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Radar theoretical study: minimum detection range and maximum signal to noise ratio (SNR) equation by using MATLAB simulation program

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Abstract: This paper deals with the minimum detection range versus maximum signal to noise ratio (SNR) for several choices parameters like (peak power percent, radar cross section (RCS), antenna gain, coherently pulses, and duty cycle) by using MATALAB simulation program, these programs have been developed to make them easy for any user of evaluating the radar range and SNR equations and make them so faster and more convenient. After enter the input these parameters, the programs will calculate the detection range and SNR of a radar system and view the result as graphically.

Keywords: Radar Range Equation, Minimum Detection, Maximum SNR, Radar Low and High PRF, Antenna Gain

1. Introduction

Radar is an instrument that radiates electromagnetic wave in the space, which detects and locates of objects. Today, it is widely used for velocity estimation, imaging, and many other functions. The principle of radar operates likes to sound wave reflection. If any wave sound incident on the object (target) (like rocky canyon and cave), it will be reflected and heard, this sound wave reflecting is called echo. If sound speed is known, we can estimate the distance and direction of the objects [2, 3, 5, 14, 10].

Radar systems are composed of a transmitter that radiates electromagnetic waves of a particular waveform and a receiver that detects the echo returned from the target. Only a small portion of the transmitted energy is re-radiated back to the radar. These echoes will processed by the radar receiver to extract target information such as (range, speed, direction, position and others). The range to the target is evaluated from the travelling time of the wave. The direction of the target is determined by the arrival angle of the echoed wave. The relative velocity of the target is determined from the Doppler shift of the returned signal [8, 14, 18, 20].

Radar can be classified in terms of ground based, air borne and ship based radar systems. Also can be classified into numerous categories based on the specific radar characteristics, such as the frequency band, antenna type, and waveforms utilized, also classified by the types of waveforms or operating frequency [9, 14, 20].

The goal of this paper includes a discussion of several forms of the radar equations, and one of the equations of radar theory is the radar range equation, including those most often used in predicting radar performance. In this paper, we use MATLAB simulation program to representing the radar range equation, and plotting it versus with the signal to noise ratio (SNR) for several choices of parameters (like radar cross section – RCS , and peak transmit power – P_t), and their effecting on the detection range and SNR performance. Also refers to low pulse repetition frequency (PRF), and high PRF.

2. Range

The distance of the target (object) is determined by traveling of electromagnetic waves at speed of light, $(c = 3\times10^8 m/\text{sec})$. The target's range (R), is computed by measuring the time delay Δt , it takes a pulse to travel two way path between the radar and the target, as following[2, 3, 6, 8, 14, 18]:

$$R = \frac{\Delta t.c}{2} \tag{1}$$

The factor of (1/2) is needed for two way time delay. If the respective travel time Δt is known, the distance (R) between a target and the radar can be calculated by using this equation. In general, most functions of radar are time dependent. The time synchronization between the transmitter and receiver of a radar is required for range measurement. Radar radiate each pulse during transmit time (T) or pulse width τ, and wait for returning echoes during listening or rest time, and then radiate the next pulse, as illustrated in figure (1). The time between the beginning of one pulse and the start of next pulse is called pulse repetition time (PRT), and is equal to inverse of pulse repetition frequency (PRF) as follows:

$$PRF = \frac{1}{PRT} \quad (2)$$

The radar transmitting duty cycle (factor) (d_t) is defined as a ratio $d_t = \tau / T$. The radar average transmitted power is:

$$P_{ave} = P_t \times d_t \quad (3)$$

Where (Pt) indicate to the radar peak transmitted power. The pulse energy is:

$$E_p = P_t \tau = P_{ave} T = \frac{P_{ave}}{f_r} \quad (4)$$

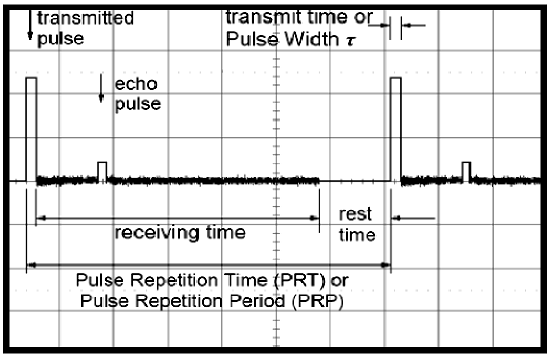


Figure 1. *Transmitted and received pulse.*

3. Theoretical Minimum Detection Range and Maximum SNR Equation

The radar range equation is not derived from first principle, but it has been developed from several steps. The total peak power (watts) developed by the radar transmitter (P_t), is applied to the antenna system. Consider the antenna had an isotropic or omnidirectional radiation pattern (one that radiates energy equally in directions), the power density (P_D) (Watt per square meter) at a distance (R) (meter) from the radiating antenna would be the total power divided by the surface area of a sphere of radius (R) [2, 3, 5, 6, 14, 8].

$$P_D = \frac{Peak\ transmitted\ power}{Sphere\ area} \quad \frac{Watt}{m^2}$$

The power density at range (R) away from the radar:

$$P_D = \frac{P_t}{4\pi R^2} \quad (5)$$

Where (P_t) is the peak transmitted power and $(4\pi R^2)$ is the area of sphere of radius (R). Radar systems use a directional antenna pattern in order to concentrate the power density in a certain direction, which is usually characterized by the gain (G), and the antenna effective aperture (A_e), they are related by:

$$A_e = \frac{G\lambda^2}{4\pi} \quad (6)$$

The antenna gain (G) is directly proportional to aperture, and the dimensions of an antenna depend on the gain (G) and wavelength (λ). The higher the frequency, the smaller the antenna, or the higher is its gain by equal dimensions.

The relationship between the effective of antenna aperture (A_e) and the physical aperture (A) is:

$$A_e = \rho A, \quad where \quad 0 \le \rho \le 1$$

ρ is indicated to aperture efficiency, when $\rho = 1$, the gain in transmitting is equal to receipting. When using a directive antenna of gain (G), then the power density is given by:

$$P_D = \frac{P_t G}{4\pi R^2} \quad (7)$$

When the incident transmitted signal collide on the target, the signal will induce time varying currents on the target so that the target now becomes a source of radio waves, part of which will propagate return at the radar, the power reflected by the target toward the radar (P_r) is defined as the product of the incident power density and the radar cross section (RCS), which is symbolized (σ) of the target, and is given by:

$$P_r = P_D \times \sigma \quad (8)$$

By substituting equation (7) in equation (8), we get:

$$P_r = \frac{P_t G \sigma}{4\pi R^2} \quad (9)$$

When the signal reflected from the target toward the radar systems over a distance (R), the power density (P_{Dr}) return at the radar is:

$$P_{Dr} = \frac{P_r}{4\pi R^2} \tag{10}$$

By substituting equation (9) in equation (10) we get:

$$P_{Dr} = \frac{P_t G\sigma}{(4\pi)^2 R^4} \tag{11}$$

The total power received (S) by antenna receiving of effective area of (A_e) from a target at range (R):

$$S = P_{Dr} \times A_e \tag{12}$$

From equations (6, 11, 12) we get:

$$S = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 R^4} \tag{13}$$

The received power (S), can be written in terms of signal-to-noise ratio (SNR), and thermal noise power (kT_0BF), where (k) is Boltzmann's constant and is equal to $1.38 \times 10^{?3}$, (T_0) is the noise temperature of the radar, (B) is the noise bandwidth of the radar receiver, and noise figure (F). Substituting in to equation (13), we get:

$$S_{\min} = kT_0BF(SNR)_{0_{\min}} \tag{14}$$

From equations (13) and (14) we get:

$$R_{\max} = \left[\frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_0BF(SNR)_{\min}}\right]^{1/4} \tag{15}$$

Or equivalently:

$$(SNR)_{\min} = \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 kT_0BFR^4} \tag{16}$$

Where $(SNR)_{\min}$ is the minimum detectable SNR of the system. It is an important relationship for radar designers. If greater detection range is desired, then significant improvements to antenna gain or transmitted power must be realized, and the other parameters are often fixed.

The (SNR) is signal to noise ratio units in decibel (*dB*), when transform the units from system international of units (*SIU*) to *dB* we use the relationship:

$$dB = 10\log_{10}(siu)$$

And inversion, when transform from (*dB*) to (*SIU*) we use:

$$siu = 10^{\left(\frac{dB}{10}\right)}$$

3.1. Low Pulse Repetition Frequency (PRF) Radar Equation

A low PRF is unambiguous in range but ambiguous in Doppler. Radar cannot be unambiguous in range and Doppler at the same time. It always depends on the (maximum range, atmospheric signal attenuation, transmitter power, etc) whether using a given PRF yields ambiguous ranges or not. In reality, a single pulse with a single return can't really be trusted as a reliable radar detection, for this reason radars usually send multiple pulses (n_p), and we can defined (n_p) as follows[1, 7, 14, 8]:

$$n_p = T_i f_r \tag{17}$$

Where (T_i) is the time that a target is illuminated by the beams, (n_p) is number of pulses that collides the target, and (f_r) is the PRF. For a low PRF, the signal pulse radar equation is:

$$(SNR)_1 = \frac{P_t G^2 \lambda^2 \sigma \times 1}{(4\pi)^3 R^4 kT_e BFL} \tag{18}$$

And for (*n*) pulses we get:

$$(SNR)_{n_p} = \frac{P_t G^2 \lambda^2 \sigma \times n_p}{(4\pi)^3 R^4 kT_e BFL} \tag{19}$$

Now , substitute equation (17) in equation (19) we get:

$$(SNR)_{n_p} = \frac{P_t G^2 \lambda^2 \sigma \times T_i f_r}{(4\pi)^3 R^4 kT_e BFL} \tag{20}$$

3.2. High Pulse Repetition Frequency (PRF) Radar Equation

High PRF radars are ambiguous in range, but is unambiguous in radial velocity. Thus when the receiver selects the central line, the spectrum is identical to the CW case. A high PRF gives greater range to the beam by putting more energy into each pulse. But, because multiple pulses will have emitted before one has returned from a long distance target, the range information cannot gives directly, for this reason we could computed from Doppler information. We consider high PRF radar that uses a periodic train of very short pulses. The pulse width is (τ) and the period is (*T*). This pulse train can be represented by using an exponential Fourier series. The central power spectrum line (DC components) for this series contains most of the signal's power. For a peak transmitted power (p_t), the average power in the central line is $(d_t)^2$, compare with the average power of the transmitted waveform of $(p_t d_t)$. Thus, the signal pulse radar equation for a high PRF radar (in terms of DC spectral power line) is[1, 4, 7, 10, 14, 19, 8, 11]:

$$SNR = \frac{P_t G^2 \lambda^2 \sigma\, d_t^2}{(4\pi)^3 R^4 kT_e BFLd_r} \tag{21}$$

Where, in this case, one can no longer ignore the receive duty factor since its value is comparable to the transmit duty factor. In fact, $(d_r \approx d_t = \tau f_r)$, it follows that:

$$SNR = \frac{P_t \tau f_r T_i G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_e F L} \quad (22)$$

and finally

$$SNR = \frac{P_{ave} T_i G^2 \lambda^2 \sigma}{(4\pi)^3 R^4 k T_e F L} \quad (23)$$

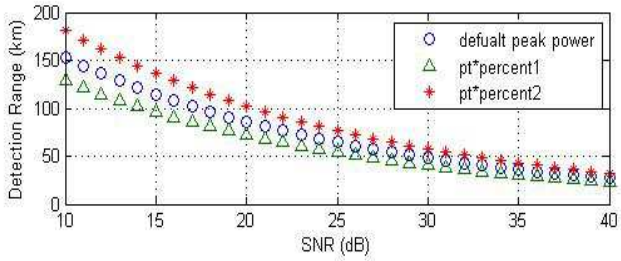
Where $P_{ave} = \tau f_r$, and $B = 1/T_i$ note the product $P_{ave} T_i$ is a "kind of energy" product, which indicates that high PRF radars can enhance the detection performance.

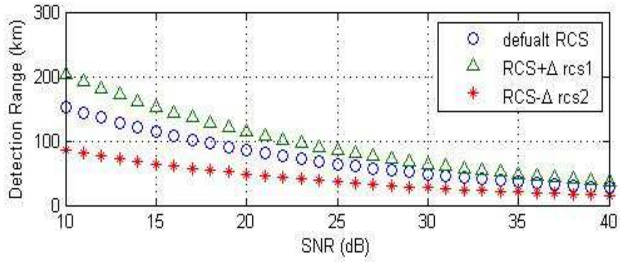
4. Results and Discussion

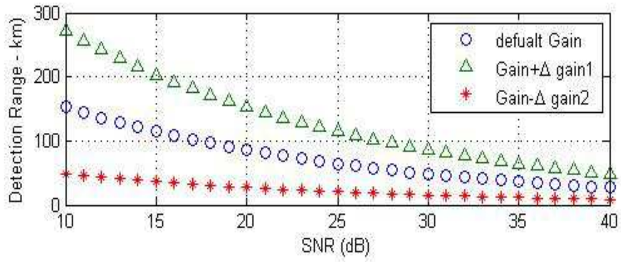
To convert from the SIU to dB, should use first the MATLAB function "*pow2db*" and/or function "*db2pow*" when to convert from dB to SIU.

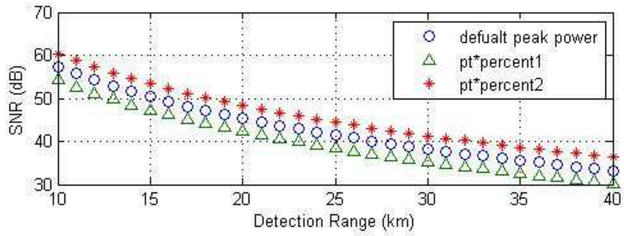
Figure (2) represents the equations (15 and 16), which are programmed by MATLAB simulation, observation of these plots shows the detection range versus SNR or (SNR versus detection range) for several choices like peak transmit power, antenna gain, and RCS. Can be seen that the detection range decreases exponentially with SNR increases, at approximately SNR= < 25dB, we observe that the curve lines are separated and spaced between them which is very clear and significant, but at SNR > = 25dB these curve lines, which represents the detection range be approaching each other and there is a little different. Also we found that the peak power has a little effect on improving the detection range or SNR when compared with other radar parameters such as RCS and antenna gain, because the transmitted and received signal power is proportional to the fourth power of the range while in communication systems is proportional to square power of the range, that means the radar's received energy drops with the fourth power of the distance. In this case, the radar needs high power often in MWatt to be effective at long range [16], for this reason, it greats difficulty in designing radar systems particularly at long range. To reduce the signal power in the radar we could use another parameters such as antenna gain, RCS, noise figure … etc.

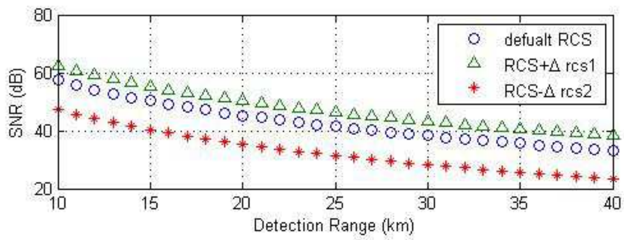
Figures (3-8) shows the detection range/SNR versus the three various choices of peak power, antenna gain and RCS, when we plotted the range versus antenna gain for three choices (values) of peak transmit power and RCS, we found that the range increases exponentially with the antenna gain increases, at approximately gain <= 40 dB we observe that the curve lines be approaching each other and there is a little different, but at gain>= 40 dB these curve lines are separated and spaced between them which is very clear and significant, while the SNR increases linearly with the gain increases. As for the rest graphics, we observed that they follow a linearly behavior. From these graphics were found that the peak transmit power has a little effect if compared with the gain and RCS, to reduce the peak power of the broadcast excessive by increasing in gain which is accompanied by a decrease in bandwidth, and is achieved by increasing the antenna size relative to the wavelength, and also depends on RCS of the target, where they are directly proportional with the peak power, or using high frequencies, therefore we observe that many of radars have operating at frequencies from a few MHz to the UV region of the spectrum[10].











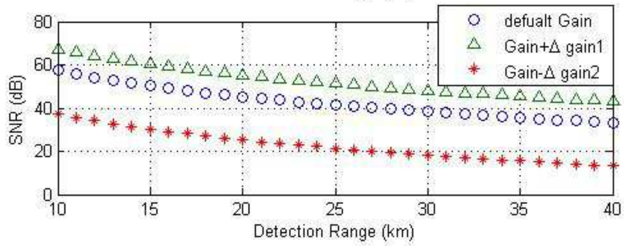


Figure 2. *Detection range versus SNR and/or SNR versus Range for several choices*

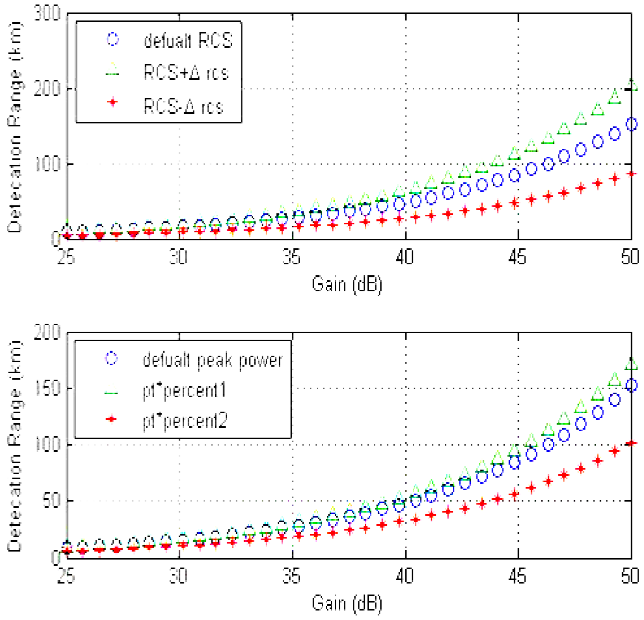


Figure 3. *Shows the detection range versus gain for three choices of peak power percent and RCS*

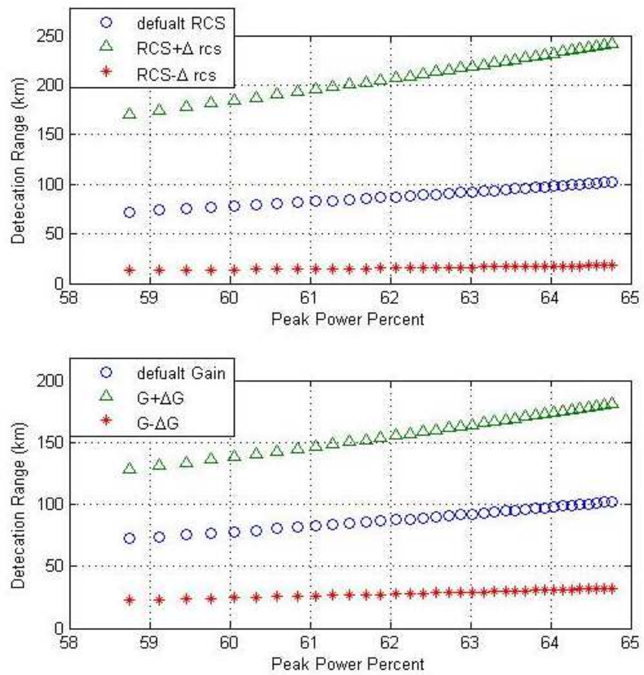


Figure 4. *Shows the detection range versus peak power percent for three choices of gain and RCS*

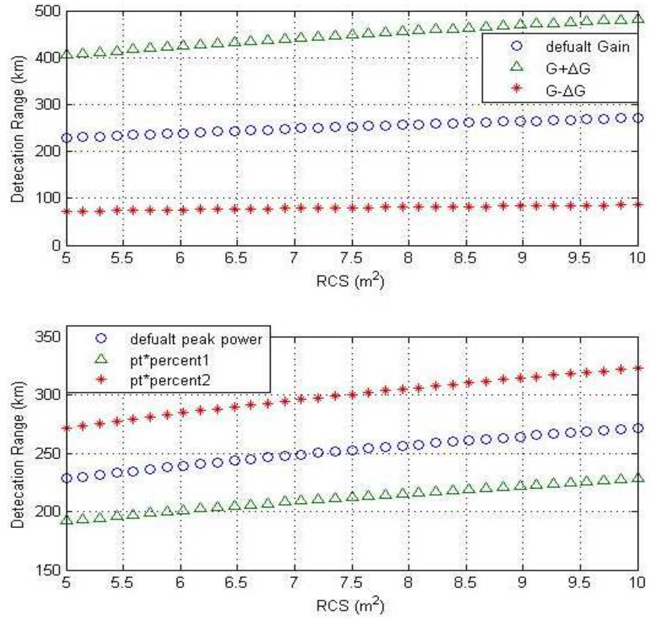


Figure 5. *Shows the detection range versus RCS for three choices of gain and peak power percent*

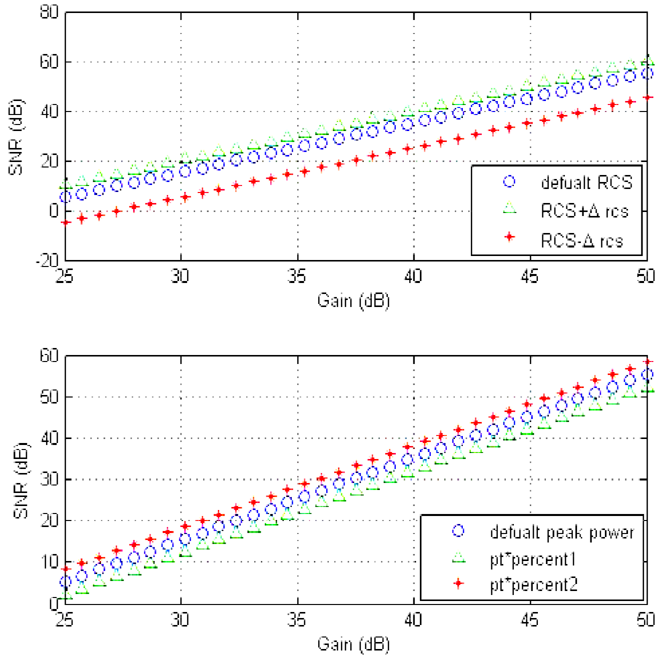


Figure 6. *Shows the maximum SNR versus gain for three choices of RCS and peak power percent*

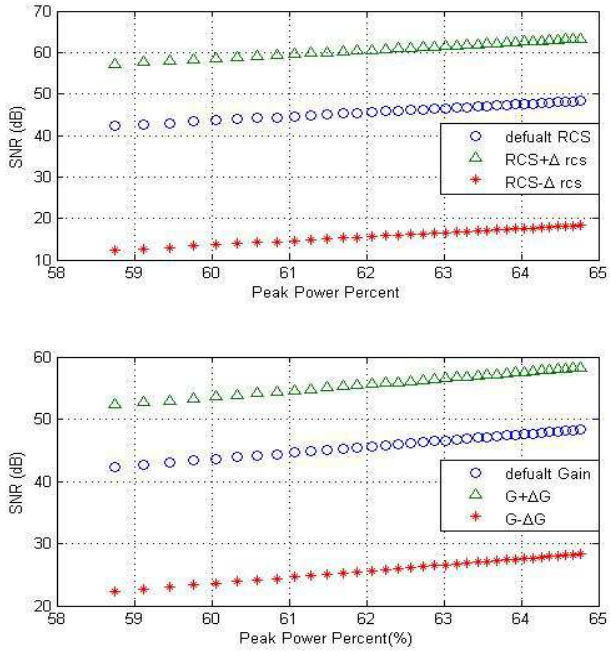


Figure 7. *Shows the maximum SNR versus peak power percent for three choices of gain and RCS*

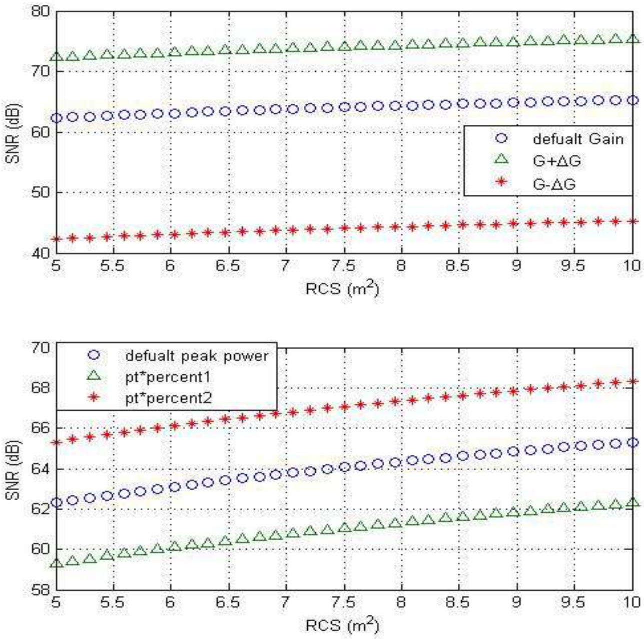


Figure 8. *Shows the maximum SNR versus RCS for three choices of gain and peak power percent*

In figure (9) represents the equation (20) which is programmed by MATLAB simulation, and shows the relationship between range and SNR for three choices of coherently pulses, the detection range is decreasing exponentially with increasing of SNR, while SNR is decreasing linearly with increasing of detection range. We observed that the short train of pulses have a clear effect on the detection range and SNR, whereas the number of pulses is directly proportional with detection range/SNR.

The relationship between pulsed radars and range measurement is how to unambiguously determine the range of the target. The radar receiver measures the time between the leading edges of the last transmitting pulse and the echo pulse. It is possible that an echo will be received from a long range target after the transmission of a second transmitting pulse. In this case, the radar will determine the wrong time interval and therefore the ambiguous range and occurs where there are strong targets at a range in excess of the pulse repetition time. The pulse repetition time defines a maximum unambiguous range. To increase the value of the unambiguous range, it is necessary to increase the PRT, and this means to reduce PRF. It is very difficult to use radar pulse to measurement the far and close targets at the same time, for this reason there are long range pulsed radars cannot measure range of close target and vice versa. Also the loss decreases with increasing numbers of pulses[17, 13]

In figure (10) shows range or SNR versus number of coherently pulses for three choices of RCS, peak power and antenna gain. We observed that the detection range increases logarithmic with number of coherent pulses. Also there is a noticeable change in the detection range increases when one of the parameters increases like (RCS, antenna gain and peak power).

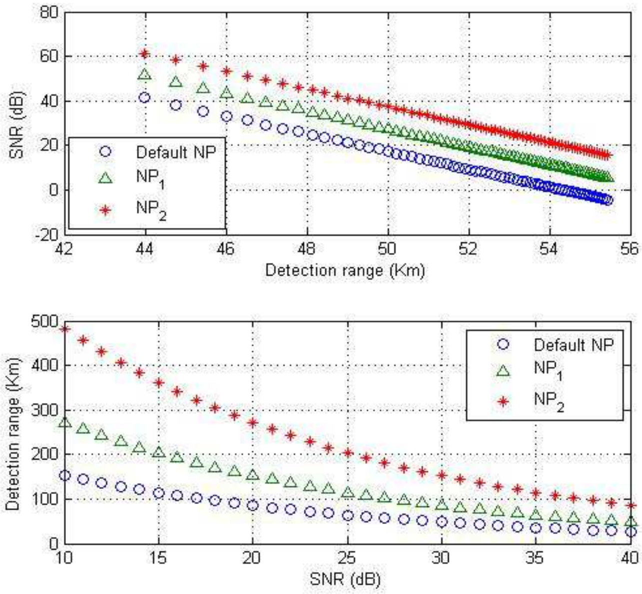


Figure 9. *Shows the detection range versus SNR for three choices of coherently pulses*

In figure (11) represents the equation (23) which is programmed by MATLAB simulation, we plot the detection range versus SNR (or SNR versus detection range) for three choices of duty cycle, we observed that the detection range decreases exponentially with the SNR. The duty cycle is defined as the ratio between the pulse duration (τ) and the period (T) of a rectangular waveform. The pulse width or pulse duration of transmitted signal is to ensure that the radar emits sufficient energy to allow that the reflected pulse is detectable by its receiver. Therefore, it constrains the maximum detection range of a target, where the pulse width is directly proportional with duty cycle $\left(d_t = \tau / T\right)$. Reduction of the bandwidth will lead to reduction in energy contained and thus leads to reduce the detection range[12, 11].

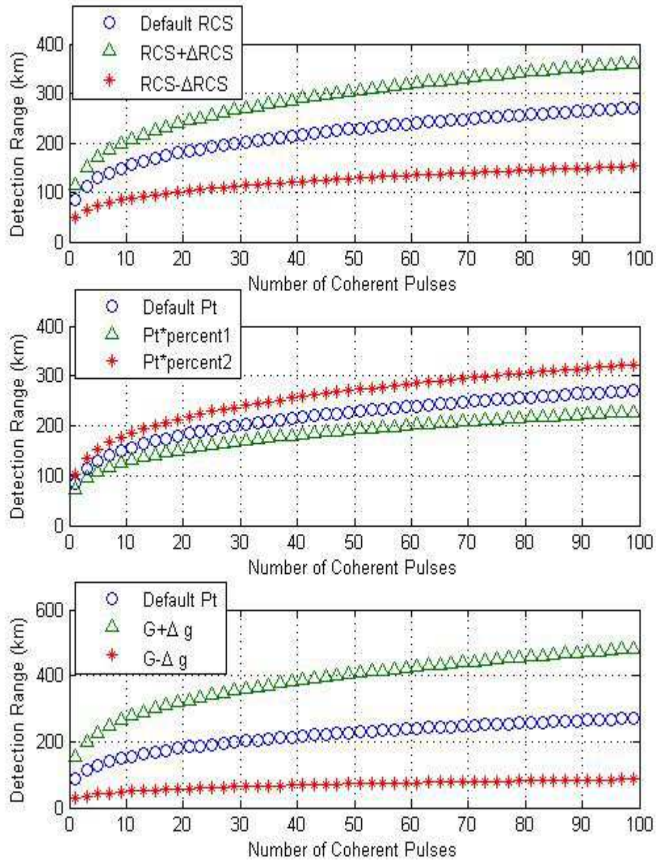


Figure 10. *Shows the range or SNR versus number of coherently pulses for several choices of RCS, peak power and antenna gain*

Figure (12) shows the range versus duty cycle for several of parameters choices like (RCS, peak power and antenna gain). we observed that the detection range increases logarithmic with duty cycle.

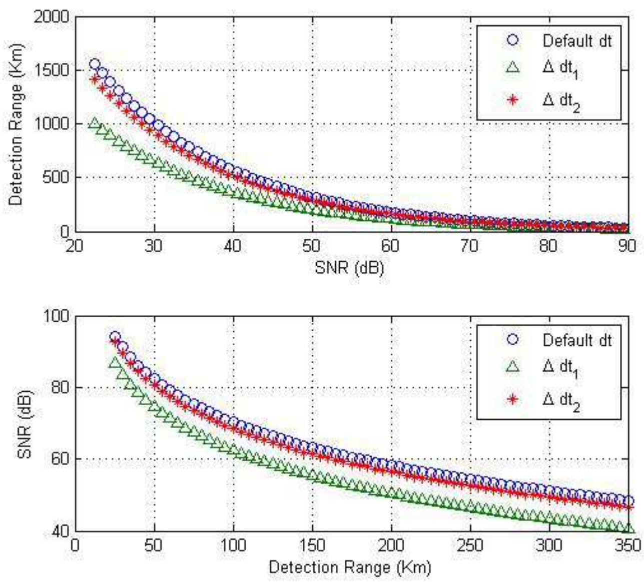


Figure 11. *Detection Range versus SNR or SNR versus range for three choices of duty cycle.*

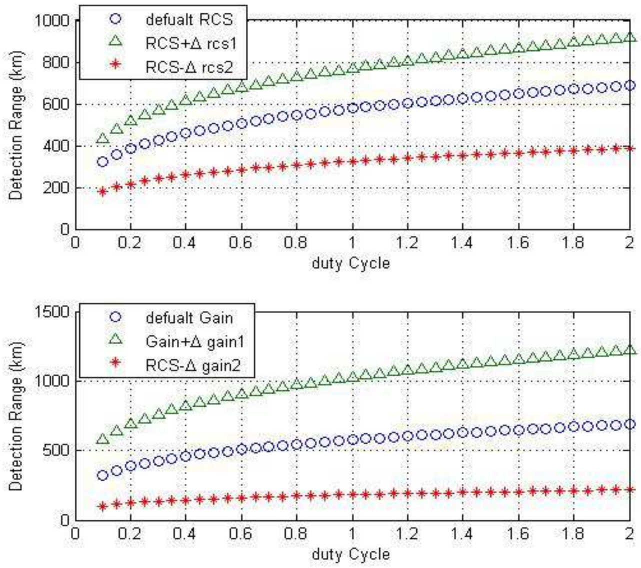


Figure 12. *Detection range versus duty cycle for several choices of RCS, peak power, and antenna gain*

5. Conclusion

These programs have been developed to make them so easy and faster for all users to determine the detection range and signal to noise ratio – SNR for several choices parameters as a function of the radar range equations. They used to plot curves of detection range versus SNR for several choices parameters. Where these programs represents the equations (15, 16, 20, 23).

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