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Application of Laplace Variation Iteration Method to Solving the Nonlinear Gas Dynamics Equation

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Abstract: In this work, we use a new analytical technique called Laplace variational iteration method to construct the exact solution of the nonlinear equation of gas dynamics. This method is based on the determination of the Lagrange multiplier in an optimal way. Application of the method to three test modeling problems from mathematical physics leads to a sequence which tends towards the exact solution of the problem. The solution procedure shows the reliability of the method and is high accuracy evident.

Keywords: Laplace Variational Iteration Method, Nonlinear Gas Dynamics Equation, Lagrange Multiplier

1. Introduction

Gas dynamics is the science of the flow of air and other gas and or the notion of badies through air and other gas, and its effects on physical systems, based on the principes of fluid mechanics and thermodynamics; This Science considers also products of combustion (and combustion). In the major studes,the speed of gases is similar to sound velocity, and some times one found a signifiant change in gas and objects temperatures.[3, 16]. Theoretical equations need for the computational of gas dynamics effects, based on: fluid dynamics principles; gas laws; gas thermodynamic properties (real gas or gas mixtures);energy equation; combustion laws.These equations are written in terms of pressure (P) and temperature (T), or volume (V) and T, and their partial derivatives.Thus Many problems of gas dynamics are governed by linear and non linear partial differential equations (PDE).In the area of non linear gas dynamics, the last 20 years have witnessed of a remarkable number of advances in the solving of PDEs based on wide variety of numerical methods.

We can list: Adomian decomposition method [15], Homotopy analysis method [18], finite difference scheme[19], reduced differential transform method [20]. Reconstruction of Variational Iteration Method [21, 24], Homotopy perturbation method [11-14], Variational iterative method [22], El-Zaki transform homotopy perturbation method [2], Natural decomposition method [23], Variational homotopy perturbation method [27], Differential transform method, Modified Homotopy perturbation method [25], Homotopy perturbation transformation method [28, 29] and so on.However, it is difficult to find the exact solutions of non linear PDEs of gas dynamics.

In this work, we use the Laplace Variational Iteration Method to find the exact solution of the gas dynamics equation[26]

$$\frac{\partial u(x,t)}{\partial t}+\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x}=u(x,t)-u^2(x,t)+g(x,t);$$
$$0\leqslant x\leqslant 1,\ t>0$$

2. Description of the Method [9,10]

Consider the general nonlinear, inhomogeneous partial differential equation.

$$Lu(x,t) + Nu(x,t) = g(x,t) \tag{1}$$

Where L is the highest power derivative which is easily invertible, N represents a general nonlinear differential operator and $g(x,t)$ is a source term. with the initial condition and here L is operator $\left(\frac{\partial}{\partial t}\right)$, $u(x,0) = h(x)$

Taking the Laplace transform to the both sides of the given equation.

$$\mathcal{L}Lu(x,t) + \mathcal{L}Nu(x,t) = \mathcal{L}g(x,t) \tag{2}$$

we obtain:

$$s\mathcal{L}u(x,t) - u(x,0) = \mathcal{L}g(x,t) - \mathcal{L}Nu(x,t) \tag{3}$$

We have

$$\mathcal{L}u(x,t) = \frac{1}{s}h(x) + \frac{1}{s}\mathcal{L}g(x,t) - \frac{1}{s}\mathcal{L}Nu(x,t) \tag{4}$$

Taking the inverse Laplace

$$u(x,t) = h(x) + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}g(x,t)\right] - \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}Nu(x,t)\right] \tag{5}$$

Derivative (5) by $\frac{\partial}{\partial t}$ both sides, we have

$$\frac{\partial}{\partial t}u(x,t) - \frac{\partial}{\partial t}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}g(x,t)\right] + \frac{\partial}{\partial t}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}Nu(x,t)\right] = 0 \tag{6}$$

The correction functional of the variational iteration method is given as

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda\left[\frac{\partial}{\partial \tau}u(x,\tau) - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}g(x,\tau)\right] + \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}Nu(x,\tau)\right]\right]d\tau \tag{7}$$

The general lagrange multiplier for (7) can be identified optimally via variation theory to get

$$1 + \lambda/_{\tau=t} = 0 \qquad \lambda^{'}/_{\tau=t} = 0 \tag{8}$$

From, (8), we obtain

$$\lambda = -1 \tag{9}$$

Substituting $\lambda = -1$ into (7), we get the iterative formula for $n = 0, 1, 2, \ldots,$ as follows

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left[\frac{\partial}{\partial \tau}u(x,\tau) - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}g(x,\tau)\right] + \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}Nu(x,\tau)\right]\right]d\tau \tag{10}$$

Start with the intial iteration

$$u_0(x,t) = u(x,0) = h(x) \tag{11}$$

The exact solution is given as a limit of the successive approximations $u_n(x,t)$, $n = 0, 1, 2, \ldots,$ in other words,

$$u(x,t) = \lim_{n \longrightarrow +\infty} u_n(x,t) \tag{12}$$

3. Numerical Examples

In this section, three problems are presented to illustrate the efficiency of the method.

3.1. Problem 1: The Following Nonlinear Homogeneous Gas Dynamics Equation [1,2,3,4,5,6,7,8,16]

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = -\dfrac{1}{2}\dfrac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t), \quad 0 \leqslant x \leqslant 1, t > 0 \\ u(x,0) = e^{-x} \end{cases} \tag{13}$$

Consider the following homogenous gas dynamic equation :

$$\frac{\partial u(x,t)}{\partial t} = -\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t) \tag{14}$$

Taking Laplace transfom on equation (14)

$$\mathcal{L}\left(\frac{\partial u(x,t)}{\partial t}\right) = \mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x}\right) + \mathcal{L}\left(u(x,t)\right) - \mathcal{L}\left(u^2(x,t)\right) \tag{15}$$

We obtain:

$$s\mathcal{L}u(x,t) - u(x,0) = \mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} - u^2(x,t)\right) + \mathcal{L}\left(u(x,t)\right) \tag{16}$$

Then the equation (16) can be written:

$$s\mathcal{L}u(x,t) - \mathcal{L}u(x,t) = u(x,0) + \mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} - u^2(x,t)\right) \tag{17}$$

We obtain :

$$\mathcal{L}u(x,t)(s-1) = u(x,0) + \mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} - u^2(x,t)\right) \tag{18}$$

The equation (18) gives:

$$\mathcal{L}u(x,t) = \frac{1}{s-1}u(x,0) + \frac{1}{s-1}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} - u^2(x,t)\right) \tag{19}$$

The inverse Laplace transform of (19)

$$\mathcal{L}^{-1}\mathcal{L}u(x,t) = \mathcal{L}^{-1}\left(\frac{1}{s-1}u(x,0)\right) + \mathcal{L}^{-1}\left(\frac{1}{s-1}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} - u^2(x,t)\right)\right) \tag{20}$$

The equation (20) gives:

$$u(x,t) = e^t u(x,0) + \mathcal{L}^{-1}\left(\frac{1}{s-1}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} - u^2(x,t)\right)\right) \tag{21}$$

Differentiating (21) with respect to t, we have

$$\frac{\partial u(x,t)}{\partial t} = e^t u(x,0) + \frac{\partial}{\partial t}\mathcal{L}^{-1}\left(\frac{1}{s-1}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} - u^2(x,t)\right)\right) \tag{22}$$

The correction functional of equation (22) is writlen

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(t,\tau)\left[\frac{\partial u_n(x,\tau)}{\partial \tau} - e^s u(x,0) - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left(\frac{1}{s-1}\mathcal{L}\left(-\frac{1}{2}\frac{\partial \tilde{u_n}^2}{\partial x} - \tilde{u_n}^2\right)\right)\right] d\tau \tag{23}$$

Like $\lambda(t,\tau) = -1$, the equation (23) gives:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left[\frac{\partial u_n(x,\tau)}{\partial \tau} - e^s u(x,0) - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left(\frac{1}{s-1}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u_n^2(x,\tau)}{\partial x} - u_n^2(x,\tau)\right)\right)\right] d\tau \tag{24}$$

We know that:

$$u_0(x,t) = u(x,0) = e^{-x} \tag{25}$$

Let us calculate $u_1(x,t)$ for $n = 0$, one has

$$u_1(x,t) = u_0(x,t) - \int_0^t \left[\frac{\partial u_0(x,s)}{\partial \tau} - e^s u(x,0) - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left(\frac{1}{s-1}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u_0^2(x,\tau)}{\partial x} - u_0^2(x,\tau)\right)\right)\right] d\tau \tag{26}$$

We obtain:

$$u_1(x,t) = u_0(x,t) - \int_0^t \left[-e^{s-x} - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left(\frac{1}{s-1}\mathcal{L}(0)\right)\right] d\tau \tag{27}$$

The equation (27) gives:

$$u_1(x,t) = e^{t-x} \tag{28}$$

By using the same procedure one obtains:

$$u_2(x,t) = u_3(x,t) = ... = e^{t-x} \tag{29}$$

Finally one has:

$$\begin{cases} u_0(x,t) = e^{-x} \\ u_1(x,t) = e^{t-x} \\ u_2(x,t) = e^{t-x} \\ \vdots \\ u_n(x,t) = e^{t-x} \end{cases} \tag{30}$$

So the desired solution is:

$$u(x,t) = \lim_{n\longrightarrow\infty} u_n(x,t); \quad \forall n \geq 0 \tag{31}$$

$$u(x,t) = e^{t-x} \tag{32}$$

3.2. Problem 2: The Nonlinear Non-Homogenous Gas Dynamic Equation [2,3,4,16]

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = -\dfrac{1}{2}\dfrac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t) - e^{t-x}, \quad 0 \leqslant x \leqslant 1, t > 0 \\ u(x,0) = 1 - e^{-x} \end{cases} \tag{33}$$

Consider the following non-homogenous, nonlinear gas dynamic equation:

$$\frac{\partial u(x,t)}{\partial t} = -\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t) - e^{t-x}, \quad 0 \leqslant x \leqslant 1, t > 0 \tag{34}$$

Taking Laplace transfom on equation (34), we obtain

$$s\mathcal{L}u(x,t) = u(x,0) + \mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t)\right) - \mathcal{L}(-e^{t-x}) \tag{35}$$

The equation (35) gives:

$$\mathcal{L}u(x,t) = \frac{1}{s}(1 - e^{-x}) + \frac{1}{s}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t)\right) - \frac{1}{s}\mathcal{L}(-e^{t-x}) \tag{36}$$

Taking inverse Laplace to equation (36) gives:

$$u(x,t) = 1 - e^{t-x} + \mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t)\right)\right] \tag{37}$$

By deriving equation (37) with respect to t, we have:

$$\frac{\partial u(x,t)}{\partial t} = -e^{t-x} + \frac{\partial}{\partial t}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t)\right)\right] \tag{38}$$

The equation (38) is reduced as follows:

$$\frac{\partial u(x,t)}{\partial t} + e^{t-x} - \frac{\partial}{\partial t}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t) - u^2(x,t)\right)\right] = 0 \tag{39}$$

The correction functional of equation (39) is writlen

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left(\frac{\partial u_n(x,\tau)}{\partial \tau} + e^{\tau-x} - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u_n^2(x,\tau)}{\partial x} + u_n(x,\tau) - u_n^2(x,\tau)\right)\right]\right) d\tau \tag{40}$$

We know that:

$$u_0(x,t) = u(x,0) = 1 - e^{-x} \tag{41}$$

Let us calculate $u_1(x,t)$ for $n=0$, one has

$$u_1(x,t) = u_0(x,t) - \int_0^t \left(\frac{\partial u_0(x,\tau)}{\partial \tau} + e^{\tau - x} - \frac{\partial}{\partial \tau}\mathcal{L}^{-1}\left[\frac{1}{s}\mathcal{L}\left(-\frac{1}{2}\frac{\partial u_0^2(x,\tau)}{\partial x} + u_0(x,\tau) - u_0^2(x,\tau)\right)\right]\right) d\tau \tag{42}$$

From (42), we get:

$$u_1(x,t) = 1 - e^{t-x} \tag{43}$$

By using the same procedure one obtains:

$$u_2(x,t) = u_3(x,t) = ... = 1 - e^{t-x} \tag{44}$$

Finally one has:

$$\begin{cases} u_0(x,t) = 1 - e^{-x} \\ u_1(x,t) = 1 - e^{t-x} \\ u_2(x,t) = 1 - e^{t-x} \\ \vdots \\ u_n(x,t) = 1 - e^{t-x} \end{cases} \tag{45}$$

Thus, the solution is:

$$u(x,t) = \lim_{n \longrightarrow \infty} u_n(x,t); \quad \forall n \geq 0 \tag{46}$$

$$u(x,t) = 1 - e^{t-x} \tag{47}$$

3.3. Problem 3: The Nonlinear Non-Homogenous Gas Dynamic Equation [2, 3, 17]

$$\begin{cases} \dfrac{\partial u(x,t)}{\partial t} = -\dfrac{1}{2}\dfrac{\partial u^2(x,t)}{\partial x} + u(x,t)\left(1 - u(x,t)\right)\ln a, \quad 0 \leqslant x \leqslant 1, t > 0, a > 0 \\ u(x,0) = a^{-x} \end{cases} \tag{48}$$

Consider the following non-homogenous, nonlinear gas dynamic equation:

$$\frac{\partial u(x,t)}{\partial t} = -\frac{1}{2}\frac{\partial u^2(x,t)}{\partial x} + u(x,t)\left(1 - u(x,t)\right)\ln a \tag{49}$$

$\Longleftrightarrow$

$$\frac{\partial u(x,t)}{\partial t} = -u(x,t)\frac{\partial u(x,t)}{\partial x} + u(x,t)\left(1 - u(x,t)\right)\ln a \tag{50}$$

Taking Laplace transfom on equation (50), we obtain:

$$s\mathcal{L}\left(u(x,t) - u(x,0) = -\mathcal{L}\left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right) + \mathcal{L}\left(u(x,t)\right)\ln a - \mathcal{L}\left(u^2(x,t)\ln a\right)\right. \tag{51}$$

The equation (51) gives:

$$\mathcal{L}\left(u(x,t)\right) = \frac{a^{-x}}{s - \ln a} - \frac{1}{s - \ln a}\mathcal{L}\left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right) - \frac{\ln a}{s - \ln a}\mathcal{L}\left(u^2(x,t)\right) \tag{52}$$

Taking the inverse Laplace transform ,we obtain:

$$u(x,t) = a^{-x+t} - \mathcal{L}^{-1}\left(\frac{1}{s - \ln a}\mathcal{L}\left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right)\right) - \mathcal{L}^{-1}\left(\frac{\ln a}{s - \ln a}\mathcal{L}\left(u^2(x,t)\right)\right) \tag{53}$$

By deriving equation (53) with respect to t, we have:

$$\frac{\partial u(x,t)}{\partial t} = \ln a\left(a^{-x+t}\right) - \frac{\partial}{\partial t}\left(\mathcal{L}^{-1}\left(\tfrac{1}{s - \ln a}\mathcal{L}\left(u(x,t)\frac{\partial u(x,t)}{\partial x}\right)\right)\right) - \frac{\partial}{\partial t}\left(\mathcal{L}^{-1}\left(\tfrac{\ln a}{s - \ln a}\mathcal{L}\left(u^2(x,t)\right)\right)\right) \tag{54}$$

The correction functional of equation (54) is written:

$$\begin{aligned} u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda(t,\tau)(\frac{\partial u(x,\tau)}{\partial \tau} - \ln a \left(a^{-x+\tau}\right) \\ + \frac{\partial}{\partial \tau}\left(\mathcal{L}^{-1}\left(\frac{1}{s-\ln a}\mathcal{L}\left(u(x,\tau)\frac{\partial u(x,\tau)}{\partial x}\right)\right)\right) \\ + \frac{\partial}{\partial \tau}\left(\mathcal{L}^{-1}\left(\frac{\log a}{s-\ln a}\mathcal{L}\left(u^2(x,\tau)\right)\right)\right))d\tau \end{aligned} \tag{55}$$

Making correction functional stationary, approximate Lagrange multiplier can be identified as $\lambda(t,\tau) = -1$, so

$$\begin{aligned} u_{n+1}(x,t) = u_n(x,t) - \int_0^t (\frac{\partial u(x,\tau)}{\partial \tau} - \ln a \left(a^{-x+\tau}\right) \\ + \frac{\partial}{\partial \tau}\left(\mathcal{L}^{-1}\left(\frac{1}{s-\ln a}\mathcal{L}\left(u(x,\tau)\frac{\partial u(x,\tau)}{\partial x}\right)\right)\right) \\ + \frac{\partial}{\partial \tau}\left(\mathcal{L}^{-1}\left(\frac{\ln a}{s-\ln a}\mathcal{L}\left(u^2(x,\tau)\right)\right)\right))d\tau \end{aligned} \tag{56}$$

Therefore,

$$\begin{cases} u_0(x,t) = a^{-x+t} \\ u_1(x,t) = u_0(x,t) = a^{-x+t} \\ u_2(x,t) = u_1(x,t) = a^{-x+t} \\ \vdots \\ u_n(x,t) = u_{n-1}(x,t) = a^{-x+t} \end{cases} \tag{57}$$

Thus, the solution is:

$$u(x,t) = \lim_{n\longrightarrow\infty} u_n(x,t) = a^{-x+t} \tag{58}$$

4. Conclusion

Through these examples, we have shown again the utility of Laplace's variational iteration method, in the search for an approximate solution of the nonlinear equation of gas dynamics. This method gives the exact solution with great precision, using the initial conditions and can be considered as a reliable refinement of existing techniques.

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