also remains deferent for deferent orbitals.

6. Solution of Schrödinger Equation for the Two Body Approximation of Helium-Like Systems

Under the above two body model approximation in which we have adopted the nullification of attractive and repulsive potentials of two isoelectrons of an IE by the new isopotential, the Schrödinger equation reads as,

$$\left(\frac{1}{2\mu_{IE}} p \times p - \frac{Z^* e^2}{r} \right) \times |\psi\rangle = E \times |\psi\rangle \quad (22)$$

It is to be noted that as we are working within the conventional Euclidean space there is no need to use iso-wave function. The use of isotopy is required only for the quantification of the isopotential between the spin-paired isoelectrons in an orbital. Moreover, in eq.(22) there we have kinetic energy of IE with respect to nucleus hence there appears reduced mass of the IE.

Now eq.(22) gets further transformed to,

$$\nabla^2 |\psi\rangle + \frac{2\mu_{IE}}{\hbar^2} \left(E + \frac{Z^* e^2}{r} \right) \times |\psi\rangle = 0 \quad (23)$$

Notice that eq.(23) resembles the Schrödinger equation for a hydrogenic atom.

Therefore, we can directly use the standard solutions already available in literature for hydrogen atom or for a single electron systems [27, 28], of course, with appropriate changes. Thus the energy E is given by,

$$E = - \frac{2\pi^2 \mu_{IE} (Z^*)^2 e^4}{n^2 \hbar^2} \quad (n = 1, 2, 3, \dots) \quad (24)$$

where n is the radial or principal quantum number that can only have integer values. Notice that the energy given by eq.(24) is for the double occupancy of an orbital because in its numerator we have reduced mass of IE (c.f. eq.(4)) which is composed of two isoelectrons. Similarly, the total wave function gets determined by three quantum numbers n , l and m_l , the conventional principal, azimuthal and magnetic quantum numbers respectively. As usual the solution of eq.(23) gives,

$$n = 1, 2, 3, \dots \quad (25)$$

$$l = 0, 1, 2, 3, \dots, (n-1) \quad (26)$$

$$m_l = l, l-1, l-2, \dots, 1, 0, -1, \dots, -(l-1), -l \quad (27)$$

We are tabulating some of the normalized spin paired doubly occupied wave functions in Tables 1 – 4 each one of them are obtained for the spin paired occupancy by two electrons.

Table 1. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{\hbar^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2 \times \mu_e$ for K shell.

Normalized spin paired doubly occupied wave functions
K Shell: $n = 1$
 $l = 0, m_l = 0$

$$\psi_{100} = \psi_{1s} \text{ or } 1s = \frac{1}{\sqrt{\pi}} \left(\frac{2Z^*}{a_0} \right)^{3/2} \exp \left(-\frac{2Z^* r}{a_0} \right)$$

Table 2. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{\hbar^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2 \times \mu_e$ for L shell.

Normalized spin paired doubly occupied wave functions
L Shell: $n = 2$
 $l = 0, m_l = 0$

$$\psi_{200} = \psi_{2s} \text{ or } 2s = \frac{1}{4\sqrt{2\pi}} \left(\frac{2Z^*}{a_0} \right)^{3/2} \cdot \left(2 - \frac{2Z^* r}{a_0} \right) \exp \left(-\frac{Z^* r}{a_0} \right)$$

$l = 1, m_l = 0$

$$\psi_{210} = \psi_{2p_z} \text{ or } 2p_z = \frac{1}{4\sqrt{2\pi}} \left(\frac{2Z^*}{a_0} \right)^{5/2} \cdot z \cdot \exp \left(-\frac{Z^* r}{a_0} \right)$$

$l = 1, m_l = 1$

Normalized spin paired doubly occupied wave functions

$$\psi_{211} = \psi_{2p_x} \text{ or } 2p_x = \frac{1}{4\sqrt{2}\pi} \left(\frac{2Z^*}{a_0} \right)^{5/2} \cdot x \cdot \exp\left(-\frac{Z^*r}{a_0}\right)$$

$$l=1, m_l=-1$$

$$\psi_{21(-1)} = \psi_{2p_y} \text{ or } 2p_y = \frac{1}{4\sqrt{2}\pi} \left(\frac{2Z^*}{a_0} \right)^{5/2} \cdot y \cdot \exp\left(-\frac{Z^*r}{a_0}\right)$$

Table 3. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{h^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional

Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2 \times \mu_e$ for M shell with $l=0$ and 1 .

Normalized spin paired doubly occupied wave functions

M Shell: $n=3$

$$l=0, m_l=0$$

$$\psi_{300} = \psi_{3s} \text{ or } 3s = \frac{1}{81\sqrt{3}\pi} \left(\frac{2Z^*}{a_0} \right)^{3/2} \left(8 \frac{(Z^*)^2 r^2}{a_0^2} - 36 \frac{Z^* r}{a_0} + 27 \right) \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

$$l=1, m_l=0$$

$$\psi_{310} = \psi_{3p_z} \text{ or } 3p_z = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{2Z^*}{a_0} \right)^{5/2} \cdot z \cdot \left(6 - \frac{2Z^* r}{a_0} \right) \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

$$l=1, m_l=1$$

$$\psi_{311} = \psi_{3p_x} \text{ or } 3p_x = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{2Z^*}{a_0} \right)^{5/2} \cdot x \cdot \left(6 - \frac{2Z^* r}{a_0} \right) \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

$$l=1, m_l=-1$$

$$\psi_{31(-1)} = \psi_{3p_y} \text{ or } 3p_y = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{2Z^*}{a_0} \right)^{5/2} \cdot y \cdot \left(6 - \frac{2Z^* r}{a_0} \right) \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

Table 4. Normalized spin paired doubly occupied wave functions of Helium atom/Helium-like systems, ψ_{nlm_l} with $a_0 = \frac{h^2}{2\pi^2 e^2 \mu_{IE}}$, the conventional

Bohr radius, where μ_{IE} is the reduced mass of the isoelectronium and is related as $\mu_{IE} = 2 \times \mu_e$ for M shell with $l=2$.

Normalized spin paired doubly occupied wave functions

M Shell: $n=3$

$$l=2, m_l=0$$

$$\psi_{320} = \psi_{3d_{z^2}} \text{ or } 3d_{z^2} = \frac{1}{81\sqrt{6}\pi} \left(\frac{2Z^*}{a_0} \right)^{7/2} (3z^2 - r^2) \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

$$l=2, m_l=1$$

$$\psi_{321} = \psi_{3d_{xz}} \text{ or } 3d_{xz} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{2Z^*}{a_0} \right)^{7/2} \cdot xz \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

$$l=2, m_l=-1$$

$$\psi_{321} = \psi_{3d_{yz}} \text{ or } 3d_{yz} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{2Z^*}{a_0} \right)^{7/2} \cdot yz \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

$$l=2, m_l=-2$$

$$\psi_{32(-2)} = \psi_{3d_{xy}} \text{ or } 3d_{xy} = \frac{\sqrt{2}}{81\sqrt{\pi}} \left(\frac{2Z^*}{a_0} \right)^{7/2} \cdot xy \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

$$l=2, m_l=2$$

Normalized spin paired doubly occupied wave functions

$$\psi_{322} = \psi_{3d_{x^2-y^2}} \text{ or } 3d_{x^2-y^2} = \frac{1}{81\sqrt{2}\pi} \left(\frac{2Z^*}{a_0} \right)^{7/2} \cdot (x^2 - y^2) \cdot \exp\left(-\frac{2Z^* r}{3a_0}\right)$$

Moreover, the shapes and orientations of these doubly occupied orbitals remains identically same as that is given by quantum mechanics for hydrogen-like systems (c.f. Figure 5). The only difference that results is the significant contraction of the orbitals in going from hydrogenic to heliumic systems (c.f. Tables 8 – 10 of Subsection 6.2.).

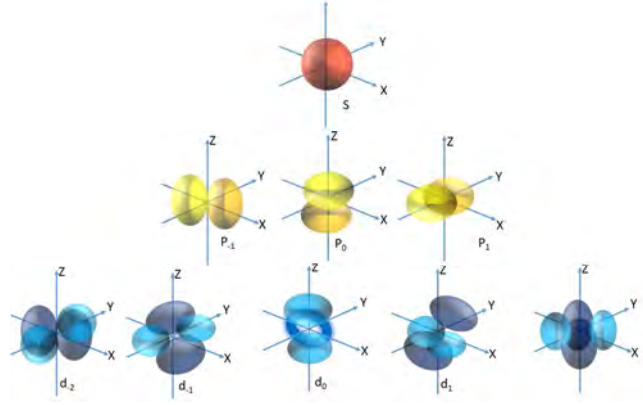


Figure 5. Various orbital shapes of hydrogenic systems.

The inspection of the expressions of wave functions given in Tables 1 – 4 establish that on occupation by spin paired two electrons none of the hydrogen like atomic orbitals change their shapes and spatial orientations. Only the size gets reduced appropriately. Hence, the change of terminology of orbitals in going from single electron orbitals to two spin paired electrons orbitals is not warranted. This is what is also reflected in Tables 1 – 4.

As the wave functions listed in Tables 1 – 4 are the double occupancy wave functions hence in going from $1s$ to any higher energy orbital represents the corresponding excited state. Thus the electronic configurations $2s^2$, $2p_x^2$, $2p_y^2$, $2p_z^2$, $3s^2$, $3p_x^2$, $3p_y^2$, $3p_z^2$, etc. are the various excited states. The term symbols for each of them is 1S_0 . Notice that these excited states would be formed by simultaneous excitation of two spin paired electrons to a single higher energy orbital retaining spin pairing. But electronic transitions are governed by Franck-Condon principle [27] hence as such, such excitations would be a rare phenomena thus they would be extremely unstable even if they are formed in traces. However, the utility of these Helium-like orbitals is in the building up of electronic configuration of successive elements of the periodic table according to the Aufbau principle, Pauli principle and Hunds rule (see for example the reference [27]).

The energy of an orbital is determined primarily by the principal quantum number, n , therefore, the energy sequence in increasing order of doubly occupied orbitals, from eq.(24), gets established as,

$$1s < 2s = 2p_x = 2p_y = 2p_z < 3s = 3p_x = 3p_y = 3p_z = 3d_{z^2} = 3d_{x^2-y^2} = 3d_{xy} = 3d_{yz} = 3d_{xz} < 4s \dots \quad (28)$$

which is identically same as that of Hydrogen-like systems.

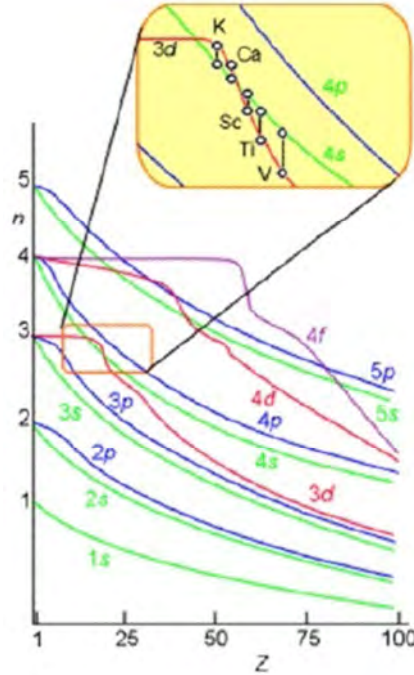


Figure 6. Energy of Hydrogen-like atomic orbitals with increasing atomic number, Z , which is identical for Helium-like atomic orbitals.

However, the helium-like wave functions given in Tables 1 – 4 are similar to the hydrogen-like wave functions and hence in the former the penetration of $2s$, $2p$, $3s$, $3p$, $3d$, etc. towards the nucleus would be similar to that we have in the

latter case (for the said penetration see for example [27]). Hence, on the same lines the degeneracy of orbitals in the given shell K , L , M , etc. gets split and the above sequence of energy broadly transforms to,

$$1s < 2s < 2p_x = 2p_y = 2p_z < 3s < 3p_x = 3p_y = 3p_z < 3d_{xy} = 3d_{yz} = 3d_{zx} = 3d_{x^2-y^2} = 3d_{z^2} < 4s \dots \quad (29)$$

Again this sequence is identically same which we have for hydrogen-like orbitals. More rigourously we will have the same energy sequence with increasing atomic number for Helium-like orbitals as that we have for Hydrogen-like orbitals depicted in Figure 6.

6.1. Effective Nuclear Charge of Helium Atom and Helium-Like Systems for $1s$ Occupancy

From eq.(4) in view of $M_{\text{nucleus}} \gg m_{\text{IE}}$ we practically have $\mu_{\text{IE}} \equiv m_{\text{IE}} = 2m_e$. Hence, eq.(24) effectively reads as,

$$E = -\frac{2\pi^2 m_e e^4}{n^2 h^2} \times 2(Z^*)^2 \quad (n = 1, 2, 3, \dots) \quad (30)$$

Therefore, for $n = 1$ eq.(27) would read as,

$$E_{1s} = E_{H(1s)} \times 2(Z^*)^2 \quad \text{where} \quad E_{H(1s)} = -\frac{2\pi^2 m_e e^4}{h^2} \quad (31)$$

which on rearrangement gives an expression of effective

nuclear charge, namely:

$$Z^* = \sqrt{\frac{E_{1s}}{2E_{H(1s)}}} \quad (32)$$

In terms of experimental ionization energies eq.(32) then reads as,

$$Z^* = \sqrt{\frac{I_{n+1} + I_n}{2 \times I_{H(1s)}}} \quad (33)$$

where I_{n+1} and I_n are the last two ionization energies.

Therefore, we have substituted experimental values of the relevant ionization energies [29] in eq.(33) that produced corresponding values of effective nuclear charges which we have tabulated in Tables 5 – 7 along with the values that is given by Slater rule [13] and those computed by Clementi et al [14-17] for direct comparison.

Table 5. Effective Nuclear Charge (Z^*) of Helium and Helium-like Systems for 1s Spin-paired Electrons from H to Ne.

Element/ Ion	Z^* Equation (33)	Z^* Slater Rule §	Z^* Clementi et al ‡
H ⁻	0.686	0.7	Not available
He	1.704	1.7	1.6875
Li ⁺	2.698	2.7	2.691
Be ²⁺	3.696	3.7	3.685
B ³⁺	4.695	4.7	4.680
C ⁴⁺	5.69	5.7	5.673
N ⁵⁺	6.695	6.7	6.665
O ⁶⁺	7.695	7.7	7.658
F ⁷⁺	8.696	8.7	8.650
Ne ⁸⁺	9.698	9.7	9.642

§ [13], ‡ [17]

Table 6. Effective Nuclear Charge (Z^*) of Helium and Helium-like Systems for 1s Spin-paired Electrons from Na to Ca.

Element/ Ion	Z^* Equation (33)	Z^* Slater Rule §	Z^* Clementi et al ‡
Na ⁹⁺	10.699	10.7	10.626
Mg ¹⁰⁺	11.702	11.7	11.609
Al ¹¹⁺	12.704	12.7	12.591
Si ¹²⁺	13.708	13.7	13.575
P ¹³⁺	14.711	14.7	14.558
S ¹⁴⁺	15.716	15.7	15.541
Cl ¹⁵⁺	16.721	16.7	16.524
Ar ¹⁶⁺	17.727	17.7	17.508
K ¹⁷⁺	18.733	18.7	18.490
Ca ¹⁸⁺	19.740	19.7	19.473

§ [13], ‡ [17]

Table 7. Effective Nuclear Charge (Z^*) of Helium and Helium-like Systems for 1s Spin-paired Electrons from Sc to Cu.

Element/ Ion	Z^* Equation (33)	Z^* Slater Rule §	Z^* Clementi et al ‡
Sc ¹⁹⁺	20.748	20.7	20.457
Ti ²⁰⁺	21.756	21.7	21.441
V ²¹⁺	22.766	22.7	22.426
Cr ²²⁺	23.776	23.7	23.414
Mn ²³⁺	24.788	24.7	24.396
Fe ²⁴⁺	25.800	25.7	25.381
Co ²⁵⁺	26.814	26.7	26.367
Ni ²⁶⁺	27.828	27.7	27.353
Cu ²⁷⁺	28.844	28.7	28.339

§ [13], ‡ [17]

The data in Tables 5, 6 and 7 clearly demonstrates that the effective nuclear charge for two isoelectrons as IE in 1s orbital in each case using eq.(33) excellently matches with that obtained using Slater rule and it nicely correlates with that of Clementi et al. Hence, it serves to demonstrate the credibility of our ad hoc assumptions that imparts a physical reality to our structured model of Helium atom and Helium-like systems. If one carefully compares the values of Z^* given in the second column with those listed in the fourth column of Tables 5, 6 and 7 it would be noticed that the values of the second column

are higher than those of fourth column and the difference between them increases as atomic number increases. To understand this trend we need to first realize that the Clementi et al values are obtained using SCF theory of quantum mechanics and in doing so a union of two electrons is not involved. Indeed, for a many body system one does not have analytic solution of Schrödinger equation and hence the SCF theory has been developed that involves the iteration technique and hence it is an approximate method. Still one can understand the said increasing difference between Z^* values. The separated two spin paired electrons of a given orbital would produce little higher shielding than the two spin paired electrons in the form of an IE. Therefore, as atomic number increases the the two electrons of 1s orbital become more tightly bound to nucleus and as a result their average distance from the nucleus continuously decreases (c.f. Tables 8 – 10 of Subsection 6.2). Therefore, it is easy to realize that if we assume the two spin paired electrons as unbound they would be more effective in shielding compared to the spin paired IE.

6.2. The Most Probable Radius of Helium Atom and Helium-Like Systems

The most probable radius, r^* , of 1s orbital of Helium atom and that of Helium-like systems is given by,

$$r^*(\uparrow\downarrow) = \frac{a_0}{2Z^*} \quad (34)$$

We have used the standard value of Bohr radius $a_0 = 52.9$ pm and the Z^* values calculated by eq.(33) using experimental ionization energies and tabulated in Tables 8 – 10. For the sake of comparison we have also tabulated the average radius $r^*(\uparrow) = a_0 / Z$ computed for the Hydrogen-like systems and tabulated in Tables 8 – 10.

Table 8. Most probable radius of Helium atom and Helium-like Systems and that of Hydrogenic Systems from H to Ne.

Element/ Ion	$r^*(\uparrow\downarrow)$ /pm	Element/ Ion	§ $r^*(\uparrow)$ /pm
H ⁻	77.11	H	52.9
He	31.04	He ⁺	26.45
Li ⁺	19.71	Li ²⁺	17.63
Be ²⁺	14.31	Be ³⁺	13.225
B ³⁺	11.27	B ⁴⁺	10.58
C ⁴⁺	9.30	C ⁵⁺	8.817
N ⁵⁺	7.90	N ⁶⁺	7.557
O ⁶⁺	6.87	O ⁷⁺	6.613
F ⁷⁺	6.08	F ⁸⁺	5.878
Ne ⁸⁺	5.45	Ne ⁹⁺	5.29

§ [27]

Table 9. Most probable radius of Helium atom and Helium-like Systems and that of Hydrogenic Systems from Na to Ca.

Element/ Ion	$r^*(\uparrow\downarrow)$ /pm	Element/ Ion	§ $r^*(\uparrow)$ /pm
Na ⁹⁺	4.94	Na ¹⁰⁺	4.809
Mg ¹⁰⁺	4.52	Mg ¹¹⁺	4.408
Al ¹¹⁺	4.16	Al ¹²⁺	4.069
Si ¹²⁺	3.86	Si ¹³⁺	3.779

Element/	$r^*(\uparrow\downarrow)$ /pm	Element/	${}^s r^*(\uparrow)$ /pm
P ¹³⁺	3.60	P ¹⁴⁺	3.527
S ¹⁴⁺	3.37	S ¹⁵⁺	3.306
Cl ¹⁵⁺	3.16	Cl ¹⁶⁺	3.112
Ar ¹⁶⁺	2.98	Ar ¹⁷⁺	2.939
K ¹⁷⁺	2.82	K ¹⁸⁺	2.784
Ca ¹⁸⁺	2.68	Ca ¹⁹⁺	2.645

§ [27]

Table 10. Most probable radius of Helium atom and Helium-like Systems and that of Hydrogenic Systems from Sc to Cu.

Element/	$r^*(\uparrow\downarrow)$ /pm	Element/	${}^s r^*(\uparrow)$ /pm
Ion	Equation (34)	Ion	
Sc ¹⁹⁺	2.55	Sc ²⁰⁺	2.519
Ti ²⁰⁺	2.43	Ti ²¹⁺	2.405
V ²¹⁺	2.32	V ²²⁺	2.30
Cr ²²⁺	2.23	Cr ²³⁺	2.204
Mn ²³⁺	2.13	Mn ²⁴⁺	2.116
Fe ²⁴⁺	2.05	Fe ²⁵⁺	2.035
Co ²⁵⁺	1.97	Co ²⁶⁺	1.959
Ni ²⁶⁺	1.90	Ni ²⁷⁺	1.889
Cu ²⁷⁺	1.83	Cu ²⁸⁺	1.824

§ [27]

If no screening effect operates then $r^*(\uparrow)$ should be equal to $2 \times r^*(\uparrow\downarrow)$. But from Tables 8 – 10 we see that it is not so. Actually we find that in all cases $2 \times r^*(\uparrow\downarrow) > r^*(\uparrow)$, that is the shrinking of the electron charge is less than if no shielding effect operated. However, as atomic number increases $2 \times r^*(\uparrow\downarrow)$ tends to approach $r^*(\uparrow)$.

It seems that our method of calculation of effective nuclear charge, that is experienced by two $1s$ electrons, via eq.(33) is superior over that of Clementi et al because the former method is based on the experimental ionization energies.

Yet another outcome of this presentation is that when we start filling atomic orbitals in accordance with the Aufbau principle we first place one electron in a hydrogen-like orbital and then when we fill the second electron guided by the Pauli principle it becomes a Helium-like orbital. Thus for example, in the case of Na the electronic configuration is $1s^2 2s^2 2p^6 3s^1$, in that $3s^1$ is the Hydrogen-like wave function and all the lower energy ones are the Helium-like wave functions. That is, in $1s^2 2s^2 2p_x^2 2p_y^2 2p_z^2$ there we have the occupancy by an IE in each orbital.

6.3. Lower Excited States of Helium Atom

We recall that energy of the three lower electronic states of He atom follows the following order, namely:

$$1s^2(\uparrow\downarrow)(^1S) < 1s^1(\uparrow)2s^1(\uparrow)(^3S) < 1s^1(\uparrow)2s^1(\downarrow)(^1S) \\ < 1s^1(\uparrow)2p^1(\uparrow)(^3P) < 1s^1(\uparrow)2p^1(\downarrow)(^1P) < \dots \quad (35)$$

We notice that the triplate state $1s^2(\uparrow\uparrow)(^3S)$ is not amongst the lower excited states. This is so because in this case there would be very strong magnetostatic repulsion. Hence both the electrostatic and magnetostatic strong repulsions would not

allow electrons to come close to form a union i.e. an IE. On the contrary the repulsive potential energy would be very high if two electrons are forced to occupy the small volume of the $1s$ orbital with parallel spins. However, from the next higher energy states the 3S state is of lower energy than the 1S state. To explain it we need to realize that in these two states the two electrons are in two different spherically symmetric hydrogenic orbitals hence no IE formation could take place. Therefore, no hadronic mechanics based explanation can be applied. Thus the conventional quantum mechanical explanation remains in order.

7. Concluding Remarks

This paper we dedicate to the memory of G. N. Lewis who coined the term coordinate covalent bond, which is formed by the donation of a lone pair of electrons to an acceptor molecule (1923) that led us to conclude that all spin paired electrons of atomic orbitals are isoelectronium.

In this paper first we have discussed in Section 1 the chemical reactivity and chemical bonding, particularly the coordinate covalent bond (G. N. Lewis), bases that strongly indicate that all spin paired electrons in each atomic orbital, indeed, exist as a quasistatic particle formed by the union of two electrons, which was named earlier by Santilli as isoelectronium.

As a result of this from hadronic physics and chemistry point of view the iso-Helium atom and iso-Helium-like systems constitute a problem in which we have to deal with two different levels of lengths and corresponding interactions. The IE formation is a phenomena occurring within the hadronic volume and because of it, not only there occurs electrostatic interaction between two electrons but also a very strong magnetostatic interaction comes into play between them. However, at the same time a new phenomena comes into play because of deep overlap of wave packets of two electrons that obviously produces correspondingly additional interaction, which has been demonstrated by Santilli as getting quantified by Hulthén potential. Whereas the kinetic energy of IE and its electrostatic interaction with the nucleus both involve the distances of atomic dimensions.

The crucial assumption permitted by the quasi-particle status imparted to an IE is that — on formation of an IE the electrostatic, the magnetostatic and Hulthén potential between the two isoelectrons of an IE produce an algebraic sum equal to zero or very small but of a constant magnitude potential. In view of it we have termed our model as an approximation. However, in this way the problem simplifies to a great extent that it resembles hydrogenic systems with a difference that instead of a single electron now we have an IE. That is, now we can use quantum mechanical tools to solve the corresponding Schrödinger equation. Thus it becomes a very simple system in which we need not to mathematically tackle various potentials between the two isoelectrons in solving Schrödinger equation. Of course, we have derived an expression of Hulthén potential under the condition of the said nullification of potentials of two isoelectrons (c.f. eq.(21)).

Indeed, under this assumption any variation in r_{12} would be insignificant compared to the magnitude of r that amounts to treating IE as a rigid union. However, it is unlikely that they would form a rigid union. That is, an IE must be an oscillating union about r_{12}^0 , the distance at which the said complete nullification of potentials takes place. This oscillatory motion we need to quantify using Schrödinger-Santilli isoequation. This we have described in Appendix C. We also see that according to eq.(21) r_{12}^0 will not be of same magnitude in different Helium-like orbitals.

Under this simplification the corresponding derived expression of energy of the atomic quantum states offered us to calculate the effective nuclear charge that is experienced by two electrons of $1s$ orbital of various Helium-like systems. Amazingly, our values match exceedingly well with earlier estimates based on SCF theory and Slater rule. Also we have calculated the most probable radius of $1s$ orbital of various Helium-like systems that as expected are smaller than those for Hydrogenic $1s$ orbitals and compares well with the radius of the latter in the sense that as atomic number increases the $1s$ orbital continuously shrinks with increasing nuclear charge.

The credibility of the approximations used in the two body model approximation of heliumic systems presented herein gets demonstrated by the calculated values of the effective nuclear charge using the input of the experimental ionization energies into eq.(33) — because the so calculated values are fantastically realistic. We have also explained the physical origin of the said assumption of null potential contribution to the total potential energy of the system.

With the demonstration herein of the credibility of Helium-like systems treated using IE and adopting corresponding implied approximations it is worth to extend the same approximation to describe

1. atomic structure of all elements of periodic table by considering every spin paired extranuclear electrons as an IE.
2. chemical bonding starting with hydrogen molecule, the simplest molecule housing an IE.

Finally a word on the difference between the Hadronic potentials of this paper and that appearing in Santilli's earlier works. In Santilli's works the hadronic potential is obtained as strongly attractive whereas that of the present paper it is strongly repulsive. As stated in the main text of this paper this seemingly opposite nature of the Hadronic potential gets easily reconciled. In the present model we have treated magnetostatic (strongly attractive) and hadronic (strongly repulsive) interactions between two isoelectrons of an IE separately. Whereas the mathematical model of Santilli's earlier works seemingly treats both the interactions as a single hadronic type. Therefore, it is no wonder that the hadronic potential of Santilli's earlier works is obtained as strongly attractive one.

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Appendix

A. Potential Energy of Two Electrons at a Distance Apart

1. The potential energy of repulsion, U_e^- , of two electrons at a distance r apart is given by,

$$U_e^- = \frac{e^2}{4\pi\epsilon_0 r} \quad (\text{A.1})$$

where e is the charge on an electron and ϵ_0 is vacuum permittivity. The standard values of the involed constants are:

$$4\pi\epsilon_0 = 1.112650 \times 10^{-10} \text{ J}^{-1} \cdot \text{C}^2 \cdot \text{m}^{-1}, \quad -e = 1.602192 \times 10^{-19} \text{ C}$$

On substitution of the numerical value of $r = 10^{-15} \text{ m}$ and using above values we obtain,

$$U_e^- = \frac{(1.602192)^2 \times 10^{-38} \text{ C}^2}{(1.112650 \times 10^{-10} \text{ J}^{-1} \cdot \text{C}^2 \cdot \text{m}^{-1})(10^{-15} \text{ m})} \quad (\text{A.2})$$

$$= 2.307122 \times 10^{-13} \text{ J} = 1.44 \text{ MeV}$$

2. The magnetostatic potential energy of a bar magnet of magnetic moment μ in the magnetic field intensity of \mathbf{B} is given by [30],

$$U = -\mathbf{B} \cdot \mu \quad (\text{A.3})$$

Hence, as above we consider that one electron is situated at a distance r in the magnetic field of the other spinning electron. Hence eq.(A.3) in this case reads as,

$$U_e^+ = -\mathbf{B}_e(r) \cdot \mu_e \quad (\text{A.4})$$

However, \mathbf{B} in terms of magnetic moment is given by,

$$\mathbf{B}(r) = -\frac{\mu_0 \mu}{2\pi r^3} \quad (\text{A.5})$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ J} \cdot \text{C}^{-2} \cdot \text{s}^2 \cdot \text{m}^{-1}$ is the vacuum permeability.

Recall that in an isoelectronium two spinning electrons have opposite spins, hence their magnetic moments are directed opposite to each other. Therefore, $\mathbf{B}_e(r)$ of eq.(A.4) is given by,

$$\mathbf{B}_e(r) = -\frac{\mu_0 (-\mu_e)}{2\pi r^3} \quad (\text{A.6})$$

However, it can be easily shown that

$$|\mu_e| = \mu_B = 9.274078 \times 10^{-24} \text{ C.s}^{-1}.\text{m}^2,$$

where μ_B is the Bohr magneton.

Hence, the potential energy of the magnetostatic attraction of two spin paired electrons on using eq.(A.4) gets expressed as,

$$U_e^+ = - \frac{\left[(9.274078)^2 \times 10^{-48} \text{ C}^2.\text{s}^{-2}.\text{m}^4 \right] (4\pi \times 10^{-7} \text{ J.C}^{-2}.\text{s}^2.\text{m}^{-1})}{2\pi (10^{-15})^3 \text{ m}^3} \quad (\text{A.8})$$

$$= -2 \times (9.274078)^2 \times 10^{-10} \text{ J} = -1.7201705 \times 10^{-8} \text{ J} = -107364.6 \text{ MeV}$$

3. Thus from eqs.(A.2) and (A.8) we see that $U_e^+ \gg U_e^-$.

4. Which gets quantitatively supported by,

$$|U_e^+ / U_e^-| = 7.5487226 \times 10^4 \quad (\text{A.9})$$

5. Whereas at $r = 1 \text{ pm}$ we have

$$U_e^+ = -1.7201705 \times 10^{-17} \text{ J} = -107.37 \text{ eV} \text{ and}$$

$U_e^- = 2.307122 \times 10^{-16} \text{ J} = 1440 \text{ eV}$. Thus the repulsive potential becomes much higher than the attractive one. It may then perhaps be treated as highly loose or unstable union.

6. At $r = 10 \text{ nm}$ we have

$$U_e^+ = -1.7201705 \times 10^{-29} \text{ J} = -1.074 \times 10^{-10} \text{ eV} \text{ and}$$

$U_e^- = 2.307122 \times 10^{-20} \text{ J} = 0.144 \text{ eV}$. That is the magnetostatic attraction becomes negligible compared to electrostatic repulsion.

7. In above calculations the null potential energy is obtained at,

$$r^2 = \frac{2 \times \mu_B^2 \times \mu_0 \times \epsilon_0}{e^2} = 7.4559 \times 10^{-26} \text{ m}^2 \quad (\text{A.10})$$

$$\Rightarrow r = 0.273055 \text{ pm}$$

and the attractive and repulsive potential energies have the magnitude of $8.4493 \times 10^{-16} \text{ J} = 5.2736 \text{ keV}$.

We present above results of calculations in Table 11.

Table 11. Electrostatic and magnetostatic potential energies of two electrons with varying distance of separation between them.

S. No.	r / m	U_e^- Repulsive (Electrostatic)	U_e^+ Attractive (Magnetostatic)
1.	10^{-15}	1.44 MeV	- 1.074 $\times 10^5$ MeV
2.	2.73055×10^{-13}	5.2736 keV	- 5.2736 keV
3.	10^{-12}	1.44 keV	- 1.0736 $\times 10^2$ eV
4.	10^{-8}	0.144 eV	- 1.9736 $\times 10^{-10}$ eV

B. The Notion of Trigger That Assist IE Formation

Let us briefly describe the concept of *trigger* that assist IE formation as elucidated by Santilli himself elsewhere [6].

1. A notion for the very existence of the IE is that of a *trigger*, namely, *external (conventional) interactions*,

$$U_e^+ = -\mu_B^2 \times \frac{\mu_0}{2\pi r^3} \quad (\text{A.7})$$

which get computed for $r = 1 \text{ fm}$ as,

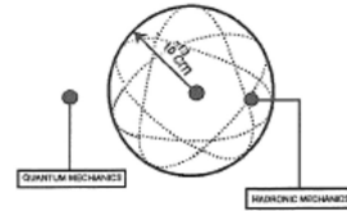


Figure 7. A schematic unit of the hadronic horizon, namely, of the sphere of radius 1 fm ($= 10^{-13} \text{ cm}$) outside which the quantum chemistry is assumed to be exactly valid, and inside which nonlinear, nonlocal and nonpotential effects are no longer negligible, thus requiring the use of hadronic chemistry for their numerical and invariant treatment.

which causes the identical electrons to move the one towards the other and penetrate into the hadronic horizon (Figure 7) against their repulsive Coulombic interactions.

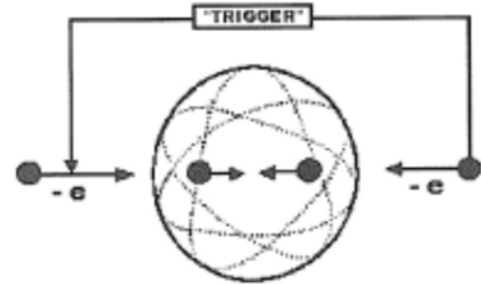


Figure 8. A schematic view of the trigger, namely, the external means suitable to force the electrons with the same charge to penetrate the hadronic barrier (c.f. Figure 7), in which the attractive hadronic forces overcome the repulsive Coulombic barrier.

Once inside the above mentioned horizon, the attractive magnetostatic force overcomes the repulsive Coulombic interaction, and at the same time a new interaction comes into play which is of hadronic origin due to the mutual overlap of wave packets of electrons, resulting in a bound state. The notion of the said *trigger* has been schematic represented in Figure 8.

2. In the case of the hadronic model of π^0 meson as a bound state of an electron and a positron at short distances, there is no need for an external trigger because the constituents naturally attract each other [26]. On the contrary, the existence of the Cooper pair does indeed require a trigger, which Santilli

and Animalu identified as being provided by the Cuprate ions [21, 22]. For the case of isolated hydrogen molecule the trigger has been identified in the two H-nuclei, which do attract the electrons [6,10] because the attraction of the electrons by the two nuclei is sufficient to cause the overlapping of the two wave packets of electrons.

C. IE as a Simple Harmonic Oscillator Within Hadronic Volume

As described in the main text and the Appendices A and B of the present paper the union of two electrons as IE is based on attractive and repulsive forces it cannot be a rigid union but would be an oscillating pair. Definitely the oscillatory motion of an IE takes place well within the hadronic volume. Therefore, an isolated harmonic oscillating IE about \hat{r}_{12}^0 , to the first approximation, will have the iso-potential energy $\hat{V}(\hat{r}_{12})$ given by,

$$\hat{V}(\hat{r}_{12}) = -U \times \frac{1}{2} k (r_{12} - r_{12}^0) \times U^? = -\left(\frac{\hat{1}}{2}\right) \hat{\times} \hat{k} \hat{\times} (\hat{r}_{12} - \hat{r}_{12}^0) \quad (C.1)$$

Therefore, the corresponding Schrödinger-Santilli isoequation applicable within the hadronic volume reads as,

$$\left[\frac{\hat{p}^2}{2\hat{\times}\hat{\mu}} \times \hat{I} - \left(\frac{\hat{1}}{2}\right) \hat{\times} \hat{k} \hat{\times} (\hat{r}_{12} - \hat{r}_{12}^0) \right] \hat{\times} |\hat{\psi}_{IE}\rangle = E_{IE} \times |\hat{\psi}_{IE}\rangle \quad (C.2)$$

where $\hat{\mu}$ is the reduced iso-mass of the IE, \hat{k} is the iso-force constant of the vibration and rest of the terms have their usual meanings. The solution of eq.(C.2) gives the following expression of the vibrational energy, namely:

$$E_{\hat{v}} = \left(\hat{v} + \left(\frac{\hat{1}}{2}\right) \right) \hat{\times} \hat{v} \quad \text{with} \quad \hat{v} = \hat{0}, \hat{1}, \hat{2}, \dots \quad (C.3)$$

and the isonormalized isowave-function reads as,

$$\hat{\psi}_{\hat{v}}(\hat{\xi}) = \left[2^{\hat{v}} \hat{\times} \hat{v}! \hat{\pi}^{\hat{1}/2} \right]^{-1/\hat{2}} \hat{\times} \hat{H}_{\hat{v}}(\hat{\xi}) \hat{\times} \exp\left(-\hat{\xi}^2 \hat{\div} 2\right) \quad (C.4)$$

where $\hat{\xi} = \hat{\beta}^{\hat{1}/2} \hat{\times} (\hat{r}_{12} - \hat{r}_{12}^0)$ is the displacement variable in iso-Hilbert space, $\hat{\beta}$ is an isotopically lifted constant $\beta = \frac{2\pi}{h} \sqrt{m_e k}$, m_e is the mass of an electron and $\hat{H}_{\hat{v}}$ is the iso-Hermite polynomial.

Now eq.(C.3) further simplifies to,

$$E_{\hat{v}} = E_v = \left(v + \frac{1}{2} \right) \hat{v} \quad (C.5)$$

where \hat{v} is the vibrational iso-quantum number. Notice that

\hat{v} is the isotopically lifted conventional vibrational quantum number v , which can have only the integer values and \hat{v} is the fundamental iso-vibrational frequency within the hadronic space of IE.

Thus we see that an IE will have a zero point energy equal to $(1/2)\hat{v}$. Also if required we can add anharmonicity terms in eq.(C.5) as we do in the case of molecular vibrations. Indeed, the vibrational energy of IE would be relatively very small compared to its orbital energy hence the orbital energy given by eq.(30) to the first approximation measures the energy of an IE in a given orbital. To ascertain this conjecture we need to design an appropriate experiment to see if there exists a vibrational fine structure in electronic spectra of say Helium atom in singlet-singlet transition. The similar fine structure in ground singlet-first excited triplet state is also expected but less prominently because in such excitation IE breaks down.

The same would be true for the rotational fine structure of IE. More so, the rotations and vibration of IEs would be constrained by the strong electrostatic attraction between the nuclear charge and IE. This aspect needs further investigation.

As stated in the main text the IE in a heliumic system is not an isolated species but is imbedded in the electrostatic field of nuclear protons hence the simple harmonic oscillation approximation within the hadronic volume needs to be modified accordingly. For the same reason the rigid rotor treatment for the heliumic IE needs to take account of this fact. However, so far we have not attended to it.

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