



# Identifying a Satisfactory Operation Point for Fuzzy Multiobjective Environmental/Economic Dispatch Problem

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**Abstract:** In this paper, reference point based neural network (NN) algorithm is proposed for solving fuzzy multiobjective environmental/economic dispatch problem (FM-EEDP). There are instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices, for this reasons a fuzzy representation of environmental/economic dispatch problem (EEDP) has been investigated. Our approach has two characteristic features. Firstly, FM-EEDP has been defuzzified. Secondly reference point based NN algorithm is implemented in such a way that the decision-maker (DM) participate early in the optimization process instead of leaving him/her alone with the final choice. The target is to identify the Pareto-optimal region closest to the DM preference so as to achieve the pollution limitations which controlled using environmental protection rules and to carry out the maximum cost limitation. Moreover to help the DM to identify the best compromise solution from a finite set of alternatives, TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) method is implemented. On the basis of the application of the standard IEEE 30-bus 6-generator test system, we can conclude that the proposed method can provide a sound optimal power flow by considering the multiobjective problem. Also, with a number of trade-off solutions in the region of interests, we proved that the DM able to make a better and more reliable decision.

**Keywords:** Environmental/Economic Dispatch Problem, Neural Network, Reference Point, Fuzzy Numbers, TOPSIS Method

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## 1. Introduction

EEDP is one of the most important optimization problems from the view point of power system to derive optimal Environmental/Economic. Traditional economic dispatch to minimize the fuel cost is inadequate when environmental emissions are also to be included in the operation of power plants. With the increase in the environmental awareness and the passage of environmental regulations, the environmental constraints are having a significant impact on the operation of power systems. The purpose of EEDP is to figure out the optimal amount of the generated power for the fossil-based generating units in the system by minimizing the fuel cost and emission level simultaneously, subject to various equality and inequality constraints including the security measures of the power transmission/distribution.

The instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices [1]

are the main reasons to fuzzy representation of the multiobjective EEDP; where the input data involve many parameters whose possible values may be assigned by the experts. In practice, it is natural to consider that the possible values of these parameters as fuzzy numerical data which can be represented by means of fuzzy subsets of the real line known as fuzzy numbers.

In this paper, an attempt is made to identify a satisfactory operation point for FM-EEDP. Based on Alpha concept, FM-EEDP is defuzzified at certain degree of  $\alpha$  ( $\alpha$ -cut level) [2-4]. Also, a combination between one of the preference based strategy and NN methodology to obtain a set of solutions near the reference points. Moreover, to help the DM to identify the best compromise solution from a finite set of alternatives, TOPSIS method is implemented. It is based upon simultaneous minimization of distance from an ideal point (IP) and maximization of distance from a nadir point (NP). Such procedures will provide the DM with a best compromise solution to achieve his/her requirements, so that

a better and a more reliable decision can be made. Simulation results are presented for the standard IEEE 30-bus system-6 generator which shows the effectiveness and potential of the proposed approach to solve EEDP.

## 2. Literature Review

Various optimization techniques, which pertaining the EEDP, have been proposed by many researchers [5-11]. Instead of simplifying the multiobjective problem to a single objective problem; there is a direction is to handle both objectives simultaneously as competing objectives [12-16].

There has been much research using the deterministic approach to solve the EEDP such as: An interactive search method, based on the golden section search technique [9], Newton-Raphson convergence technique [17], an improved Box complex method [18] and epsilon-constraint method [19]. More recently, multiobjective evolutionary techniques have been applied to solve the EEDP. Abido has pioneered this research by applying NSGA [12], NPGA [20] and SPEA [13] to the standard IEEE 30-bus system. In fact, it has been shown that NSGA-II can obtain minimum solutions comparable to tabu search [21].

The introduction of the environmental consideration in the economic dispatch problem is led to develop the heuristics-based multiobjective optimization techniques and use them to solve EEDP. Heuristics-based techniques use a population in their search and multiple Pareto-optimal solutions can be found in one single run. These techniques can be efficiently used to eliminate the most of difficulties in classical methods [22-30]. For instance, Xie et al. [24] adopted the fuzzy theory to convert the multiobjective EEDP into single-objective problem, and tackled it through dynamic programming algorithm. In [25] the authors use Simulated Annealing (SA) algorithm where it is an optimization technique inspired from the process of annealing in thermodynamics. On the other hand, by applying the weighted sum method (WSM) and the conic scalarization method (CSM), multiobjective EEDP is transformed into a single-objective EEDP. Then, the pseudo spot price of electricity algorithm (PSPA) is used to solve the transformed problem [26]. In [27] the EEDP has been addressed using Artificial Bee Colony (ABC) and Particle Swarm Optimization (PSO); which are two efficient optimization methods. Furthermore, Mousa and Kotb [29] presented a hybrid approach; which is a combination between two optimization techniques, genetic algorithm (GA) and local search. Additionally, Gargeya 1 et al. [30] presented a hybrid algorithm, GA and pattern search methods and the equality constraint is satisfied by penalty approach method.

Newly, many approaches are introduced to solve EEDP such as: modified PSO technique [31], reference point based multiobjective optimization using a combination between trust region algorithm and PSO [32], Grey wolf optimization [33], flower pollination algorithm [34], Kinetic gas molecule optimization [35], Colonial competitive differential evolution [36], chaotic teaching-learning-based optimization with Lévy flight (CTLBO) [37], one rank cuckoo search algorithm [38],

Hybrid Ant Colony Optimization (ACO), ABC and Harmonic Search (HS) algorithms [39], and differential evolution particle swarm optimizer [40].

Neural networks (NNs) are massively paralleled distributed computation, fast convergence and can be considered as an efficient method to solve real-time optimization problems. Due to the parallel mechanism and massive computing unit-neurons of NNs, the large-scale optimization problem can be solved efficiently [41, 42]. NNs have become widely used tools in many fields such as decision support tool, pattern recognition and secure communication [43-46].

The EEDP:

- (1) Have complex, non-smooth, nonlinear, non-convex characteristics, large number of equality and inequality constraints [23].
- (2) Very complex to solve by conventional methods because of non-linear objective function [18].
- (3) Evolutionary techniques suffer from the large number of solutions in the Pareto set; where the DM must be identifying one solution of the problem [13].
- (4) Fuzzy representation of EEDP makes it more realistic; where there are instabilities in the global market, and the rapid fluctuations of prices [14].

To overcome these reasons, this paper intends to present a novel methodology to solve the EEDP. In the new methodology, a reference point based NN with fuzziness is presented. In such a way, the DM participate early in the process of optimization instead of leaving him/her alone in the final choice. The aim is to identify the Pareto-optimal region nearest to the preference of DM to achieve the pollution restrictions which controlled using environmental preservation rules and to carry out the maximum cost limitation.

## 3. Description of the EEDP

EEDP aims to find a solution; which minimize two competing objective functions and satisfy several equality and inequality constraints. In general the EEDP is described as follows:

### 3.1. Objective Functions

- Fuel Cost Objective:

$$\text{Min } f_1(x) = \sum_{i=1}^N (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \text{ \$ / hr}; \quad (1)$$

where

$a_i, b_i, c_i$  : Fuel cost coefficients of generator  $i$ ,

$P_{Gi}$  : Power generated (p.u) by generator  $i$ ,

$N$ : Number of generators.

- Emission Objective:

$$\text{Min } f_2(\cdot) = \sum_{i=1}^N [10^{-2} (\alpha_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) + \xi_i \exp(\lambda_i P_{Gi})] \text{ ton / hr} \quad (2)$$

where

$\alpha_i, \beta_i, \gamma_i, \xi_i, \lambda_i$  : Coefficients of the  $i$ th generator's emission characteristic.

### 3.2. Constraints

- Limits Of Power Generation:

$$P_{Gi \min} \leq P_{Gi} \leq P_{Gi \max}, \quad i = 1, \dots, N; \quad (3)$$

where

$P_{Gi \min}$  : Minimum power generated

$P_{Gi \max}$  : Maximum power generated.

- Power balance constraint

$$\sum_{i=1}^n P_{Gi} - P_D - P_{Loss} = 0 \quad (4)$$

where

$P_D$  : Total load demand (p.u.),

$P_{loss}$  : Transmission losses (p.u.) [47].

- Security Constraints:

$$S_\ell \leq S_{\ell \max}, \quad \ell = 1, \dots, n_\ell; \quad (5)$$

where

$n_\ell$  : The number of transmission lines.

The system is considered as losses and the security constraint is released.

## 4. Fuzzy Multiobjective Optimization Problem

Fuzzy multiobjective optimization problem (F-MOP) with fuzzy parameters in the objective functions and constraints takes the following form:

$$\begin{aligned} \text{Minimize : } & \{f_1(X, \tilde{a}), f_2(X, \tilde{a}), \dots, f_M(X, \tilde{a})\} \\ \text{subject to: } & g_i(X, \tilde{a}) \leq 0, \quad i = 1, 2, \dots, r \end{aligned} \quad (6)$$

where

$X$ : Vector of decision variable,

$f_m(X, \tilde{a})$  : The  $m$ th objective function,

$g_i(X, \tilde{a}), i = 1, 2, \dots, r$  : The  $r$ th constraint vector,

$\tilde{a} = (\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n)$  : Vector of fuzzy parameters.

Fuzzy parameters are described by real fuzzy numbers. The real fuzzy numbers  $\tilde{a}$  form a convex continuous fuzzy subset of the real line whose membership function  $\mu_{\tilde{a}}(a)$  is defined by:

- 1) a continuous mapping from  $R^1$  to the closed interval  $[0, 1]$ ;
- 2)  $\mu_{\tilde{a}}(a) = 0$  for all  $a \in (-\infty, a_1]$ ;
- 3) strictly increasing on  $[a_1, a_2]$ ;
- 4)  $\mu_{\tilde{a}}(a) = 1$  for all  $a \in [a_2, a_3]$ ;
- 5) strictly decreasing on  $[a_3, a_4]$ ;
- 6)  $\mu_{\tilde{a}}(a) = 0$  for all  $a \in [a_4, +\infty)$ .

Definition 1. ( $\alpha$ -level set). The  $\alpha$ -level set ( $\alpha$ -cut) of the fuzzy numbers  $\tilde{a}$  is defined as the ordinary set  $L_\alpha(\tilde{a})$  for which the degree of their membership functions exceeds the level  $\alpha \in [0, 1]$ :

$$L_\alpha(\tilde{a}) = \{a \mid \mu_{\tilde{a}}(a) \geq \alpha\}. \quad (7)$$

For a certain degree  $\alpha$ , the (F-MOP) can be represented as a non-fuzzy as follows:

$$\begin{aligned} \text{Minimize : } & \{f_1(X, a), f_2(X, a), \dots, f_M(X, a)\} \\ \text{subject to: } & g_i(X, a) \leq 0, \quad i = 1, 2, \dots, r \\ & X = (x_1, x_2, \dots, x_n), \\ & a = (a_1, a_2, \dots, a_n) \\ & L_{\alpha i} \leq a_i \leq U_{\alpha i}; \end{aligned} \quad (8)$$

where constraint  $L_{\alpha i}$  and  $U_{\alpha i}$  are the lower and upper bound for the parameters  $a_i$ .

Definition 2. ( $\alpha$ -Pareto optimal solution).  $x^* \in X$  is said to be an  $\alpha$ -Pareto optimal solution to the ( $\alpha$ -VMP), if and only if there exist no other solution  $x \in X$  and  $a \in L_\alpha(\tilde{a})$  such that  $f_m(x, a) \geq f_m(x^*, a^*)$ ,  $\forall m = 1, 2, \dots, M$ , with strictly inequality holding for at least one  $i$ ; where the corresponding values of parameters  $a_i^*$  are called  $\alpha$ -level optimal parameters.

## 5. Solution Methodology

In this section, a framework for the proposed approach that involves three phases was presented. The first one defuzzified the F-MOP to the crisp multiobjective optimization problem (C-MOP) by using Alpha-cut, while the second phase employs a reference point based on NNs algorithm to solve the crisp optimization problem. Finally, identifies the best compromise solution from a finite set of alternatives using TOPSIS Technique in the phase three.

*Phase 1: Defuzzified the F-MOP*

Step 0: Formulate F-MOP

$$\begin{aligned} \text{Minimize : } & \{f_1(X, \tilde{a}), f_2(X, \tilde{a}), \dots, f_M(X, \tilde{a})\} \\ \text{subject to: } & g_i(X, \tilde{a}) \leq 0, \quad i = 1, 2, \dots, r \end{aligned}; \quad (9)$$

Step 1: Defuzzified F-MOP into C-MOP using Alpha-Level cut as follows:

$$\begin{aligned} \text{Minimize : } & \{f_1(X, a), f_2(X, a), \dots, f_M(X, a)\} \\ \text{subject to: } & g_i(X, a) \leq 0, \quad i = 1, 2, \dots, r \\ & X = (x_1, x_2, \dots, x_n) \\ & a = (a_1, a_2, \dots, a_n) \\ & L_{\alpha i} \leq a_i \leq U_{\alpha i} \end{aligned} \quad (10)$$

*Phase 2: Reference point based on NNs algorithm*

Step 2: Creating an achievement scalarizing problem using preferred reference point, where the DM plays an important role in the problem domain.

- Minimize and maximize the objective functions individually in the feasible region. This information must be given to the DM.
- The DM suggest preferred reference point  $\bar{z}$ , the reference point is a feasible or infeasible point in the

objective space. The suggested reference point is used to derive achievement scalarizing functions as follows:

$$\begin{aligned} \text{Minimize : } & \left( \sum_{i=1}^m w_i (f_i(X) - \bar{z}_i)^p \right)^{1/p} \\ \text{subject to : } & g_i(X, a) \leq 0, i = 1, 2, \dots, r \\ & X = (x_1, x_2, \dots, x_n), \\ & a = (a_1, a_2, \dots, a_n) \\ & L_{ai} \leq a_i \leq U_{ai}; \end{aligned} \quad (11)$$

To make the procedure interactive and useful in practice, Wierzbicki [48] suggested a procedure in which the obtained solution  $z'$  is used to create  $m$  new reference points, as follows:

$$z^{(j)} = \bar{z} + (z' - \bar{z}).e^{(j)}; \quad (12)$$

where  $e^{(j)}$  is the  $j$ -th coordinate direction vector.

Step 3: NN method is implemented for solving convex nonlinear programming problem (CNPP), which formulated in the previous step. The distinguishing features of the proposed network are that the primal and dual problems can be solved simultaneously [49].

I- Let the following be a general convex Nonlinear Programming (CNPP) problem:

$$\begin{aligned} u(t) &= \nabla_X E(z) = \lambda^T g(X).g(X)^T \lambda + \nabla g(X)^T [g(X) - |g(X)|] + \nabla_{XX}^2 L(z) \nabla_X L(z) + A^T (AX - b) \\ v(t) &= \nabla_\lambda E(z) = \lambda^T g(X).g(X) - \nabla g(X) \nabla_X L(z) + [\lambda - |\lambda|] \\ w(t) &= \nabla_\mu E(z) = -A \nabla_X L(z) \end{aligned} \quad (17)$$

V- States Updating:

$$\begin{aligned} X(t + \Delta t) &= X(t) - \Delta t u(t), \quad \lambda(t + \Delta t) = \lambda(t) - \Delta t v(t), \\ \mu(t + \Delta t) &= \mu(t) - \Delta t w(t). \end{aligned} \quad (18)$$

VI- Calculation:

$$s_1 = \sum_{i=1}^n u_i^2(t), \quad s_2 = \sum_{j=1}^r v_j^2(t), \quad s_3 = \sum_{j=1}^p w_j^2(t). \quad (19)$$

VII- Stopping criteria: if  $s_1 < \varepsilon$ ,  $s_2 < \varepsilon$  and  $s_3 < \varepsilon$  then

$$\begin{aligned} \text{Minimize : } & f(X), X \in R^n \\ \text{subject to : } & g_i(X) \geq 0, i = 1, 2, \dots, r \\ & h_i = a_i^T X - b_i, j = 1, 2, \dots, p \quad (p < n); \end{aligned} \quad (13)$$

where  $f(X)$  and  $g_i(X)$  are convex functions.

II- The dual problem of CNPP is formulated as follows [50]:

$$\begin{aligned} \max_{X, \lambda, \mu} & L(X, \lambda, \mu), X \in R^n \\ \text{subject to : } & \nabla_X L(X, \lambda, \mu) = 0 \\ & \lambda \geq 0; \end{aligned} \quad (14)$$

where  $\lambda = (\lambda_1, \lambda_2, \dots, \lambda_r)^T$ ,  $\mu = (\mu_1, \mu_2, \dots, \mu_p)^T$ ,

$$L(X, \lambda, \mu) = f(X) - \sum_{i=1}^r \lambda_i g_i(X) - \sum_{j=1}^p \mu_j h_j(X) \equiv L(z) \quad (15)$$

and

$$\nabla_X L(X, \lambda, \mu) = \nabla f(X) - \sum_{i=1}^r \lambda_i \nabla g_i(X) - \sum_{j=1}^p \mu_j \nabla h_j(X). \quad (16)$$

III- Parameter Initialization, Let  $t=0$ . Arbitrary choose initial vector  $x(t) \in R^n$ ,  $\lambda(t) \in R^r$ ,  $\mu(t) \in R^p$ ,  $\Delta t > 0$  ( $\Delta t = 0.0001$ ) and error  $\varepsilon = 10^{-9}$ .

IV- Computation of gradient:

output  $X(t + \Delta t), \lambda(t + \Delta t), \mu(t + \Delta t)$  otherwise  $t = t + \Delta t$  and go to step IV.

New Pareto optimal solutions are then found by forming new achievement scalarizing problems. If the DM is not satisfied with any of these Pareto-optimal solutions, a new reference point is suggested and the above procedure is repeated.

*Phase III: Identifying a Satisfactory operation point*

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**Input** data for  $N$  alternatives over  $Q$  attributes  $\leftarrow x_{ij} (i = 1, \dots, N, j = 1, \dots, Q)$ .

**Calculate** normalized rating  $r_{ij}$ .

**Develop** a set of weights  $W_j$  for each attribute; which are usually reflective of relative importance  $\leftarrow v_{ij} = w_j r_{ij}$ .

**Identify** the ideal alternative  $S^+ \leftarrow S^+ = \{v_1^+, v_2^+, \dots, v_j^+, \dots, v_q^+\} = \begin{cases} \min v_{ij} | j \in J_1 \\ \min v_{ij} | j \in J_2 \end{cases}, i = 1, \dots, N; \begin{matrix} J_1 : \text{emission attributes} \\ J_2 : \text{cost attributes} \end{matrix}$

**Identify** the nadir alternative  $S^- \leftarrow S^- = \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_{mq}^-\} = \begin{cases} \min v_{ij} | j \in J_1 \\ \min v_{ij} | j \in J_2 \end{cases}, i = 1, \dots, N$

**Calculate** the distance between each attribute and both ideal ( $D^+$ ) and nadir ( $D^-$ )

**For** each alternative, determine the ratio  $R \leftarrow R = \frac{D^-}{D^- + D^+}$

**Rank** alternative according to ratio  $R$  in descending order.

**Recommend** the alternative with the maximum ratio

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Fig. 1. The pseudo code of TOPSIS.

Optimization of the above-formulated problem using reference point based NN method yields a set of Pareto optimal solutions closed to the preferred reference point. To determine one operating points or identify satisfactory operation point (best

compromise solution) that satisfies different goals of the DM, TOPSIS method given by Hwang and Yoon [51, 52] is used. The Pseudo code of TOPSIS can be expressed in figure 1, while Figure 2 shows the flow chart of the proposed algorithm.

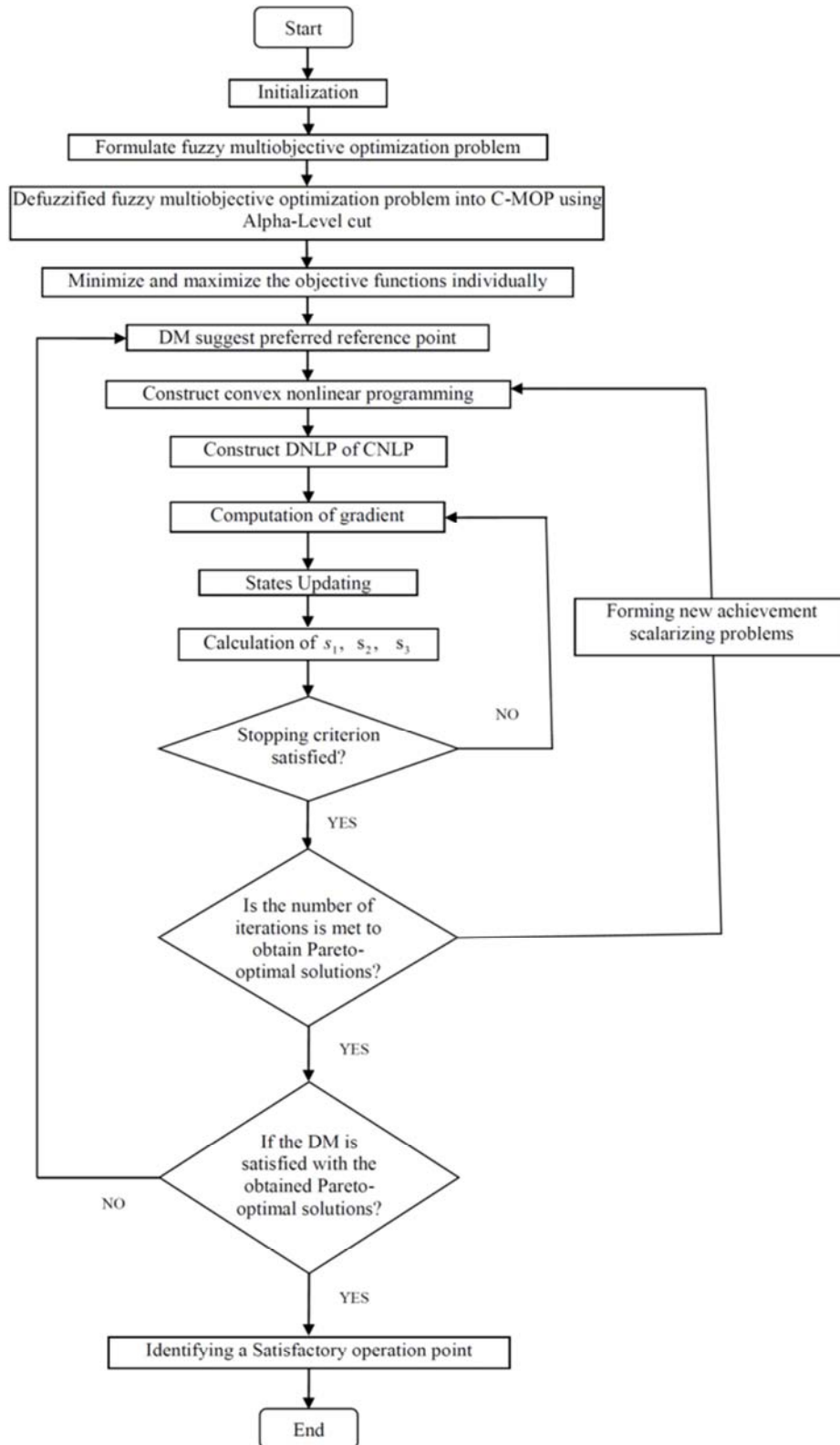


Fig. 2. The flow chart of the proposed algorithm.

## 6. Implementation of the Proposed Approach

The proposed approach has been applied to the standard IEEE 30-bus 6-generator test system. The single-line diagram of this system is shown in Figure 3 while the detailed data are given in Table 1 [53-55].

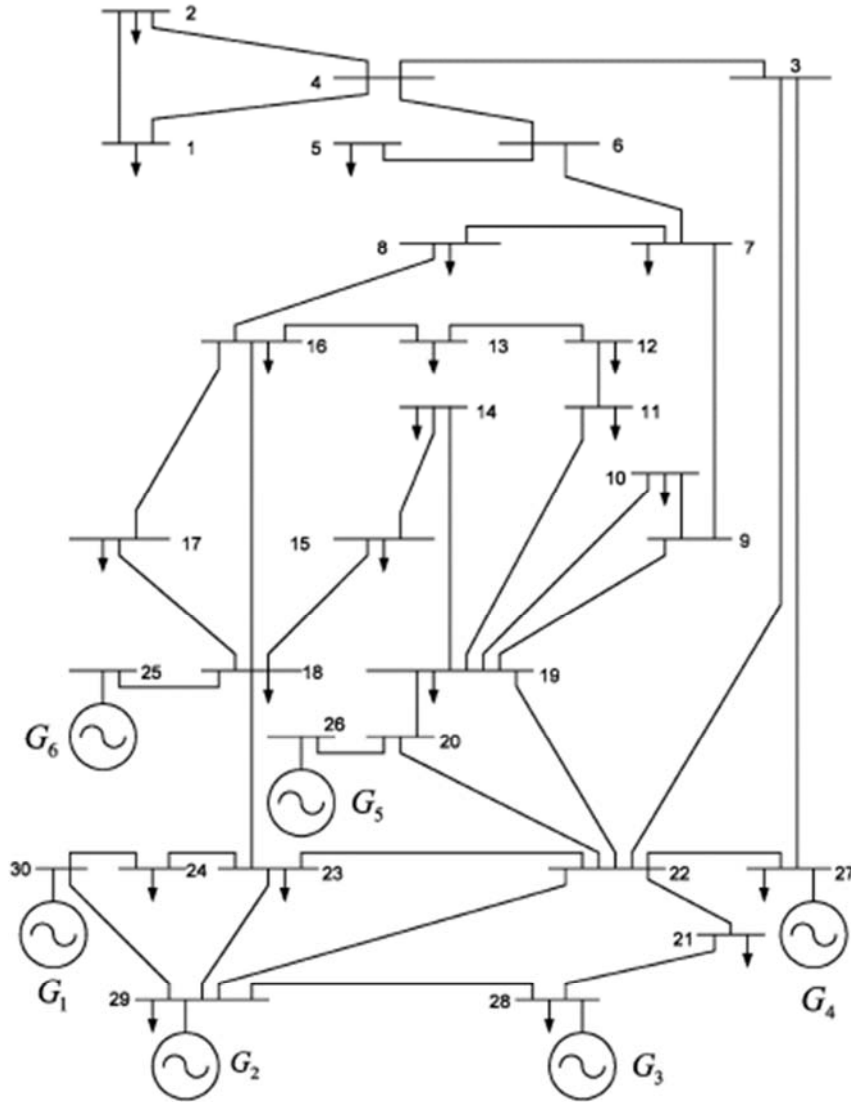


Fig. 3. Single line diagram of IEEE 30-bus 6-generator test system.

Table 1. Generator cost and emission coefficients.

		G1	G2	G3	G4	G5	G6
Cost	a	10	10	20	10	20	10
	b	200	150	180	100	180	150
	c	100	120	40	60	40	100
Emission	$\alpha$	4.091	2.543	4.258	5.426	4.258	6.131
	$\beta$	-5.554	-6.047	-5.094	-3.550	-5.094	-5.555
	$\gamma$	6.490	4.638	4.586	3.380	4.586	5.151
	$\zeta$	2.0E-4	5.0E-4	1.0E-6	2.0E-3	1.0E-6	1.0E-5
	$\lambda$	2.857	3.333	8.000	2.000	8.000	6.667

These data (cost and emission coefficients) have many controlled parameters, that values are vague and uncertain. So, each numerical value can be assigned by a specific grade of membership; where 0 represents the smallest possible grade of membership, and 1 is the largest one. Figure 4 show the fuzzy numbers that have been obtained from observing

the instabilities in the global market and rate of prices fluctuations or from interviewing DMs.

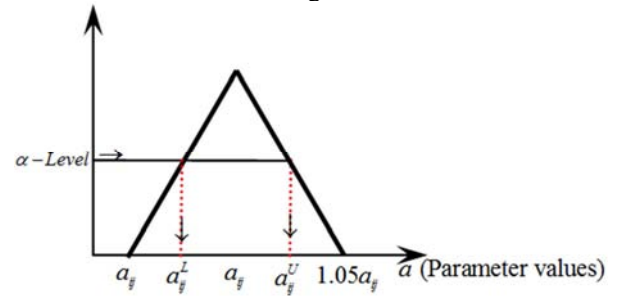


Fig. 4. Fuzzy numbers of the effectiveness of resource.

To transform a problem with these fuzzy parameters to a crisp version  $\alpha$ -cut level is used with the following

membership function:

$$\mu(a_{ij}) = \begin{cases} 1, & a = a_{ij} \\ \frac{20a}{a_{ij}} - 19 & 0.95a_{ij} \leq a \leq a_{ij} \\ 21 - \frac{20a}{a_{ij}} & a_{ij} \leq a \leq 1.05a_{ij} \\ 0 & a < 0.95a_{ij} \text{ or } a > 1.05a_{ij} \end{cases} \quad (20)$$

By this way, the fuzzy parameters can be transformed to a crisp one having upper and lower bounds  $[a_{ij}^L, a_{ij}^U]$ , which declared in figure 4. Consequently, each  $\alpha$ -cut level can be represented by the two end points of the Alpha level.

## 7. Validation and Evaluation of Results

In order to study the influence of fuzzy parameters on the obtained Pareto optimal solutions, all the range of the

parameter fluctuation (cost and emission coefficients) were scanned, two bounds of Alpha value have been considered  $\alpha = 0, \alpha = 1$  with some values between these bounds  $\alpha = 0.2, 0.4, 0.6, 0.8$ .

The DM plays an important role; where he/she expected to be an expert in the problem domain and provide us with different preferred reference point for each case as in figures (5-10). Figures (5-10) declare partial set of nondominated solutions which obtained by exploring the optimal Pareto frontier using different  $\alpha$ -cut level and certain preferred reference point. For every reference point, partial set of the Pareto frontier were found closely to the preferred reference point. Graphical presentations of the experimental results are presented in figures (5-10) for six instances ( $\alpha = 0, 0.2, 0.4, 0.6, 0.8, 1$ ) with different three preferred reference point. Also, it is obvious from figures (5-10), that the results maintain the diversity and convergence for all  $\alpha$ -cut level.

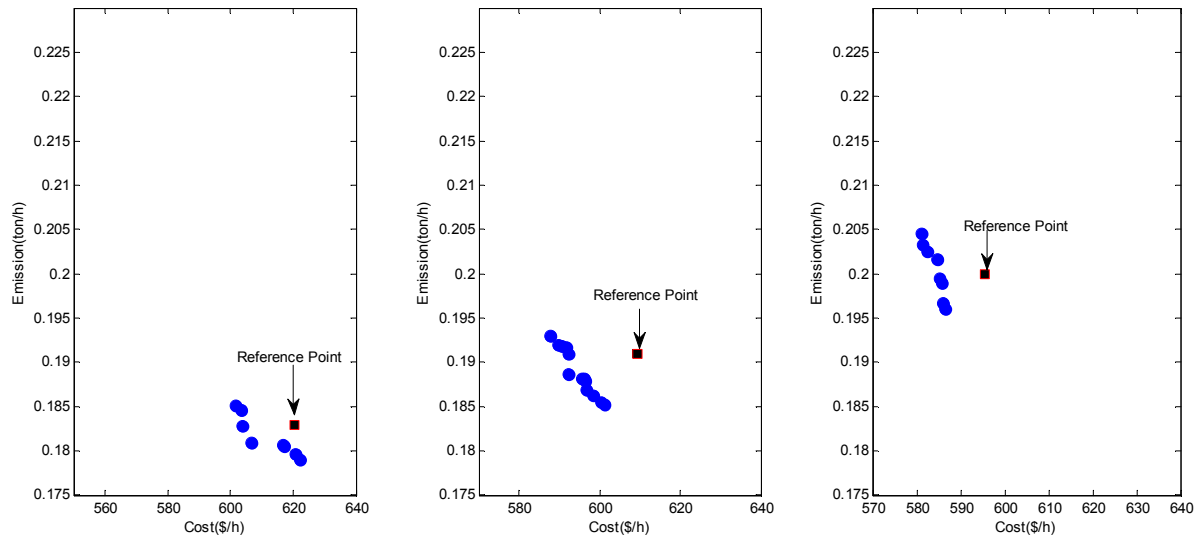


Fig. 5. Pareto optimal set for  $\alpha$  cut level = 0.

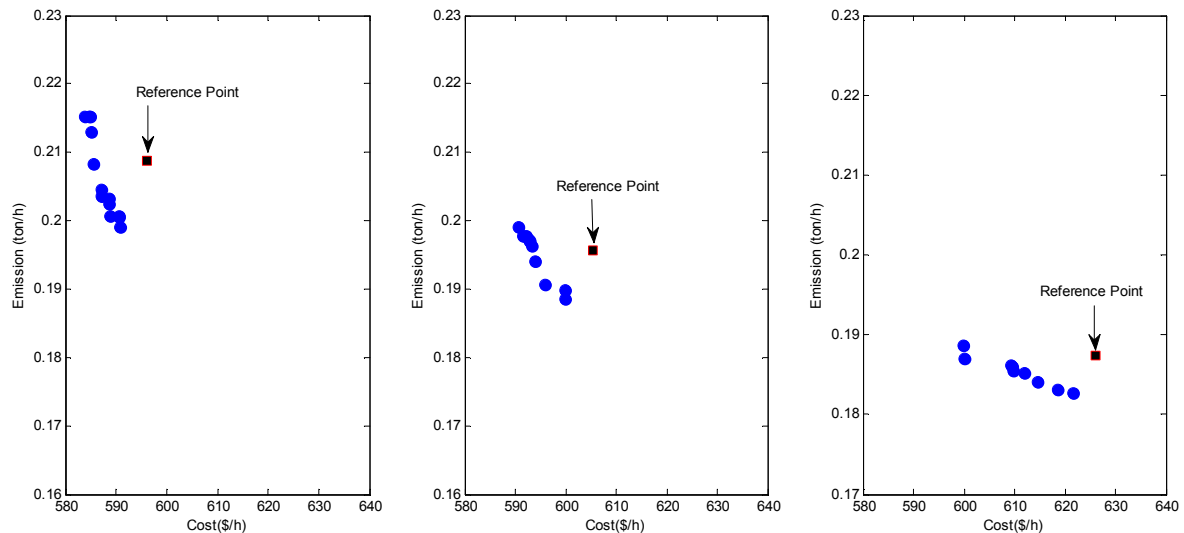


Fig. 6. Pareto optimal set for  $\alpha$  cut level = 0.2.



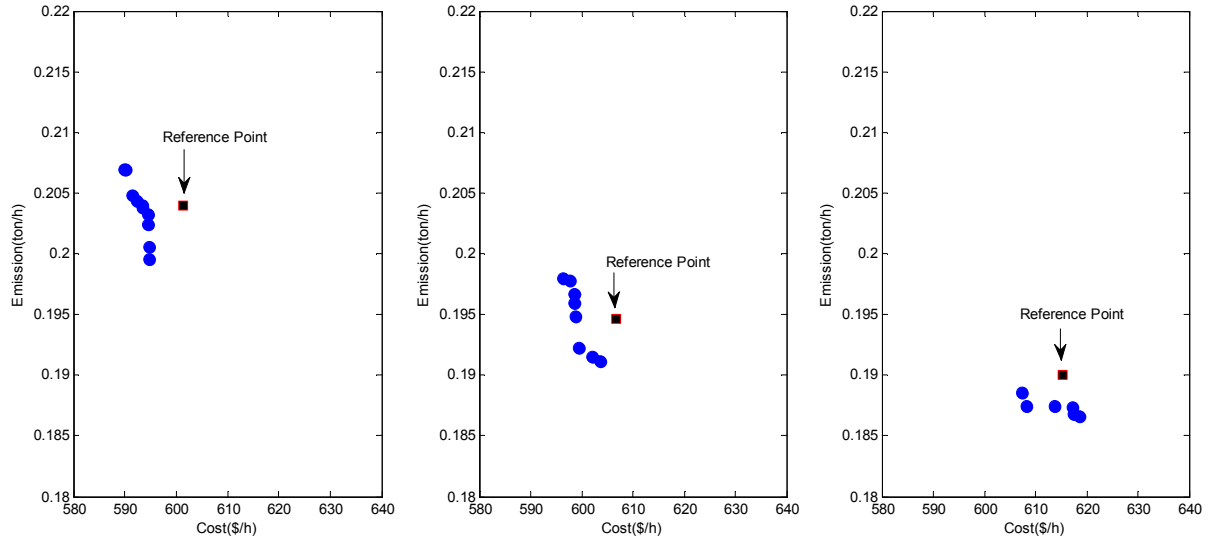


Fig. 7. Pareto optimal set for  $\alpha$  cut level =0.4.

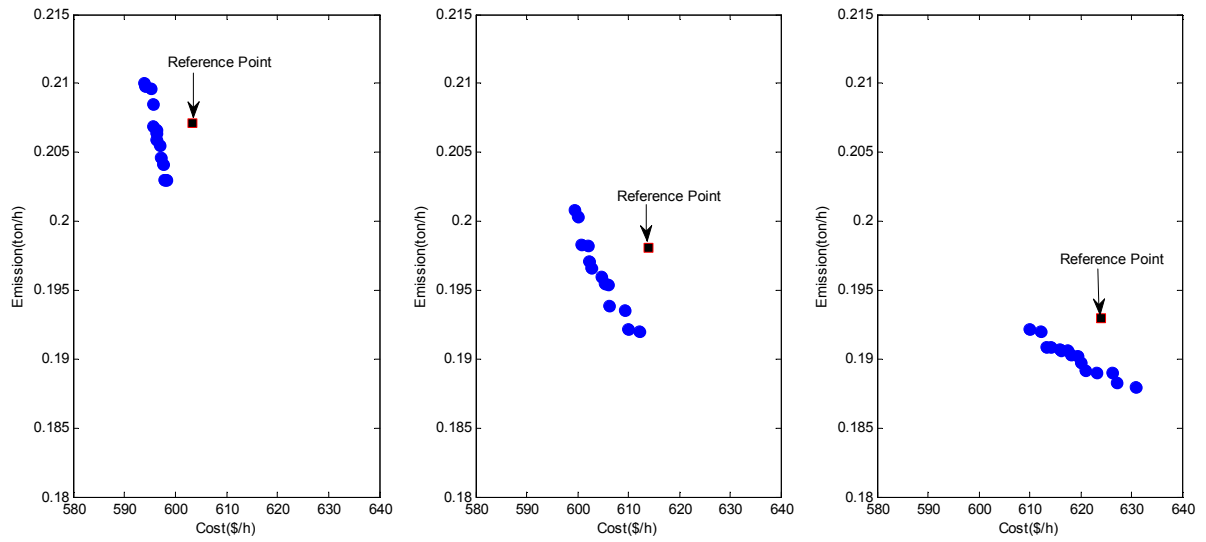


Fig. 8. Pareto optimal set for  $\alpha$  cut level =0.6.

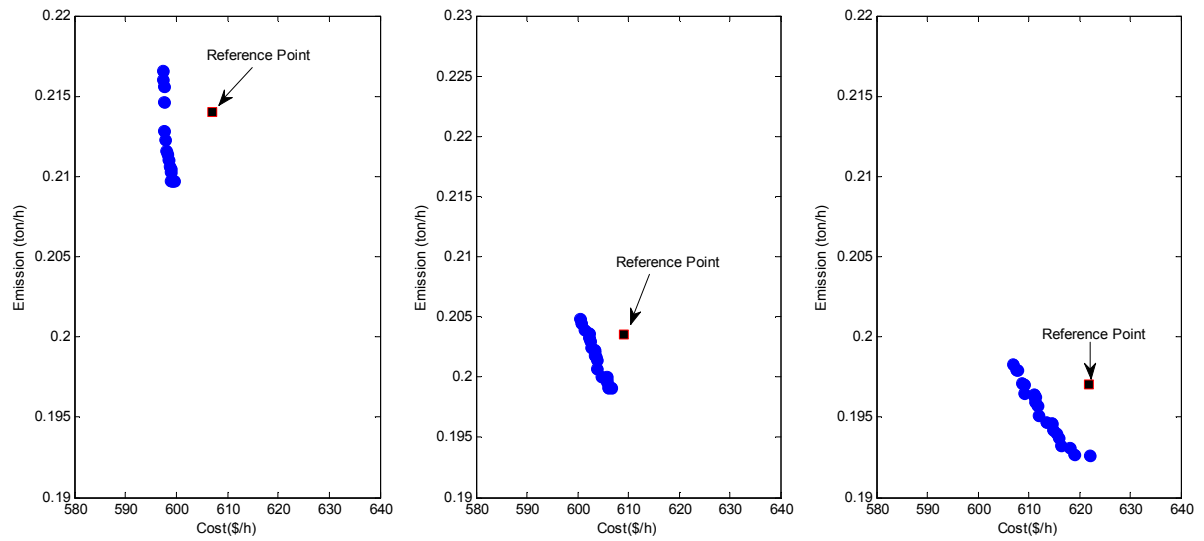


Fig. 9. Pareto optimal set for  $\alpha$  cut level =0.8.



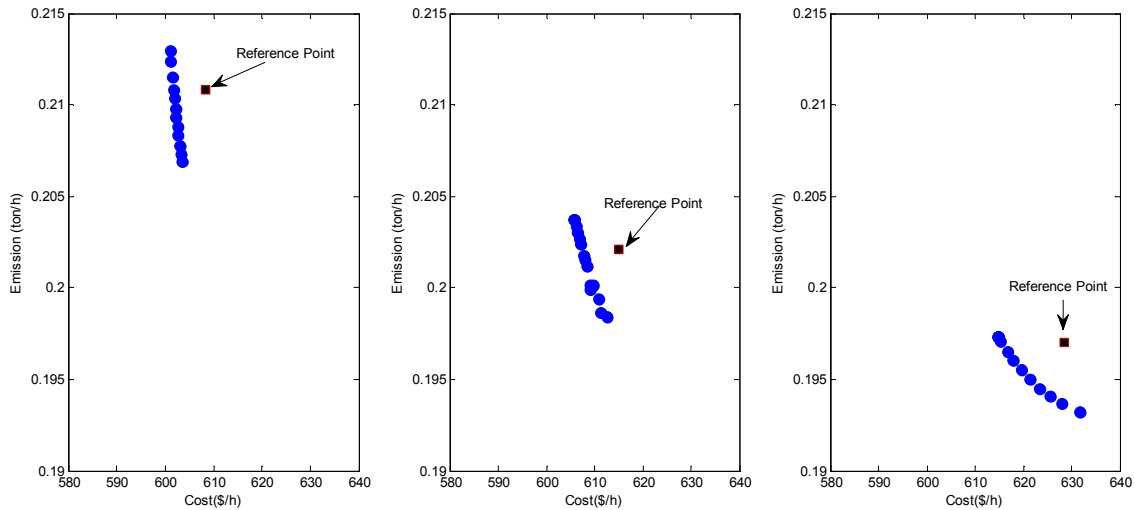


Fig. 10. Pareto optimal set for  $\alpha$  cut level = 1.0.

We can say that, such procedure will provide the DM with a set of Pareto solutions near her/his preference; which allows the DM to concentrate only to those regions on the Pareto optimal frontier which are of interest to her/him preference. However the proposed procedure is able to find solutions near the supplied reference points, it can't find one operation point on the entire Pareto-optimal front corresponding to the preferred reference point. A technique to identify best compromise solution is implemented, which mean that the task of choosing a single preferred Pareto optimal solution from the resulting partial set is also an important task which has discussed below.

### 7.1. Identifying a Satisfactory Solution

For this practical application (EEDP), we need to select one solution, which will satisfy the different goals, such a solution is called best compromise solution. These goals are to avoid breaching environmental protection rules, or the generating cost must not exceed allowable limitation. To select the best compromise solution, TOPSIS method is used.

TOPSIS method has the ability to identify the best alternative from a finite set of alternatives quickly, where it can incorporate relative weights of criterion importance

according DM preference and environmental protection rules. Here, the human DM plays an important role; where he/she expected to be an expert in the problem domain. The effect of changing the weights on the fuel cost and emission was studied. In each case one weight is changed linearly, and the other weight are generated in such a way that  $w_1 + w_2 = 1$ . In contrast, we observed the weights and the corresponding values of  $f_1(\cdot), f_2(\cdot)$  to conclude best compromise operating point. In each case, one weight is changed linearly taking six values. Consequently, six solutions of the objective functions is obtained corresponding to the six weights are drawn vs. weights for the three cases as shown in Figures (11-16). From Figures (11-16) the following points may be concluded:

- (1) One operation point has been selected, depending on the user defined weights.
- (2) From a finite set of alternatives, the proposed algorithm has the ability to identify the best operating point quickly.
- (3) The best compromise solution which is identified by the proposed scheme satisfies all the different goals given by the DM.
- (4) From the computational results, this scheme saved the time taken by the DM to select best compromise solution, and this due to reducing the Pareto set to a manageable size.

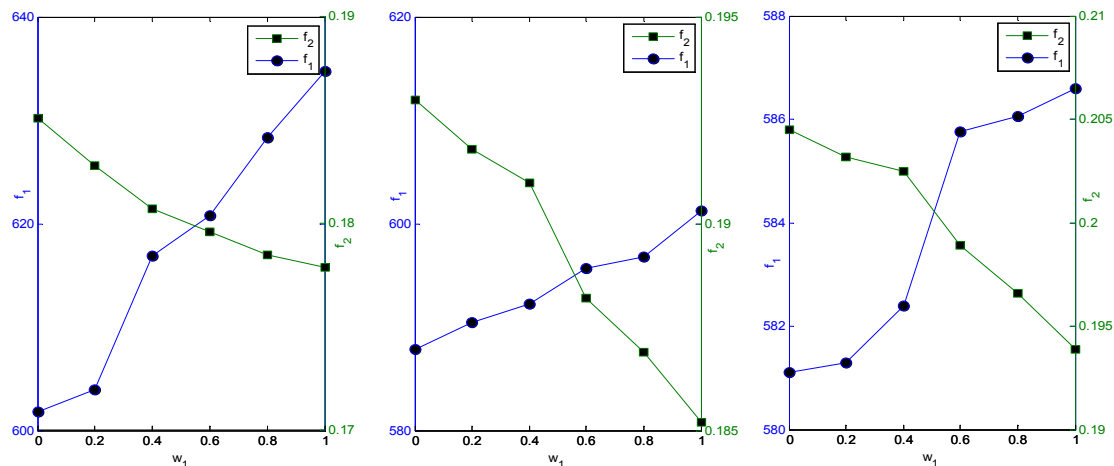
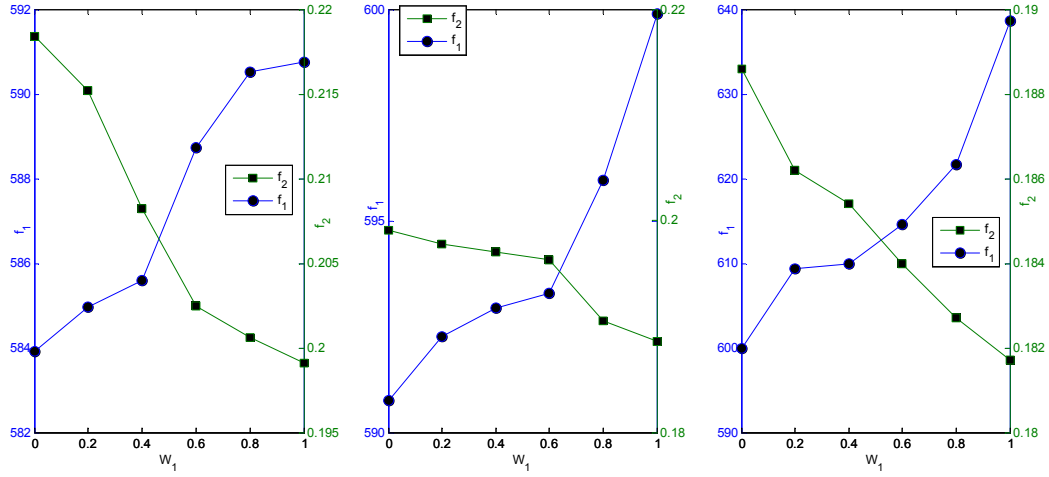
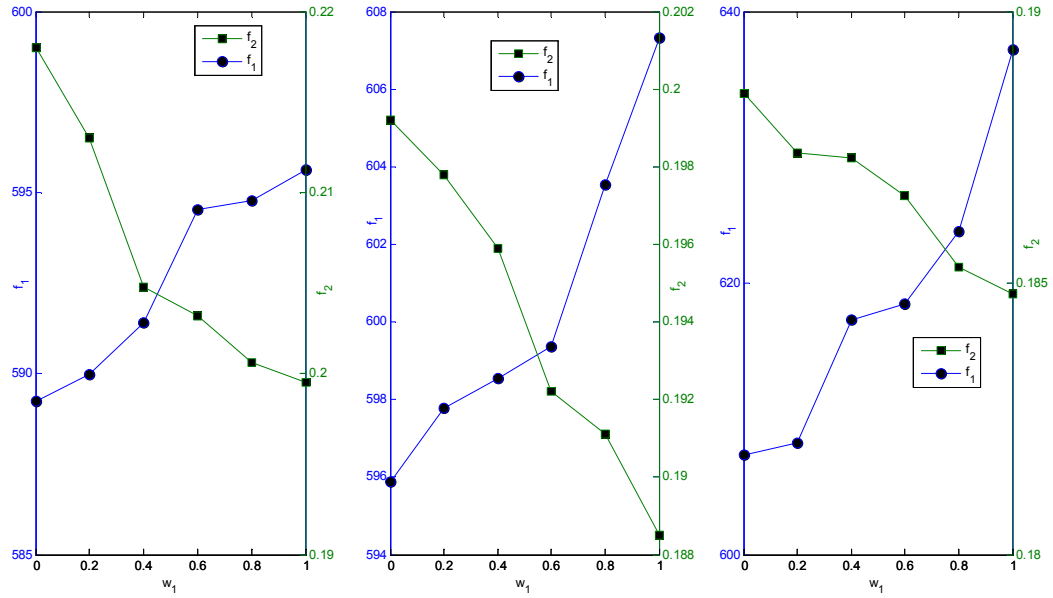
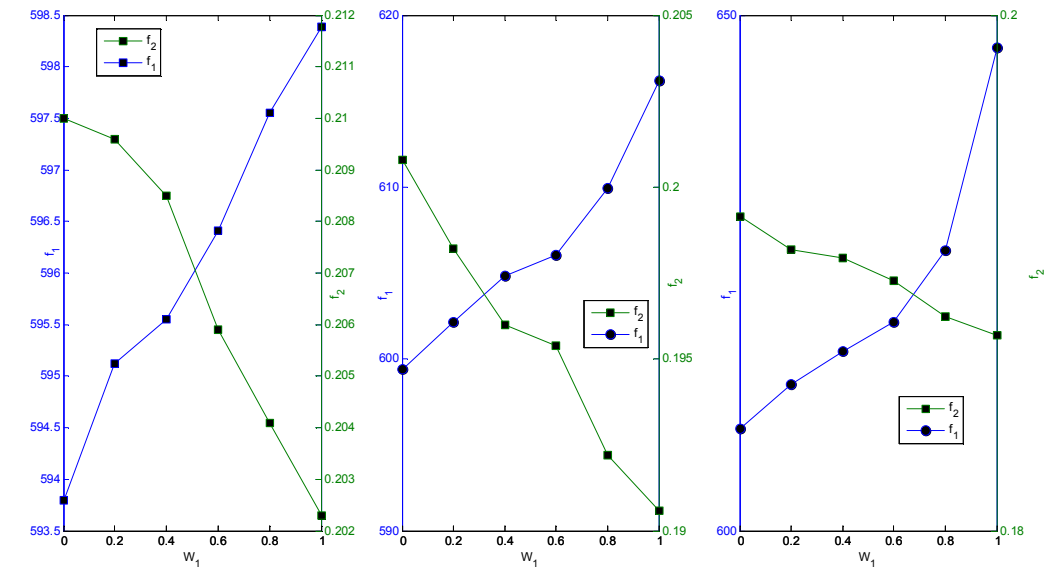


Fig. 11. Best compromise solution for different weights for  $\alpha$  cut level = 0.

Fig. 12. Best compromise solution for different weights for  $\alpha$  cut level =0.2.Fig. 13. Best compromise solution for different weights for  $\alpha$  cut level=0.4.Fig. 14. Best compromise solution for different weights for  $\alpha$  cut level=0.6.

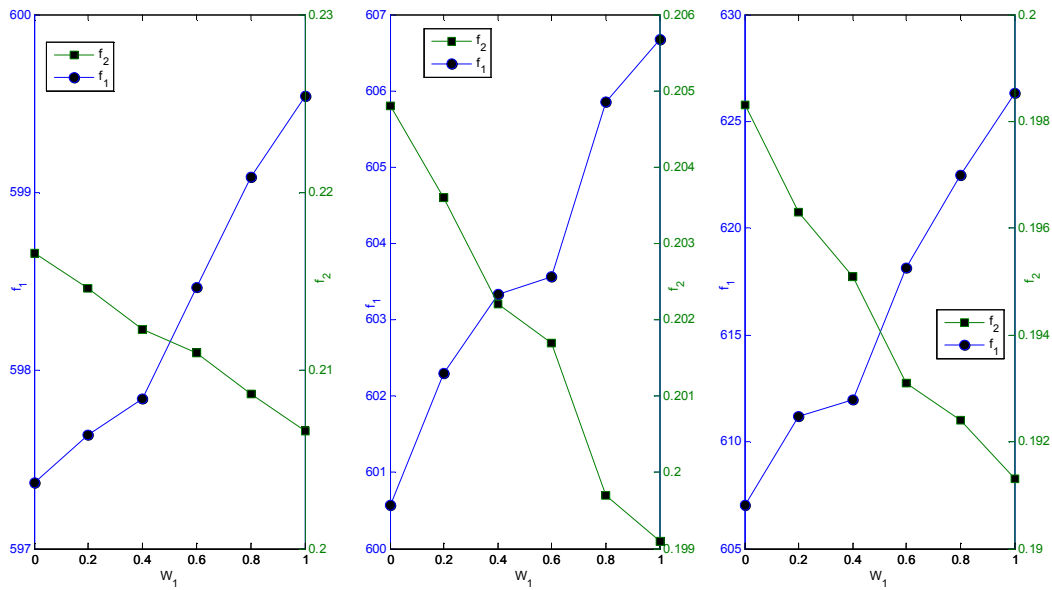


Fig. 15. Best compromise solution for different weights for  $\alpha$  cut level=0.8.

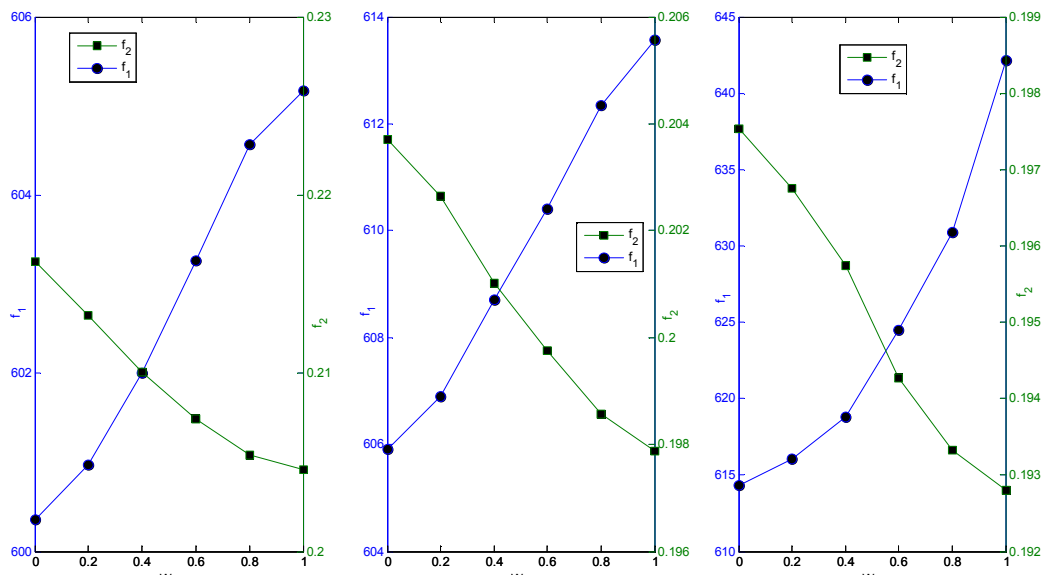


Fig. 16. Best compromise solution for different weights for  $\alpha$  cut level=1.

## 7.2. Comparative Study

In this subsection, a comparative study has been carried out to assess the proposed approach. Firstly, our approach uses only the objective function information, not derivatives or other auxiliary knowledge. Therefore it can deal with the non-smooth, non-continues and non-differentiable functions which are actually existed in practical optimization problems. Furthermore, most of the conventional techniques give single point (at each iteration) of problem solving by converting the multiobjective problem to a single objective problem by linear combination of different objectives as a weighted sum. On the other hand, the proposed approach generates a set of a manageable size accordingly to DM preference.

Additionally, evolutionary techniques suffer from the large size of the Pareto set, where the DM must identify one

alternative solution. Therefore, the proposed approach has been used to reduce the Pareto set to a manageable size closed to the DM preference. Moreover, our approach goal is not only to prune a given set, but also to generate a representative subset, which maintains the characteristics of the general Pareto set and take the DM preference into consideration.

In addition, the proposed algorithm by using TOPSIS technique has the ability to identify the best operating compromise point from a finite set of alternatives quickly, which will satisfy the different goals, given by the DM. So we can say that this scheme saved the time taken by the DM to select the best compromise solution.

Finally, our approach has many major advantages which can be mentioned in the following points:

- Suitable to handle EEDP
- Simple concepts.
- Easy implementation.
- Less execution efforts.
- More flexible and adaptive to a wide variety of problems and robust than the conventional methods.
- Can handle implications of global financial crisis, instabilities in the global market, and the rapid fluctuations of prices.
- Able to reduce the Pareto set to a manageable size closed to the DM preference.
- Able to identify the best operating point from a set of alternatives quickly.
- Satisfy the different goals, such as environmental protection rules, and allowable limitation of the generating cost.
- Its limitation that it have time consuming for solving EEDP, but this not represent any problem to us; where the EEDP is solved off-line before operating the system.

## 8. Conclusions

In this paper, reference point based NN algorithm is proposed for solving FM-EEDP. The fuzzy representation of the EEDP introduced because there are instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices. Firstly, FM-EEDP has been converted to C-MOP using  $\alpha$ -cut level. Secondly reference point based NN algorithm is implemented in such a way that the DM participates early in the optimization process instead of leaving him/her alone with the final choice. Moreover to help the DM to identify the best compromise solution from a finite set of alternatives, TOPSIS method is implemented. Such procedure gives the DM a better and more reliable decision. Simulation results are presented for the standard IEEE 30-bus system-6 generator which shows the effectiveness and potential of the proposed approach to solve EEDP. The main features of the proposed algorithm could be summarized as follows:

- Our approach can deal with the non-smooth, non-continues and non-differentiable functions which are actually existed in practical optimization problems.
- Our approach suitable to handle EEDP, simple concepts, easy implementation, less execution efforts, more flexible and adaptive to a wide variety of problems and robust than the conventional methods.
- Fuzzy representation of the EEDP make our approach can deal with instabilities in the global market, implications of global financial crisis and the rapid fluctuations of prices.
- From the computational results, our approach saved the time taken by the DM to select best comprise solution, and this due to reducing the Pareto set to a manageable size.
- Using TOPSIS technique has the ability to identify the best operating compromise point from a finite set of alternatives quickly, which will satisfy the different

goals, given by the DM.

- The proposed approach can identify the best operating point without applying the method again and again.

In future works, more complex real-world applications is tested by the proposed algorithm. In addition, conduct research on the parallel mechanism of multi-reference point algorithms and multi-criteria decision group problems to improve the efficiency of such approaches; which are very relevant for real- world scenarios.

## References

- [1] L. Alcorta, F. Nixon, The Global Financial Crisis and the Developing World :Impact on and Implications for the Manufacturing Sector, United nations: Industrial development organization, Vienna (2011) 1-51.
- [2] A. A. Mousa, I. M. El\_Desoky, Stability of Pareto optimal allocation of land reclamation by multistage decision-based multipheromone ant colony optimization, Swarm and Evolutionary Computation 13 (2013) 13–21.
- [3] R. Bellman, L. Zadeh, Decision Making in a fuzzy environment, Management Science 17 (1970) 141-164.
- [4] M. Sakwa, Fuzzy sets and Interactive Multiobjective Optimization, Plenum Press, New York (1993).
- [5] S. F. Brodesky, R. W. Hahn, Assessing the influence of power pools on emission constrained economic dispatch, IEEE Trans. Power Syst. 1 (1) (1986) 57–62.
- [6] A. Farag, S. Al-Baiyat, T. C. Cheng, Economic load dispatch multiobjective optimization procedures using linear programming techniques, IEEE Trans. Power Syst. 10 (2) (1995) 731–738.
- [7] C. S. Chang, K. P. Wong, B. Fan, Security-constrained multiobjective generation dispatch using bicriterion global optimization, IEE Proc. Gen. Transm. Distrib. 142 (4) (1995) 406–414.
- [8] J. X. Xu, C. S. Chang, X. W. Wang, Constrained multiobjective global optimization of longitudinal interconnected power system by genetic algorithm, IEE Proc. Gen. Transm. Distrib. 143 (5) (1996) 435–446.
- [9] J. Zahavi, L. Eisenberg, Economic–environmental power dispatch, IEEE Trans. Syst. Man Cybern. SMC 5 (5) (1985) 485–489.
- [10] Y. T. Hsiao, H. D. Chiang, C. C. Liu, Y. L. Chen, A computer package for optimal multiobjective VAR planning in large scale power systems, IEEE Trans. Power Syst. 9 (2) (1994) 668–676.
- [11] B. S. Kermanshahi, Y. Wu, K. Yasuda, R. Yokoyama, Environmental marginal cost evaluation by non-inferiority surface, IEEE Trans. Power Syst. 5 (4) (1990) 1151–1159.
- [12] M. A. Abido, A novel multiobjective evolutionary algorithm for environmental/economic power dispatch, Electr. Power Syst. Res. 65 (2003) 71–81.
- [13] M. A. Abido, Environmental/economic power dispatch using multiobjective evolutionary algorithms, IEEE Trans. Power Syst. 18 (4) (2003) 1529–1537.

- [14] A. A. Galal, A. A. Mousa, B. N. Al-Matrafi, Ant Colony Optimization Approach Based Genetic Algorithms for Multiobjective Optimal Power Flow Problem under Fuzziness, *Applied Mathematics* 4 (2013) 595-603, doi: 10.4236/am.2013.44084.
- [15] M. Azzam and A. A. Mousa, Using genetic algorithm and topsis technique for multiobjective reactive power compensation, *Electric Power Systems Research* 80 (2010) 675–681.
- [16] M. S. Osman, M. A. Abo-Sinna, and A. A. Mousa, IT-CEMOP: An Iterative Co-evolutionary Algorithm for Multiobjective Optimization Problem with Nonlinear Constraints, *Journal of Applied Mathematics & Computation (AMC)* 183 (2006) 373-389.
- [17] M. R. Gent, J. W. Lamont, Minimum-Emission Dispatch, *IEEE Transactions on Power Apparatus and Systems PAS-90* (6) (1971) 2650-2660.
- [18] J. Nanda, D. P. Kothari, K. S. Lingamurthy, Economic-Emission Load Dispatch through Goal Programming Techniques, *IEEE Transactions on Energy Conversion* 3 (1) (1988) 26-32.
- [19] J. S. Dhillon, S. C. Parti, D. P. Kothari, Multiobjective Optimal Thermal Power Dispatch. *Electrical Power and Energy Systems*, 16 (6) (1994) 383-389.
- [20] M. A. Abido, A Niched Pareto Genetic Algorithm for Multiobjective Environmental/Economic Dispatch, *Electrical Power and Energy Systems* 25 (2) (2003) 97-105.
- [21] R. T. F. Ah King, H. C. S. Rughooputh, Elitist Multiobjective Evolutionary Algorithm for Environmental/Economic Dispatch, *IEEE Congress on Evolutionary Computation*, Canberra, Australia 2 (2003) 1108-1114.
- [22] M. A. El-Shorbagy, A. A. Mousa, S. M. Nasr, A chaos-based evolutionary algorithm for general nonlinear programming problems, *Chaos, Solitons and Fractals* 85 (2016) 8-21.
- [23] X.-S. Yang, S. S. S. Hosseini, A. H. Gandomi, Firefly Algorithm for solving non-convex economic dispatch problems with valve loading effect, *Applied Soft Computing* 12 (2012) 1180–1186.
- [24] L. Xie, S. Wang, Z. Wu, Study on Economic, Rapid and Environmental Power Dispatch Based on Fuzzy Multi-objective Optimization, *Modern Applied Science* 3 (6) (2009) 38-44.
- [25] K. K. Vishwakarma, H. M. Dubey, M. Pandit and B. K. Panigrahi, Simulated Annealing Approach For Solving Economic Load Dispatch Problems With Valve Point Loading Effects, *International Journal of Engineering, Science and Technology* 4 (4) (2012) 60-72.
- [26] C. Yaser, A pseudo spot price of electricity algorithm applied to environmental economic active power dispatch problem, *Turk J Elec Eng & Comp Sci* 20 (6) (2012) 990-1005.
- [27] E. D. Manteaw, N. A. Otero, Multi-objective environmental/economic dispatch solution using ABC\_PSO hybrid algorithm, *International Journal of Scientific and Research Publications* 2 (12) (2012) 1-7.
- [28] A. A. Mousa, M. A. El-Shorbagy, W. F. Abd El-Wahed, Local search based hybrid particle swarm optimization for multiobjective optimization, *International journal of Swarm and evolutionary computation* 3 (2012) 1-14.
- [29] A. A. Mousa, K. A. Kotb, Hybrid multiobjective evolutionary algorithm based technique for economic emission load dispatch optimization problem, *Scientific Research and Essays* 7 (25) (2012) 2242-2250.
- [30] M. A. Gargeya, S. P. Pabba, Economic Load Dispatch Using Genetic Algorithm And Pattern Search Methods *International Journal Of Advanced Research In Electrical, Electronics And Instrumentation Engineering* 2 (4) (2013) 1203-1212.
- [31] Hardiansyah, A Modified Particle Swarm Optimization Technique for Economic Load Dispatch with Valve-Point Effect, *I. J. Intelligent Systems and Applications* 7 (2013) 32-41.
- [32] A. A. El-Sawy, Z. M. Hendawy, M. A. El-Shorbagy, Reference Point Based TR-PSO for Multi-Objective Environmental/Economic Dispatch, *Applied Mathematics* 4 (2013) 803-813.
- [33] M. Pradhan, P. K. Roy b, T. Pal, Grey wolf optimization applied to economic load dispatch problems, *Electrical Power and Energy Systems* 83 (2016) 325–334.
- [34] A. Y. Abdelaziz, E. S. Ali, S. M. Abd Elazim, Implementation of flower pollination algorithm for solving economic load dispatch and combined economic emission dispatch problems in power systems, *Energy* 101 (2016) 506-518.
- [35] M. Basu, Kinetic gas molecule optimization for nonconvex economic dispatch problem, *Electrical Power and Energy Systems* 80 (2016) 325–332.
- [36] M. Ghasemi, M. Taghizadeh, S. Ghavidel, A. Abbasian, Colonial competitive differential evolution: An experimental study for optimal economic load dispatch, *Applied Soft Computing* 40 (2016) 342–363.
- [37] X. He, Y. Rao, J. Huang, A novel algorithm for economic load dispatch of power systems, *Neurocomputing* 171 (2016) 1454–1461.
- [38] T. T. Nguyen, D. N. Vo, The application of one rank cuckoo search algorithm for solving economic load dispatch problems, *Applied Soft Computing* 37 (2015) 763–773.
- [39] T. Sen, H. D. Mathur, A new approach to solve Economic Dispatch problem using a Hybrid ACO–ABC–HS optimization algorithm, *Electrical Power and Energy Systems* 78 (2016) 735–744.
- [40] M. Ghasemi, J. Aghaei, E. Akbari, S. Ghavidel, L. Li, A differential evolution particle swarm optimizer for various types of multi-area economic dispatch problems, *Energy* 107 (2016) 182-195.
- [41] Q. Liu, J. Wang, A one-layer recurrent neural network with a discontinuous hard-limiting activation function for quadratic programming, *IEEE Transactions on Neural Networks* 19 (2008) 558–570.
- [42] Q. Liu, J. Wang, Finite-time convergent recurrent neural network with a hard-limiting activation function for constrained optimization with piecewise linear objective functions, *IEEE Transactions on Neural Networks* 22 (2011) 601–613
- [43] Y. Yang, J. Cao, X. Xu, J. Liu, A generalized neural network for solving a class of minimax optimization problems with linear constraints, *Applied Mathematics and Computation* 218 (2012) 7528–7537.

- [44] R. Furtuna, S. Curteanu, and F. Leon, An elitist non-dominated sorting genetic algorithm enhanced with a neural network applied to the multi-objective optimization of a polysiloxane synthesis process, *Engineering Applications of Artificial Intelligence* 24 (2011) 772–785.
- [45] E. N. Dragoi, S. Curteanu, A. Galaction, D. Cascaval, Optimization methodology based on neural networks and self-adaptive differential evolution algorithm applied to an aerobic fermentation process, *Appl. Soft Comput. J.* 13 (1) (2013) 222–238.
- [46] X. Hu, Applications of the general projection neural network in solving extended linear-quadratic programming problems with linear constraints, *Neuro computing* 72 (2009) 1131–1137.
- [47] B. S. Kermanshahi, Y. Wu, K. Yasuda, R. Yokoyama, Environmental marginal cost evaluation by non-inferiority surface, *IEEE Trans. Power Syst.* 5 (4) (1990) 1151–1159.
- [48] A. P. Wierzbicki, The use of reference objectives in multiobjective optimization, *Multiple Criteria Decision Making Theory and Applications*, Berlin: Springer-Verlag 177 (1980) 468–486.
- [49] K.-z. Chen, Y. Leung, K. S. Leung, X.-b. Gao, A Neural Network for Solving Nonlinear Programming Problem, *Neural Computing & Applications* 11 (2) (2002) 103–111.
- [50] O. L. Mangasarian, nonlinear programming, McGraw-Hill Book Company, New York (1994).
- [51] C. L. Hwang, K. Yoon, *Multiple Attribute Decision Making: Methods and Applications*, Springer-Verlag, New York, 1981.
- [52] D. L. Olson, Comparison of weights in TOPSIS models, *Math. Comput. Model.* 40 (2004) 721–727.
- [53] J. Carpentier, Contribution to the economic dispatch problem. *Bulletin Society Francaise Electriciens* 3 (8) (1962) 431–447.
- [54] A. J. Wood, F. Bruce, *Power generation operation and control*. Wollenberg: John Wiley & Sons, Inc, 1984.
- [55] R. Zimmerman, D. Gan, MATPOWER: A Matlab power system simulation package, Available: <http://www.pserc.cornell.edu/matpower/>.