Boundary Layer Flow of a Nanofluid Through a Permeable Medium Due to Porous Plate

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Abstract: In the present article, an attempted have been made to study the behavior of boundary layer viscous fluid flow and heat transfer containing some nanosized solid particles flowing through a permeable porous medium. The problem was first modeled into a coupled system of nonlinear partial differential equations of conservation of mass, momentum and nanoparticle concentration. The system of coupled nonlinear boundary layer partial differential equations governing the flowing fluid momentum and heat transfer characteristics are reduced to a new simplified coupled nonlinear system of ordinary differential equations by means of a suitable similarity transformation. The transformed set of nonlinear coupled ordinary differential equations is than solved numerically by means of the fourth order numerical scheme the Runge-Kutta shooting method. The effects of important involved parameters that control the flow field and heat transfer characteristics, that is the viscosity parameter, the convection parameter, the Porosity parameter, the Prandtl number and the Lewis number have been obtained and discussed. Numerical solutions for velocity and temperature are sketched and graphically analyzed. The graphical results observed are indicating that by increasing the values of the non-dimensional viscosity parameter, the dimension less fluid flow profile increases, while for increasing values of the nanoparticles Brownian motion parameter, the nanoparticle concentration profile increases.

Keywords: Boundary Layer Flow, Permeable Medium, Porous Plate

1. Introduction

The term "nanofluid" refers to a liquid containing a suspension of submicronic solid particles (nanoparticles). The term nanofluid was introduced by Choi [1] and this area which has attracted the attention of the researchers. The characteristic feature of nanofluids is thermal conductivity enhancement, a phenomenon observed by Masuda et al. [2]. The flow due to porous sheet has numerous applications due to its practical and cost related advantages, it has been extensively used in many engineering fields and industrial manufacturing processes such as the aerodynamic extrusion of plastic sheets, bundle wrapping, hot rolling, extrusion of sheet material, wire rolling etc. Vadasz [3] studied the concept of porous medium and explore some emerging topics in heat and mass transfer and developed some useful results on that medium. Ingham and Pop [4] studied the phenomena of transportation of fluid and heat in porous medium. Cheng [5] investigated problem of natural convection heat transfer of non-Newtonian fluids in porous media from a vertical cone under mixed thermal boundary conditions and developed useful underlying results with the help of numerical techniques. Ahmad and Pop [6] studied important results and properties of mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids. Cheng [7] further discuss about on soret and dufour effects on heat and mass transfer by natural convection from a vertical truncated cone in a fluid-saturated porous medium with variable wall temperature and concentration. Sheikholeslami et al. [8] studied the effects of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium. Abbasi et al. [9] analyze the phenomenon which is based on Peristaltic transport of copper–water nanofluid saturating porous medium. Sheikholeslami et al. [10] further observed some

basic results for the problem with numerical modeling for alumina nanofluid magnetohydrodynamic convective heat transfer in a permeable medium using Darcy law. Hassan et al. [11] investigated the properties and results of convective heat transfer flow of nanofluid in a porous medium over wavy surface. Reddy et al. [12] studied the influence of chemical reaction, radiation and rotation on MHD nanofluid flow past a permeable flat plate in porous medium. Few other related works regarding the boundary layer flows, fluids containing nanoparticles and heat transfer analysis are cited in [13-16].

A boundary layer is the layer of fluid in the immediate vicinity of a bounding surface where the effects of viscosity are significant [17-19]. The present paper studies the problem of boundary layer of a nanofluid through a porous sheet. The governing equations are transformed into nonlinear coupled ordinary differential equations which depends on the viscosity ratio parameter Λ, Prandtl number Pr. The obtained nonlinear coupled ordinary differential equations are solved numerically using shooting iteration technique [20-21]. The velocity, temperature, and concentration distributions are discussed and presented graphically, and the skin-friction coefficient, the surface heat, and mass transfer rate at the sheet are investigated.

2. Mathematical Formulation

We consider a two-dimensional problem. Consider a steady two-dimensional flow of an incompressible viscous nanofluid caused by a porous medium. A flat plate is called porous if fluid can be entered and/or leave the control volume through the surface of the plate. We select a coordinate frame in which the *x*-axis is aligned vertically upwards. We consider a vertical plate at $y = 0$. The fluid is occupying the region $x > 0$. If the plate is such that fluid can enter from the region $x < 0$ into $x > 0$ (injection/blowing flow) or fluid can cross from $x > 0$ and enters $x > 0$ (suction flow) then plate is termed as porous. The velocity for the blowing case is taken positive and for that of the suction is negative. The uniform temperature of the plate raised to $T_w (> T_\infty)$, which is thereafter maintained constant, where T_w is temperature at the wall and T_∞ is temperature far away from the plate

For the problem of fluid flowing over a porous plate the velocity flied is of the form

$$V = \left(u(x,y), v(x,y), 0 \right), \ T = T(x,y), \ C = C(x,y) \quad (1)$$

Then the governing equations of conservation of mass, momentum, heat transfer and concentration are [22-25]

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \tilde{v}\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\tilde{v}\phi_p}{k_0}u \quad (3)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\rho^* c_p^*}{\rho c_p}\left(D_T\frac{\partial T}{\partial y}\frac{\partial C}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2 \right) \quad (4)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \quad (5)$$

Subject to the boundary conditions

$$v = v_w, \ T = T_w(x), \ C = C_w(x) \ \text{ at } \ y = 0 \quad (6)$$

$$u \to \infty, \ T \to T_\infty, \ C \to C_\infty \ \text{ as } \ y \to \infty \quad (7)$$

Where (u, v) are the velocity components along (x, y) axes, $\tilde{v}$ is the kinematic viscosity of the base fluid, T is temperature of the base fluid, T_w is the surface temperature of the sheet, c_p^* is the nanoparticles effective heat, the Brownian diffusion constant is D_B, the thermophoretic diffusion constant is D_T, ϕ_p is the porosity of permeable space, and k_0 is the penetrability of permeable space.

Introduce the following similarity transformations [26-28] in *Equations (8-10)*

$$\eta = \frac{y}{x}\sqrt{\frac{u_o}{2vL}}, \quad \psi = \sqrt{2vu_o L}\, f(\eta) \quad (8)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \ \varphi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad (9)$$

$$u = \frac{\partial \psi}{\partial y} = u_o f', \quad v = -\frac{\partial \psi}{\partial x} = -\sqrt{\frac{vu_o}{2L}}(f - \eta f') \quad (10)$$

Apply these similarity transformations over *Equations (2-7)* to reduce the partial differential equations into a new set of ordinary differential equations of the form

$$\Lambda f''' + ff'' - 2f'^2 + 2\lambda_t\theta - k_p f' = 0 \quad (11)$$

$$\theta'' + \text{Pr}(f\theta' - f'\theta) + N_b\theta'\varphi' + N_t\theta'^2 = 0 \quad (12)$$

$$\varphi'' + Le(f\varphi' - f'\varphi) + \left(\frac{N_t}{N_b}\right)\theta'' = 0 \quad (13)$$

Where Λ is the viscosity ratio parameter, λ_t is the convection parameter, k_p is the porosity parameter, Pr is the Prandtl number Le is the non-dimensional Lewis number, $N_b = \dfrac{(\rho c)D_B(C_w - C_\infty)}{v(\rho c)_f}$ is the Brownian motion parameter and $N_t = \dfrac{(\rho c)_p D_T(T_w - T_\infty)}{vT_\infty(\rho c)_f}$ is he Thermophoresis parameter. The reduced boundary conditions are

$$f = s, \ f' = 1, \ \theta = 1, \ \varphi = 1 \text{ at } \eta = 0 \quad (14)$$

$$f' = 0, \quad \theta = 0, \quad \varphi = 10 \, \text{as} \quad \eta \to \infty \qquad (15)$$

3. Solution of the Problem

The system of ordinary differential equations in *Equations (11-13)* subject to the boundary conditions in *Equations (14-15)* is solved numerically using the fourth order shooting method [29-31]. The graphical behaviors of the different important involved parameters are graphically presented in *Figures (1-6)*. *Figure 1* represents the behavior of convection parameter λ_t over the non-dimensional velocity profile $f'(\eta)$. From the graph it is observed that with an increase in the convection parameter λ_t the non-dimensional velocity profile $f'(\eta)$ increases. *Figure 2* denotes the behavior of viscosity ratio parameter Λ over the non-dimensional velocity profile $f'(\eta)$. From the graph it is noted that with an increase in the viscosity ratio parameter Λ the non-dimensional velocity profile $f'(\eta)$ increases. *Figure 3* represents the behavior of Prandtl numbers Pr over the non-dimensional temperature profile $\theta(\eta)$. From the graph it is observed that with an increase in Prandtl numbers Pr the non-dimensional temperature profile $\theta(\eta)$ decreases. *Figure 4* represents the behavior of thermophoresis parameter N_t over the non-dimensional temperature profile $\theta(\eta)$. From the graph it is observed that with an increase in the thermophoresis parameter N_t the non-dimensional temperature profile $\theta(\eta)$ increases. *Figure 5* represents the behavior of thermophoresis parameter N_b over the non-dimensional concentration profile $\varphi(\eta)$. From the graph it is observed that with an increase in the thermophoresis parameter N_b the non-dimensional tempe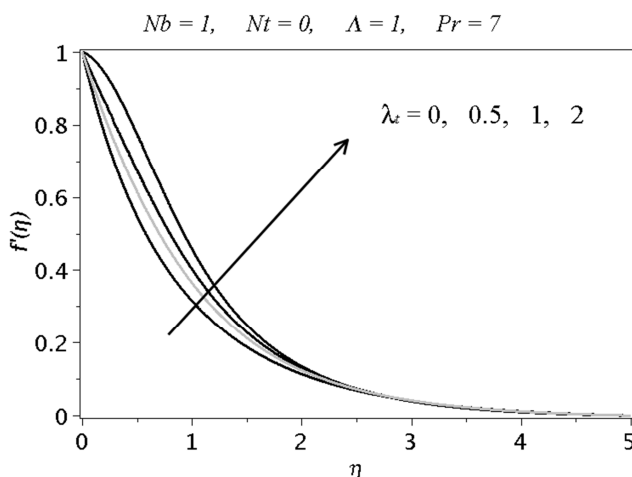rature profile $\varphi(\eta)$ decreases. *Figure 6* represents the behavior of thermophoresis parameter N_t over the non-dimensional concentration profile $\varphi(\eta)$. From the graph it is observed that with an increase in the thermophoresis parameter N_t the non-dimensional temperature profile $\varphi(\eta)$ increases.

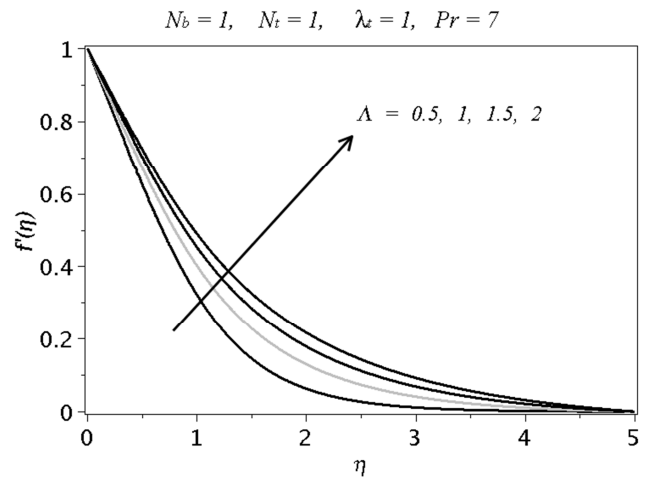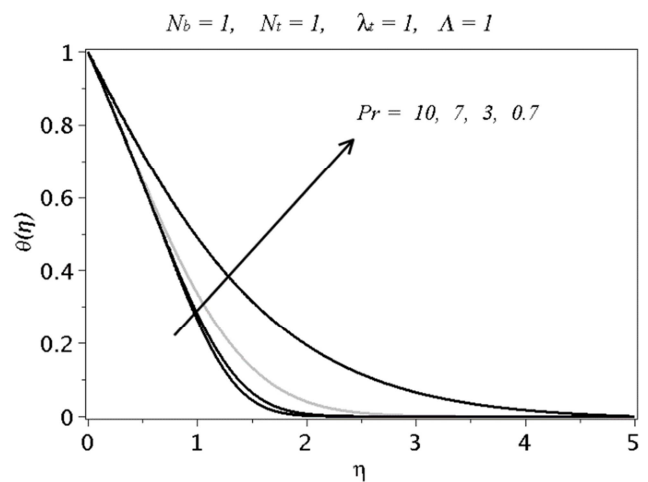**Figure 2.** *Velocity profile $f'(\eta)$ for different values of Λ.*

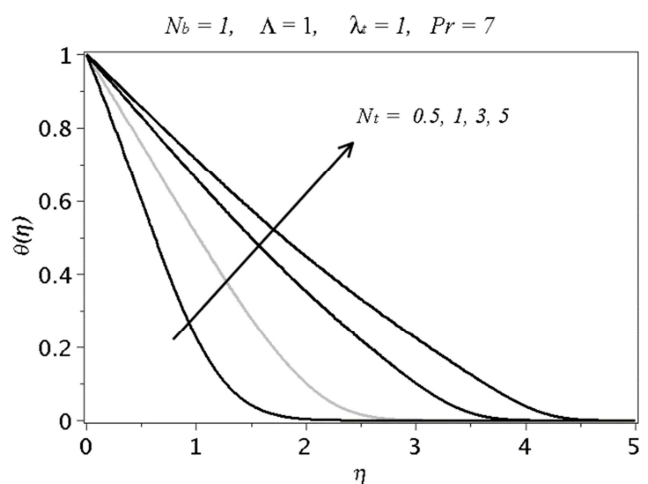**Figure 3.** *Temperature profile $f'(\eta)$ for different values of Pr.*

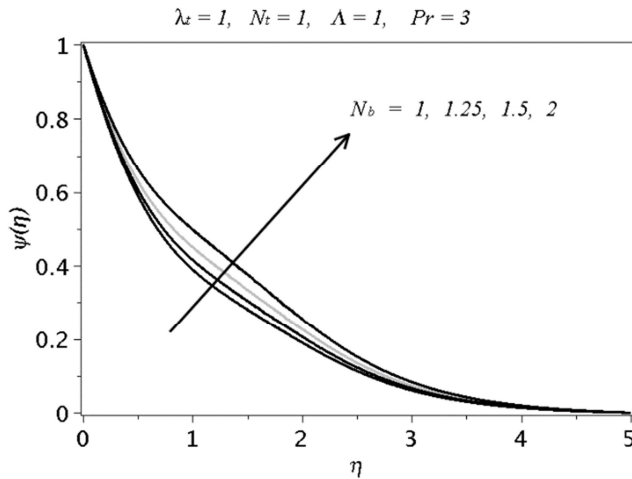**Figure 4.** *Temperature profile $\theta(\eta)$ for different values of N_t.*

Figure 1. *Velocity profile $f'(\eta)$ for different values of λ_t.*

$\lambda_t = 1,\quad N_t = 1,\quad \Lambda = 1,\quad Pr = 3$

Figure 5. *Temperature profile* $\varphi(\eta)$ *for different values of* N_b.

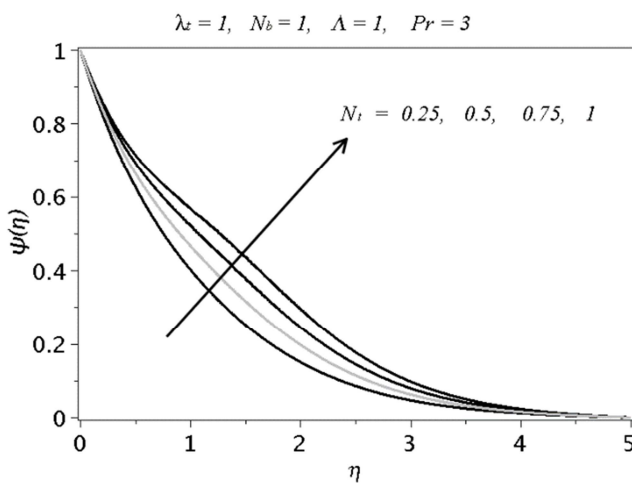$\lambda_t = 1,\quad N_b = 1,\quad \Lambda = 1,\quad Pr = 3$

Figure 6. *Temperature profile* $\varphi(\eta)$ *for different values of* N_t.

4. Conclusion

The main conclusions obtained from the analysis are:
1) With increase in the viscosity parameter, the velocity profile increases.
2) With increase in the Prandtl number, the temperature profile decreases.
3) With increase in the Brownian motion parameter, the nanoparticle concentration profile increases.

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