



# Detection of the Magnetic Field Induced by the Wake of a Moving Submerged Body Using Simple Models

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**Abstract:** Wakes induced by the motion of submerged bodies may extend for several kilometers and the magnetic field induced by the motion of conductive sea water in the wake of a moving body may be detected in certain condition. Numerical simulations are used to compute the characteristic of wake magnetic field and compare it with wind wave's magnetic fields, which can be detected using the modern magnetometers. Using the spectral analysis of the samples, the results show that the spectrum has a sharp peak, which can be detected in principle depending on sensitivity of the detector, body disturbance and environmental conditions.

**Keywords:** Marine Electromagnetic Field, Wake, Wind Waves

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## 1. Introduction

As long as nearly 200 years ago, wave motion or moving conductor seawater at sea through the geomagnetic field can induce electromagnetic field predicted by Faraday and verified by Wollaston in 1851. Since then, many theory studies have been conducted, the marine electromagnetic field is generated by ocean wave or current, that is wind waves or swells [1], ocean internal wave [2, 3], ocean currents [4-6]. Meanwhile, with the development of magnetometers, many measurements have been conducted and verified it [7]. In the 1990s, the research of electromagnetic field induce by the motion of the seawater has been studied more and more maturely [8, 9], the examples are the tsunami and currents detection using the magnetometers were reported [10, 11]. Recently years, detection the ships and submerged bodies using the magnetic abnormal arise [12-16], theory and methods to detect the ships, also present the likelihood for detection the submerged body in the sea.

Madurasinghe and Tuck [12, 13] may be the first people who investigate detection feasibility using the magnetic induced by the motion wakes of the ocean targets. The

principle of the detection is same as the moving seawater, the targets generate magnetic field consists in seawater cutting the geomagnetic field. The wakes of the ships or submerged bodies induced magnetic field anomaly in the geomagnetic field, which can be passive sensing of a high-sensitivity magnetometer or superconducting quantum interference devices gradiometer carried by a moving airplane. Zou and Nehorai [14] give a method to detect ships using an airborne magnetic transducer; Yaakobi et al [15] give the detection feasibility in a moderate sea state, considering the wind wave noise magnetic field in the detection scene.

For a long time, the detection of submerged bodies through measurement of the induced electromagnetic field played only an assistant role, usually using sound techniques. But due to the fact that silencing of modern submarines has greatly reduced the range of both passive and active acoustic detection, the demand to use other means of detection keeps growing rapidly. One of the method employ the wakes detection, however, the wake of the submerged bodies is a very large feature, and may extend for tens of kilometers and exists for hours under certain conditions. For the wake detection, there are often two techniques, one of them is SAR remote sensing, and the other is airborne magnetic magnetometers or gradiometers remote sensing. For the SAR

remote sensing, it takes account of the internal waves induced by the moving submerged bodies which modulate the surface gravity-capillary wave [17], the sea surface electromagnetic scattering characteristics changing simultaneity, the SAR can identify it at certain conditions. In this paper, we will focus on the magnetometers remote sensing the wake magnetic field assuming an airborne flying platform, and considering the real seas, the measurement magnetic field include the submerged body wake and wind waves, the first quantity is considered as a signal to be determined, whereas the second is treated as noise. The ratio of signal-to-noise or the magnetic spectrum serves as an important characteristic parameter of the present problem.

## 2. Electromagnetic Field Generated by Moving Submerged Body and Wind Wave

### 2.1. Magnetic Field Coordinate System and Equations

For calculating the marine electromagnetic field from the moving submerged body or wind wave in the marine environment, the Cartesian coordinate systems are introduced firstly. Considering the fact that, at the free surface, the amplitude of the wind wave is much smaller than the wavelength, the mean sea free surface is taken to be plane  $z = 0$  in a rectangular coordinate system, with the  $z$  axis directed upward into air, the  $x$  axis is directed in the opposite direction to the submerged body's course; and the  $y$  axis perpendicular to the  $x$  axis to form a right handed triad.

Assuming the geomagnetic field  $\mathbf{B}_E$  is constant everywhere in concern zone. The formula of it is [12]:

$$\mathbf{B}_E = F \cdot \mathbf{F} = F(\mathbf{i} \cos I \cos \gamma + \mathbf{j} \cos I \sin \gamma - \mathbf{k} \sin I) \quad (1)$$

Where  $F$  is the magnitude of  $\mathbf{B}_E$ ;  $\mathbf{i}$ ,  $\mathbf{j}$ ,  $\mathbf{k}$  are unite vectors along the three coordinate axes;  $I$  is the dip angle, and  $\gamma$  is angle between the  $x$  axis and magnetic north.

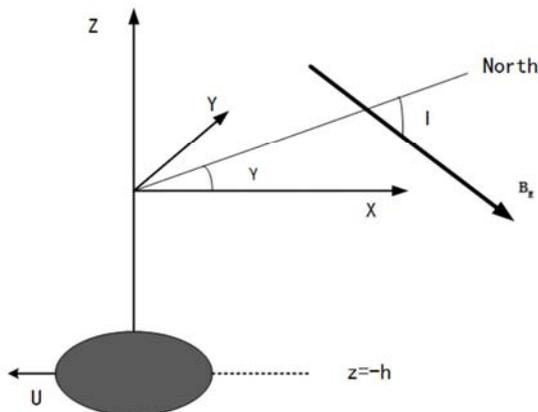


Figure 1. Coordinate system and mathematical model.

The electromagnetic field generated by the moving seawater satisfies the Maxwell equation, substituting the induced velocity of the seawater wave into the Maxwell

equation. Assuming the seawater is incompressible, and the amplitude of the wave is much smaller than its wavelength, disregard the effect of the ocean bottom by supposing the seawater occupies the entire half-space  $z < 0$ , the induced magnetic field equation [14] is:

$$\nabla^2 \mathbf{H} - \mu_0 \sigma \frac{\partial \mathbf{H}}{\partial t} - \epsilon \mu_0 \frac{\partial^2 \mathbf{H}}{\partial t^2} + \sigma \nabla \times (\mathbf{q} \times \mathbf{B}_E) = 0 \quad (2)$$

For  $z < 0$ , and

$$\nabla^2 \mathbf{H} = \epsilon \mu_0 \frac{\partial^2 \mathbf{H}}{\partial t^2} \quad (3)$$

for  $z > 0$ , where  $\mathbf{H}$  is the induce magnetic field, the magnetic permeability in vacuum is taken to  $\mu_0 = 4\pi \times 10^{-7}$  in both the air and sea, and  $\epsilon$  is the dielectric constant, in the air  $\epsilon = \epsilon_0 = 10^{-9}/36\pi$ , and in the seawater  $\epsilon = \epsilon_0 \epsilon_r$ , usually  $\epsilon_r = 80$ ,  $\sigma$  is the conductivity of the sea water, in the air  $\sigma = 0$ ;  $\mathbf{q}$  is the velocity field of the seawater wave. Assuming seawater is incompressible, to solve for the  $\mathbf{H}$ , equations (1)-(2) must be supplement the sea water velocity  $\mathbf{q}$  using the fluid dynamics equation, including a specification of boundary condition and the Earth's magnetic field force, for our concern, the boundary condition is that normal and tangential magnetic field components must be continuous across the air-sea boundary.

### 2.2. Magnetic Field Generated by a Moving Submerged Body

The fluid is assumed to be incompressible, inviscid, homogeneous and electrically conductive. In order to have the exact solution to the magnetic field generated by the moving submerged body, also assuming the density of the seawater is constant. In this case, the undersea moving bodies create only a surface wake with its corresponding flow field. The magnetic field, both above and within the sea, is governed by the linearized Maxwell equation, which is coupled with the fluid velocity. The solutions for the magnetic field are obtained by solving the differential equations using the appropriate matching conditions for both tangential and normal magnetic components. According to the slender body theory and the Rankin ovoid as a submerged body, the induced magnetic field  $\mathbf{H}$  is [12]:

$$\mathbf{H}(x, y, z, t) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} S(\theta) h_\theta(z) e^{-i(\omega_0 t + k_0 x \cos \theta + k_0 y \sin \theta)} d\theta \quad (4)$$

Where

$$h_\theta(z) = P e^{\delta z} - \frac{a(\theta)}{(k_0^2 - \delta^2)} e^{k_0 z} \quad (5)$$

For  $z < 0$ , and

$$h_{\theta}(z) = \left[ P - \frac{a(\theta)}{(k_0^2 - \delta^2)} \right] e^{k_0 z} \quad (6)$$

for  $z > 0$ . And  $S(\theta)$  is the control function between the amplitude of the wave velocity and the Rankine ovoid model

$$S(\theta) = \frac{2UV}{\pi L} \sin(k_0 \cos \theta L/2) \quad (7)$$

Where the  $U$  is the submerged body uniform speed in the '-x' direction,  $V$  and  $L$  is volume and length of a submerged slender body. Wave number  $k_0$  and wave frequency  $\omega_0$  are given by

$$k_0(\theta) = (g/U^2) \sec^2 \theta \quad (8)$$

$$\omega_0(\theta) = k_0 U \cos \theta \quad (9)$$

Where  $g$  is the acceleration of gravity;  $\theta$  is the angle between the wave propagation direction and the  $x$  axis; and  $a(\theta)$ ,  $P$  and  $\delta$  are intermediate variables;  $i = \sqrt{-1}$ , is the complex number symbol, the real part of the complex expressions is to be taken. Usually, the induced magnetic field is measured using the magnetometers, the scalar magnetometer senses the magnetic anomaly of the induced magnetic fields  $B$  to the ambient geomagnetic field, that is parallel to the earth's main magnetic field, defined by

$$\mathbf{B} = \mu \mathbf{H} \cdot \mathbf{F} = \mu \mathbf{H} \cdot (\mathbf{i} \cos I \cos \gamma + \mathbf{j} \cos I \sin \gamma - \mathbf{k} \sin I) \quad (10)$$

### 2.3. Magnetic Fields Induced by Wind Waves

In 1965, J. T Weaver analyzed the magnetic field induced by the motion of ocean waves across the earth's magnetic field, and gives a simpler method than that used by previous authors considering the wind driven waves and the ocean swell. The theoretical model was subsequently testified with experiment data. The theoretical model also suppose that the seawater occupies the entire half-space  $z < 0$ , although it is easy to extend to the case of a sea of finite depth, in order to compare with the magnetic field induced by the submerged body, we also assume that the seawater occupies the entire half-space  $z < 0$ , and the ocean monochromatic wind wave with the propagation angle  $\theta_p$ , and the corresponding magnetic field disturbance parallel to the earth's geomagnetic field in the height is expressed

$$\mathbf{H}_p = \frac{agk_p}{\omega_p} e^{-k_p z_m} \left[ h_B e^{i(\omega_p t + k_p \Omega_p)} \right] \quad (11)$$

Where  $a$ ,  $k_p$  and  $\omega_p$  are the wave amplitude, wave number, and wave frequency, respectively. The dispersion relation between  $k_p$  and  $\omega_p$  is described by

$$\omega_p^2 = gk_p \tanh k_p d \quad (12)$$

And  $d$  is the depth of sea,  $h_B$  and  $\Omega_p$  can be written as

$$h_B = \frac{k_p \sigma B_E}{(\delta_p + k_p)^2} [\cos^2 I \cos^2(\gamma - \theta_p) - \sin^2 I + i \sin(2I) \cos(\gamma - \theta_p)] \quad (13)$$

$$\delta_p^2 = k_p^2 - i\sigma\mu_0\omega_p \quad (14)$$

$$\Omega_p = x \cos \theta_p + y \sin \theta_p \quad (15)$$

In real sea environment, ocean waves produced by wind can be represented as a sum of monochromatic waves with random phase distributed uniformly in the range  $[0, 2\pi]$ . In order to simply the computation, assuming the waves are unidirectional spectrum, and the direction of wind and wind-wave propagation are collinear. Therefore, the magnetic field induced by wind waves can be given as a sum of the corresponding components:

$$\mathbf{H}_p = \sum_n a_n h_B(k_n) e^{-k_n z_m} \frac{gk_n}{\omega_n} e^{i(\omega_n t + k_n \Omega_n + \epsilon_n)} \quad (16)$$

Where the relation between  $k_n$  and  $\omega_n$  is the same as  $k_p$  and  $\omega_p$ , and  $\Omega_n$  is defined the same as  $\Omega_p$ , the wave amplitude  $a_n$  of the  $n$ -component wave was calculated using the sea spectrum depending on the parameters that one tries to represents. There are many sea spectrums in the history of the ocean wave researching, perhaps, one of these is that proposed by Pierson and Moskowitz [18]. For a fully developed ocean, the one-dimensional surface spectrum as a function of the wind speed is [19]:

$$W(k) = \frac{\alpha}{4|k|^3} \exp \left[ -\frac{\beta g^2}{U_{19.5}^4 k^2} \right] \quad (17)$$

Where  $k$  is the surface wave number,  $U_{19.5}$  is the wind speed at a height of 19.5 m,  $\alpha = 8.1 \times 10^{-3}$  and  $\beta = 0.74$  are constants. Then, for the unidirectional wind waves, the wave amplitude  $a_n$  can be expressed as

$$a_n = \sqrt{2w(k)dk} \quad (18)$$

Substituting (17), (18) into (16), the waves magnetic field can be computed.

## 3. Numerical Results and Discussion

### 3.1. Numerical Calculation of the Induced Magnetic Field

First, the magnetic field induced by undersea submerged vehicle is calculated. The size of the vehicle with the maximum radius  $R = 20$  m, length  $L = 100$  m, and the traveling speed is  $U = 10$  m/s in the depth of 100 m. The sea environment parameters are: ocean conductivity  $\sigma_0 = 5$  mho,

relative dielectric constant  $\epsilon_r = 81$ , the Earth's magnetic induction  $F = 5 \times 10^4$  nT, dip angle  $I = 70^\circ$ , vehicle's track-parallel to the magnetic north ( $\gamma = 0$ ). Figure 2 shows the magnetic distributions at the instant  $t = 0$  parallel to the earth's main field along the x axis in the air ( $z = 30$ ) and undersea ( $z = -30$ ). We can see that induced magnetic field drops exponentially at first, and then remains constant for up to several kilometers. In the figure 3, the simulated results of the induced magnetic field varied with the measurement height, submerged body's speed and depth. These show that the magnetic field drops quickly with the moving speed decrease and depth or measurement height increase.

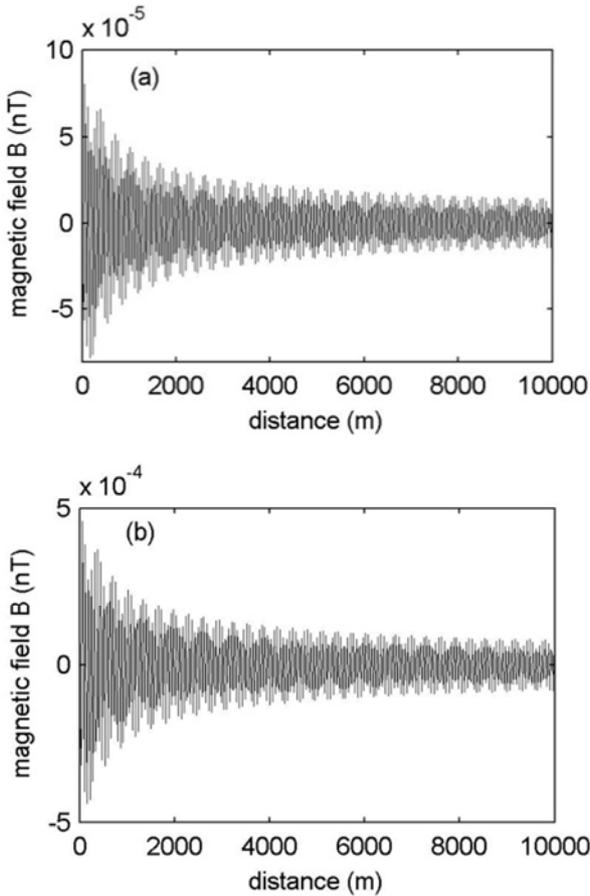


Figure 2. Induced magnetic field in the direction of the earth's main field along a track to the path of the submerged body. (a) 30 m above sea surface, (b) 30 m below sea surface.

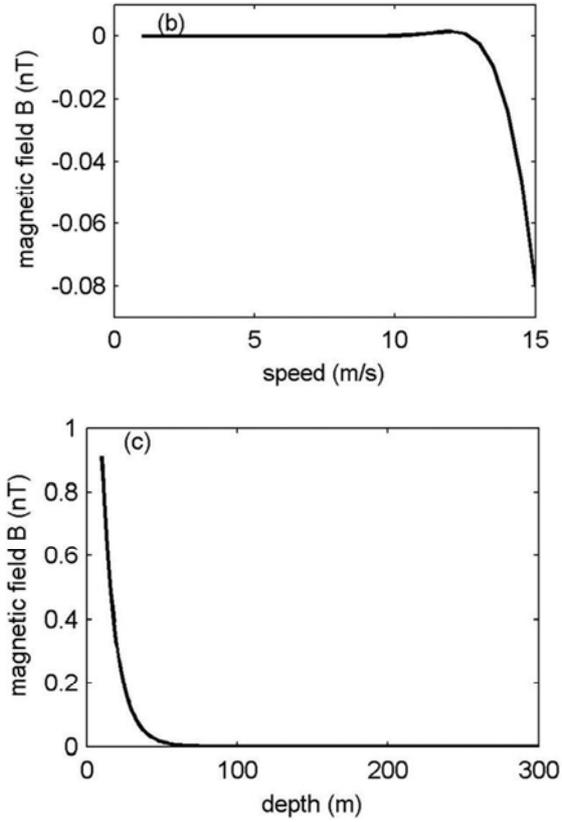
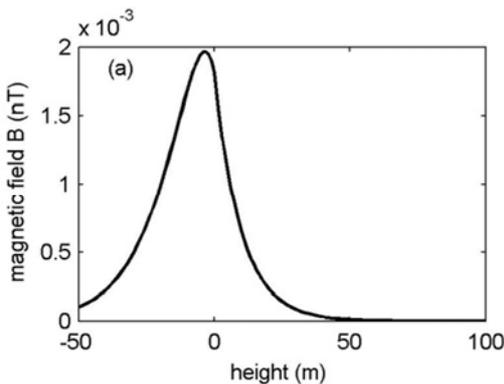


Figure 3. Induced magnetic field attention property. (a) Changing measurement height, (b) Changing moving speed, (c) changing location depth.

Second, for the wind waves, the geomagnetic and sea environment parameters are the same as the above mention, the speed and direction of the wind is assumed to be  $U_{19.5} = 10$  m/s, and  $\theta_p = 0^\circ$ , the wave numbers in the range  $[0.001, 6]$ , with equally spaced wave numbers. Figure 4 shows the magnetic distributions at the instant  $t = 0$  parallel to the earth's main field along the x axis in the air ( $z = 30$  m) and Figure 5 shows the magnetic anomaly along the  $z$ -axis attenuates. The attenuation in the air is fast, the induced magnetic field can hardly detect above 50 meters above the sea surface.

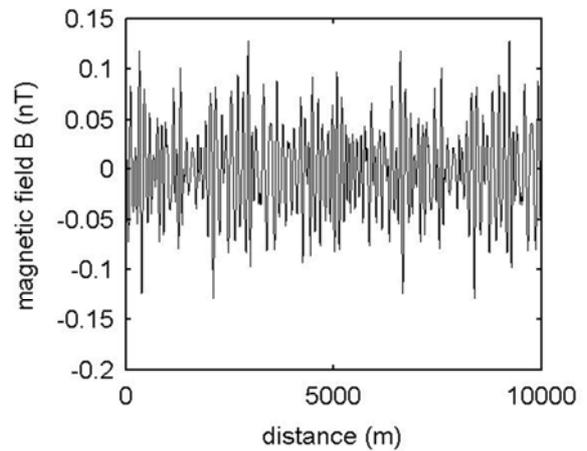


Figure 4. Magnetic anomaly induced by wind waves.

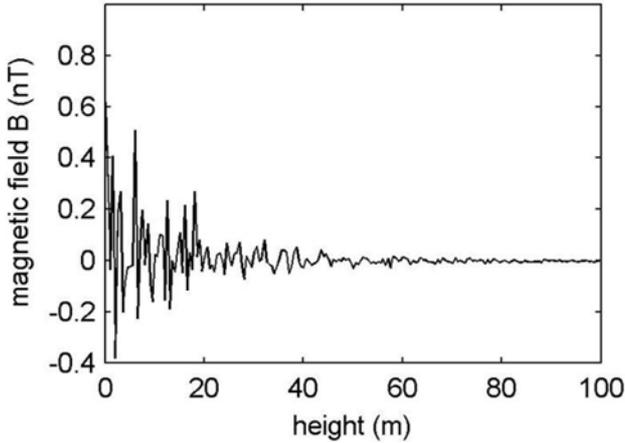


Figure 5. Magnetic field attenuation property of the measurement height increase.

### 3.2. Detection Scenario Simulation

In the real sea magnetic field abnormally detection, using a plane flying over the sea carrying a scalar magnetometer records the magnetic field fused with wind waves and submerged body's waves. In this paper, supposing an airplane, flying at a uniform speed  $v_m$  and a positive acute angle  $\beta$  with the  $x$ -axis at a fixed height. In a measurement duration time  $T$ , the fling distances can be write as

$$S = \sqrt{(x_0 + v_m T \cos \beta)^2 + (y_0 + v_m T \sin \beta)^2} \quad (19)$$

Where and  $y_0$  is the airplane initial plane coordinate,  $v_m$  is the amplitude of  $v_m$ . In order to simulate expediently, set  $x_0 = y_0 = 0$ . Usually, the vehicle moving speed  $U \ll v_m$ , and when  $T = 0$ , the submerged body is at  $(0,0,-h)$ , it may assume  $t = 0$  in the entire duration of the entire duration of the flight. The magnetic field vary with the time can be express the same as the distances along the flying route. Figure 3 shows the magnetic distributions along the flying trace at the height  $z_m = 30$  m, the acute angle  $\beta = \theta_p = 15^\circ$ , with the following specification: the size of the submerged body, geomagnetic and sea environment parameters are the same as above mentation with the traveling speed is  $U = 10$  m/s in the depth of 50 m, and  $U_{19.5} = 6$  m/s.

Figure 6 shows that two wave systems induced by the submerged body waves and wind waves can induced the same magnetic field amplitude, therefore, the ration of signal-to-noise is low, in order to detect the moving object it is essential to compare the spectral densities of the signal and noise. The corresponding plot of the magnetic spectral density for the samples is shown in figure 7 using the standard Fourier transforming spectrograms.

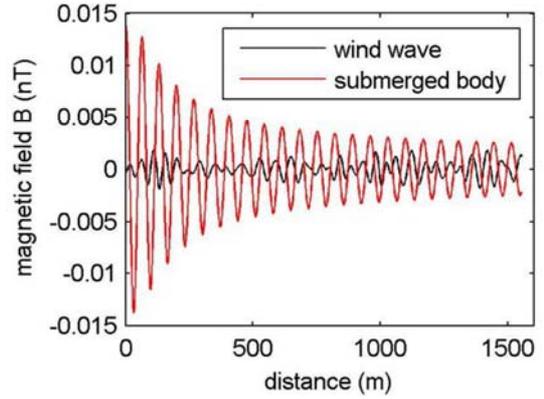


Figure 6. The induced magnetic field comparison between the wind wave and submerged body.

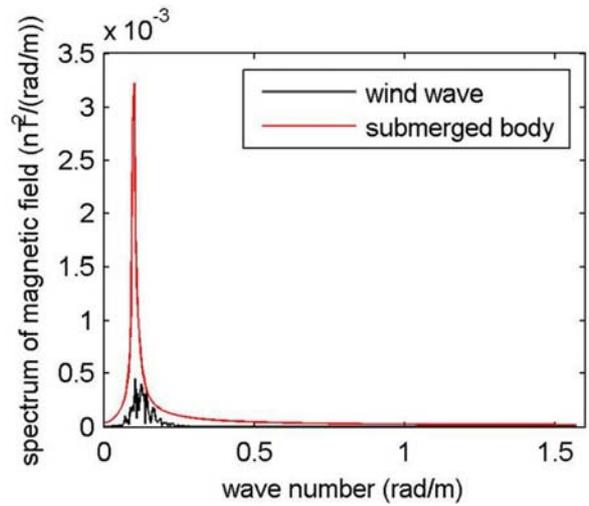


Figure 7. Spectral density comparison of the induced magnetic field.

From the figure 7, the spectrum of the submerged body is very different from the magnetic field of wind waves which is wideband with wave. Hence, the existence of the submerged body could be detected using the signal processing algorithms technology at certain condition.

## 4. Conclusions and Future Work

This paper has computed and compared the magnetic fields induced by the submerged body and wind waves in sea environment to research the detection feasibility using an airplane carrying modern magnetometers. In an unstratified conducting fluid, at a fixed flying height, the magnetic field drops nearly exponentially with the moving speed decrease and depth increase. At certain condition, the magnetic field can be detected using the modern magnetometers which have the sensitivity about  $10^{-6}$  nT, also, the magnetic remains nearly constant for several kilometers. Considering the real sea conditions, the magnetic field due to wind waves is the main fluctuations fused with the submerged body's wake magnetic, which decays with wind speed decrease and heights increase, especially in the high wave number. Then, when assuming a detection scenario, even the magnitude of the two magnetic field are nearly in the same order or different, but,

the spectrum of the two magnetic fields are different, the submerged body's spectrum is rather narrow, which has a sharp peak. The wakes can be in principle detected depending on sensitivity of the detector, body disturbance and environmental conditions.

The detection of the wake magnetic field induced by moving submarine has become a method ever-increasing interest in the last decade. However, from the presents modeling computation, there many questions remain open with regard to both theory and practical application: (i) The simulation assume the density of the seawater is constant, but in the real ocean, the seawater density varies with depth and stratified, which can produce internal waves when the submarine moving, however, the internal waves control equation are complex, at present, there are no uniform solutions considering all the formation mechanism, although it may enlarge the magnetic amplitude. (ii) For a sea sate and detection scenario, the magnetic field induced by a submarine changing quickly when the submarine parameters and flying height or acute angle, which imply the fact that the detection feasibility is depend on the sensitivity of the detector, the signal processing algorithms implementing. (iii) A method has been proposed using a multi-sensor arrangement of magnetic transducers to estimate the vessel traveling direction [20], but can't infer the submarine's depth or speed, those are important in antisubmarine assessment and may be further explored the wake spectrum to determine it, hence, there a lot of thing need to do for the practical application.

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