American Journal of Applied Psychology
2014; 3(3): 72-79
Published online June 20, 2014 (http://www.sciencepublishinggroup.com/j/ajap)
doi: 10.11648/j.ajap.20140303.15

A multiple mediator model: Power analysis based on Monte Carlo simulation

Ze-wei Ma[1, 2], Wei-nan Zeng[2, *]

[1]Thye Hua Social Work Service Center, Luogang District, Guangzhou 510555, China
[2]Department of Psychology, School of Humanities and Management, Guangdong Medical College, Dongguan 523808, China

Email address:
mazeweimac@gmail.com (Ze-wei Ma), weinanzeng@hotmail.com (Wei-nan Zeng)

To cite this article:
Ze-wei Ma, Wei-nan Zeng. A Multiple Mediator Model: Power Analysis Based on Monte Carlo Simulation. *American Journal of Applied Psychology.* Vol. 3, No. 3, 2014, pp. 72-79. doi: 10.11648/j.ajap.20140303.15

Abstract: Most of the applied psychological researchers usually conduct studies requiring application of advanced mediation models, such as multiple mediator models. However, in designing research, most of the applied researchers largely ignore the statistical power of their studies. As a result, power analyses are ignored when researchers report their results. It is well recognized that low power is one possible reason for no statistically significant result being identified in a study. Moreover, studies with low statistical power have been labeled "scientifically useless". The current study describes how to apply Monte Carlo simulation to test the type I error rates and statistical power of mediating effects in a multiple mediator model. Findings from the current simulation study indicated that the effect sizes of mediating effects and sample sizes were two important factors influencing type I error rates of indirect effects in a multiple mediator model. Furthermore, the requirement of sample size and desired power level were strongly depended on the effect size of the indirect effect.

Keywords: Mediation, Multiple Mediator Models, Statistical Power, Monte Carlo Simulation, M*plus*

1. Introduction

One central goal of science is to understand how processes work rather than simply to establish whether a total effect exists [1]. In other words, it is important for applied researchers to investigate whether the cause-effect relation between two variables is accounted for by any intervening variables [1]. As a result, the analyses of mediating effects are being commonly and widely discussed in psychological studies and sociological researches, and other research areas such as clinical medicine and epidemiology [2, 3].

Researchers from different disciplines have different understandings about mediating effects [4]. Psychological researchers interpret the $X \rightarrow W \rightarrow Y$ relation as mediation [5], and such indirect relation is often termed as indirect effect by researchers from sociology [6]. Furthermore, the "intermediate endpoint effect" is usually used for describing the intervening relation by researchers from epidemiology [7].

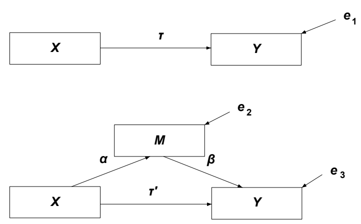

Figure 1. *A single mediator model*

A single mediator model is presented in Figure 1 [5, 8]. The total effect of X on Y is shown in the top panel of Figure 1, and the path coefficient τ is used for describing the relation between X and Y. In the bottom panel of Figure 1, a new variable M is added into the relation between X and Y, and this new variable is called mediating variable or mediator. The direct effect of X on Y is described by path coefficient τ'. Moreover, the path coefficient α is used for describing the $X \rightarrow M$ relation, and the $M \rightarrow Y$ relation is represented by the path coefficient β. Additionally, the indirect effect of the initial variable X on the outcome variable Y via the intervening or mediator variable M is defined as $\alpha\beta$, which is the product of path coefficient α and path coefficient β. The total effect of X on Y is expressed by the sum of the indirect effect and the direct effect: $\tau = \alpha\beta + \tau'$. The following regression equations are used for describing the single mediator model that presented in Figure 1:

$$Y = i_1 + \tau X + e_1 \quad (1)$$

$$M = i_2 + \alpha X + e_2 \quad (2)$$

$$Y = i_3 + \tau' X + \beta M + e_3 \quad (3)$$

Though the single mediator model proposed by Baron and Kenny is widely applied [5], most of the hypotheses of actual social science studies or psychological studies are complicated. That is to say the cause-effect relation between two variables is accounted for by several mediators. In this manner, a single mediator model should be replaced by a multiple mediator model [9].

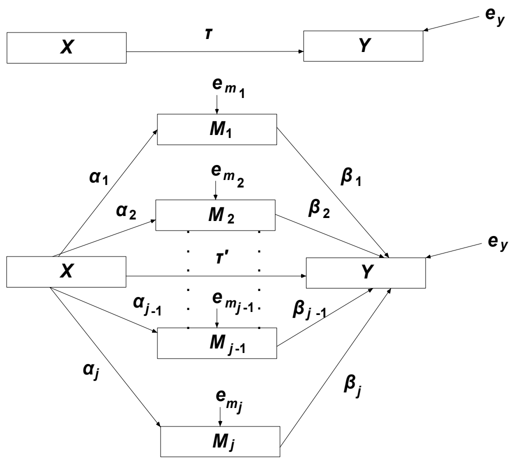

Figure 2. *A multiple mediator model*

A multiple mediator model with j mediators is shown in Figure 2 [10]. The upper panel of Figure 2 represents the total effect of X on Y (path coefficient τ). Moreover, the lower panel of Figure 2 expresses the direct effect of X on Y (path τ'). Additionally, the indirect effects of X on Y through the j mediators are shown in the lower panel of Figure 2. The mediating effect of each mediated pathway in a multiple mediator model is defined as $\alpha_i\beta_i$ (i = 1 to j), which is the product of path coefficient α_i and path coefficient β_i (i = 1 to j) [10]. The total mediating effect of a multiple mediator model is defined as $\sum_i (\alpha_i \beta_i)$ (i = 1 to j), and the total effect of X on Y is calculated as $\tau = \tau' + \sum_i (\alpha_i \beta_i)$. Of note, multiple mediator models have more advantages than single mediator models [11]. For example, the student drug addiction prevention strategy based on school level includes a plurality of intermediaries, such as resistance skills, social norms, attitude about drugs and communication skills. The multiple mediator model is likely to provide a more accurate assessment of mediation effects in many research contexts. [11]. However, most of the applied researchers only focus on how to fit their data to the models that are in accordance with their theoretical assumptions, but they largely ignore the statistical power of their studies when designing their researches. As a result, power analyses are ignored when researchers report their results [12].

It is well recognized that low power is one possible reason for no statistically significant result being identified in a study [13]. Studies with low statistical power have been labeled "scientifically useless" [13-15]. Discussions about statistical power of single mediator models are commonly seen in mediation literature (e.g. [4, 8, 11, 16, 17]). However, only a handful of published studies focus on statistical power of complex mediation models, such as multiple mediator models [9, 18]. Only with adequate power, the sample size in a study will be sufficient to find and confirm a significant mediating effect which is of small effect size [9]. As a result, it is important to perform power analyses for multiple mediator models.

Statistical power, which means the probability of accepting an alternative hypothesis after rejecting a false null hypothesis, is an important concept of statistics. Accordingly, statistical power is defined as "1 — probability of a type II error" [19, 20] [21]. Moreover, in psychology, a desirable power is at 0.8 [8]. One unique function of statistical power is to calculate required sample size to reach 0.8 power to reject a false null hypothesis [9]. If the power in a study is less than 0.8, a researcher should consider increasing the sample size [22]. Under certain circumstances, higher power is also desired [9, 23]. In fact, power analyses are only meaningful when conducted prior to data collection [24]. In light of this, it is important for a researcher to carry out power analysis to confirm how many samples are required in the hypothesis phase of a study [23].

As a result, it is of importance to perform power analysis in the design phase of a research to avoid choosing a sample size that is too small to find or underestimate the existing small real effect, which possibly resulting in a study with inadequate sensitivity, or too large to be costly [9]. Previous research has discussed on how to apply Monte Carlo simulation to detect mediating effects for complex mediator models [9], the current study is purported to investigate statistical power for a multiple mediator model with three mediators.

2. Monte Carlo Simulation

In this study, power analysis was performed by Monte Carlo simulation via M*plus* [25]. Monte Carlo simulation is a flexible and ease of implementation method [26] [9], which enables researchers to flexibly study on how sample sizes, different settings of population parameters, normality of distributions and missingness of data may affect statistical power [27]. This powerful and recommendable method can be easily conducted in software programs that have simulation capabilities, such as M*plus*, SAS, LISREL and EQS [9].

Muthén and Muthén suggested that when performing Monte Carlo simulation, a researcher should first specify which model to be studied. Once the model is chosen, it is important to select population values and have the population parameters fixed to the selected values. Of note, the selection of population values should base on theory or previous research. Muthén and Muthén suggested that previous studies can provide researchers the best estimates of population values [25].

Once the model is chosen, and after all parameters are fixed to the selected values, numerous samples are drawn from the specified population. Each sample drawn from the population is fitted to the chosen model, and then the parameters of the chosen model are estimated as well as recorded. By repeating this process thousands of times, some of the parameter estimates in some samples will be significant, whereas others will not. When replicating the sampling processes adequately, the power estimate is the empirical rate of statistical significance averaged across all replications [9, 25]. Muthén and Muthén suggested that at least 500 replications should be performed, and with increasing replications (such as 10,000 replications), the result of the Monte Carlo simulation converges will be more precise [28].

2.1. Simulation Study

Mackinnon suggested a simulation study with a binary independent variable led to the same results as for the continuous independent variable case [17]. In light of this, the independent variable in the current simulation study was simulated as a binary independent variable. The multiple mediator model investigated in the current simulation study had one independent variable *X*, three parallel mediator variables M_1, M_2 and M_3, and a single dependent variable *Y*. All mediating effects tested in the current study had causal paths that begin with the initial variable *X*, for a total of three two-path mediating effects and one direct effect. Generally, the relationship between two variables can be either positive or negative. In the current study, all relations were set to be positive.

As presented in Figure 3, all variables and paths in the multiple mediator model were named according to the common identification schemes in the mediation literature [18, 29]. The paths linking *X* to the three mediators M_1, M_2, M_3 were denoted as α_1, α_2, and α_3 respectively. The residuals of M_1, M_2 and M_3 were termed as e_{21}, e_{22} and e_{23} respectively. The paths linking M_1, M_2 and M_3 to *Y* were denoted as β_{11}, β_{12} and β_{13} respectively. The direct effect of *X* on *Y* was denoted as τ', and e_3 was termed as the residual of *Y*. Furthermore, the mediating effect of *X* on *Y* through M_1 was expressed as $\alpha_1\beta_{11}$. Moreover, $\alpha_2\beta_{12}$ represented the indirect effect of *X* on *Y* through M_2. Additionally, the indirect effect of *X* on *Y* through M_3 was defined as $\alpha_3\beta_{13}$. For clarity, the intercepts were omitted in the current simulation study [18].

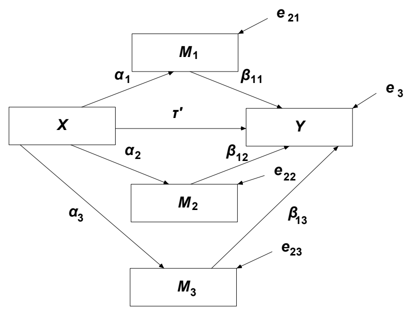

Figure 3. *A multiple mediator model with three mediators*

The three mediators were represented by the following regression equations.

The first mediator M_1 was given by:

$$M_1 = \alpha_1 X + e_{21} \tag{4}$$

The second mediator M_2 was given by:

$$M_2 = \alpha_2 X + e_{22} \tag{5}$$

The third mediator M_3 was given by:

$$M_3 = \alpha_3 X + e_{23} \tag{6}$$

Previous simulation study showed that, when the mediator was controlled, the direct effect of *X* on *Y* had no influences on the effect size of the mediating effect. Accordingly, the path coefficient τ' was fixed constant at a value of 0 to simplify the model in the current simulation study [17]. The following regression equation was given to represent dependent variable *Y*:

$$Y = \beta_{11}M_1 + \beta_{12}M_2 + \beta_{13}M_3 + e_3 \tag{7}$$

2.2. Population Values

In the current study, the independent variable *X* was simulated as a binary variable with an even split. Furthermore, the three mediator variables and the dependent variable *Y* were generated as continuous variables. In order to simplify the model, the means and variances of all the continuous variables were generated as 0 and 1 respectively. The residual variances of the dependent variables were fixed so that the total variance of all variables was 1 [9]. Cohen's guidelines for R^2 metric, the amount of explained variance in the outcome, were used to generate the path coefficients [19]. Some covariance algebra was used in the current study to determine the proper values for the residual variances and effect sizes in M*plus* [9]. The following path coefficients were chosen to show the computational details and to express how the path coefficient and residual variance of each variable were generated. The R^2 of α_1, β_{11}, α_2, β_{12}, α_3, β_{13} were set as 26% (large effect), 13% (medium effect), 2% (small effect), 26% (large effect), 13% (medium effect), 2% (small effect) respectively. Additionally, the direct effect of *X* on *Y* was fixed to 0.

The independent variable simulated in the current study was a binary variable with an even split. As a result, the proportions of the two possible values of the binary variable were 50% and 50% respectively. The residual variance of X was:

$$Var(X) = 0.5 \times (1 - 0.5) = 0.25 \quad (8)$$

According to equation (4), $Var(M_1)$ was given by:

$$Var(M_1) = \alpha_1^2 \times Var(X) + Var(e_{21}) \quad (9)$$

As noted previously that $Var(M_1) = 1$, thus:

$$\alpha_1 = \frac{\sqrt{1 - Var(e_{21})}}{\sqrt{Var(X)}} \quad (10)$$

$R^2 = 0.26$, and $Var(M_1) = 1$, thus:

$$Var(e_{21}) = Var(M_1) - R^2 = 1 - 0.26 = 0.74 \quad (11)$$

According to equation (10), α_1 was solved as 1.02.

By using the same formulas shown above, the $Var(e_{22})$, α_2, $Var(e_{23})$ and α_3 were solved as 0.87, 0.721, 0.98 and 0.283 respectively.

The equation of the variance of Y was expressed as:

$$Var(Y) = \beta_{11}^2 Var(M_1) + \beta_{12}^2 Var(M_2) + \beta_{13}^2 Var(M_3) + Var(e_3) \quad (12)$$

The variances of all continuous variables were fixed equally to a value of 1, thus $Var(Y)$ was solved as:

$$1 = \beta_{11}^2 \times 1 + \beta_{12}^2 \times 1 + \beta_{13}^2 \times 1 + Var(e_3) \quad (13)$$

The R^2 chosen for β_{11}, β_{12} and β_{13} were 0.26, 0.13, 0.02 respectively, thus:

$$\beta_{11} = \sqrt{0.26} = 0.51 \quad (14)$$

$$\beta_{12} = \sqrt{0.13} = 0.36 \quad (15)$$

$$\beta_{13} = \sqrt{0.02} = 0.14 \quad (16)$$

Put the results back to equation (13), $Var(e_3)$ was solved as 0.59.

The distribution trend of the data was simulated as normal distribution in the current simulation study. The significance of the mediating effect was detected by the sobel test that M*plus* performs by default. The simulation method used in the current study can be employed in actual applied settings, but researchers should note that the selection of estimates of parameters should base on pilot studies or previous researches. When more information is available, the small, medium, and large categorization should also be avoided [9].

2.3. Type I Error Rates

The values of the mediating effects were fixed to zero ($\alpha_i\beta_i = 0$, $i = 1$、2、3) to test the type I error rates of the indirect effects in all combinations when sample sizes were 100, 200 and 500 respectively. In the first combination, all the path coefficients were fixed to 0 ($\alpha_1 = \alpha_2 = \alpha_3 = \beta_{11} = \beta_{12} = \beta_{13} = 0$). In the second combination, the path coefficients of the independent variable X to the three mediator variables were fixed equally to 0 ($\alpha_1 = \alpha_2 = \alpha_3 = 0$), but the path coefficients of the three mediator variables to the dependent variable Y were generated as large effect, medium effect and small effect respectively ($\beta_{11} = 0.51$, $\beta_{12} = 0.36$ and $\beta_{13} = 0.14$). In the third combination, α_1, α_2 and α_3 were generated as large effect, medium effect and small effect respectively ($\alpha_1 = 1.02$, $\alpha_2 = 0.721$, $\alpha_3 = 0.283$), but the path coefficients β_{11}, β_{12} and β_{13} were generated as 0 ($\beta_{11} = \beta_{12} = \beta_{13} = 0$). In this study, 10,000 replications were used for each combination to insure that stability has been reached [25]. The type I error rate of the indirect effect in Monte Carlo simulation is regarded as the proportion of 10,000 replications for which the 95% confidence interval will not contain the value of zero. Accordingly, the empirical type I error rates would be expected to be closer or equal to the true alpha 0.05, or fulfill Bradley's liberal criterion that type I error rates should range between 0.025 and 0.075 [30].

The results of the type I error rates of the 3 combinations over the 10,000 replications were presented in Tables 1, 2 and 3 respectively. When $\alpha_i = \beta_i = 0$ ($i = 1$、2、3), the type I error rates of all mediating effects in the first combination were quite below the value of 0.05, and did not fulfill Bradley's liberal criterion as well. Although the sample size was increased to 500, the type I error rates of all indirect effects in Combination 1 were 0. The type I error rates of $\alpha_1\beta_{11}$, $\alpha_2\beta_{12}$ and $\alpha_3\beta_{13}$ in Combination 2 and Combination 3 were 0.044, 0.030, 0.004, 0.047, 0.028 and 0.002 respectively when the sample size was 100. Furthermore, whether in small samples (N = 100) or large samples (N = 500), the type I error rates of $\alpha_1\beta_{11}$ in Combination 2 and Combination 3 were quite close to 0.05. Additionally, the type I error rates of $\alpha_2\beta_{12}$ in Combination 2 and Combination 3 in all sample sizes fulfilled Bradley's liberal criterion [30]. However, the type I error rates of $\alpha_3\beta_{13}$ in Combination 2 and Combination 3 were far from 0.05 in all sample sizes. Furthermore, when the sample size was increased, all type I error rates in Combination 2 and Combination 3 were elevated as well. Moreover, when increasing the sample size to 500, the type I error rates of $\alpha_1\beta_{11}$ and $\alpha_2\beta_{12}$ in Combination 2 and Combination 3 reached or closed to an alpha of 0.05 and the type I error rate of $\alpha_3\beta_{13}$ in Combination 2 fulfilled Bradley's liberal criterion. However, the type I error rate of $\alpha_3\beta_{13}$ in Combination 3 was 0.014 when sample size was 500, which was too conservative.

These findings indicated that different effect sizes of α_i and β_i and different sample sizes were important factors influencing the type I error rates of mediating effects in a multiple mediator model.

Table 1. *Type I Error Rates in Combination 1*

Combination 1	**Sample Size**		
	N = 100	**N = 200**	**N = 500**
	Type I Error Rate	**Type I Error Rate**	**Type I Error Rate**
Total indirect effect = 0	0.001	0.000	0.000
$\alpha_1 = \beta_{11} = 0$	0.000	0.000	0.000
$\alpha_2 = \beta_{12} = 0$	0.000	0.000	0.000
$\alpha_3 = \beta_{13} = 0$	0.000	0.000	0.000

Table 2. *Type I Error Rates in Combination 2*

Combination 2	**Sample Size**		
	N = 100	**N = 200**	**N = 500**
	Type I Error Rate	**Type I Error Rate**	**Type I Error Rate**
Total indirect effect = 0	0.047	0.045	0.045
$\alpha_1 = 0,\ \beta_{11} = 0.51$	0.044	0.049	0.048
$\alpha_2 = 0,\ \beta_{12} = 0.36$	0.030	0.041	0.043
$\alpha_3 = 0,\ \beta_{13} = 0.14$	0.004	0.009	0.025

Table 3. *Type I Error Rates in Combination 3*

Combination 3	**Sample Size**		
	N = 100	**N = 200**	**N = 500**
	Type I Error Rate	**Type I Error Rate**	**Type I Error Rate**
Total indirect effect = 0	0.044	0.049	0.050
$\alpha_1 = 1.02,\ \beta_{11} = 0$	0.047	0.049	0.050
$\alpha_2 = 0.72,\ \beta_{12} = 0$	0.028	0.038	0.046
$\alpha_3 = 0.283,\ \beta_{13} = 0$	0.002	0.004	0.014

2.4. Power Analysis

Three combinations, Combination 1 ($\alpha_1 = 1.02$ (large effect), $\alpha_2 = 0.721$ (medium effect), $\alpha_3 = 0.283$ (small effect), $\beta_{11} = 0.51$ (large effect), $\beta_{12} = 0.36$ (medium effect), $\beta_{13} = 0.14$ (small effect)); Combination 2 ($\alpha_1 = 0.283$ (small effect), $\alpha_2 = 0.721$ (medium effect), $\alpha_3 = 1.02$ (large effect), $\beta_{11} = 0.36$ (medium effect), $\beta_{12} = 0.14$ (small effect), $\beta_{13} = 0.51$ (large effect)) and Combination 3 ($\alpha_1 = 1.02$ (large effect), $\alpha_2 = 0.283$ (small effect), $\alpha_3 = 0.721$ (medium effect), $\beta_{11} = 0.14$ (small effect), $\beta_{12} = 0.51$ (large effect) , $\beta_{13} = 0.14$ (small effect)) were generated in this simulation study. The power of every indirect effect in each combination was estimated over 10,000 replications in sample sizes of 100, 200 and 500 respectively.

Tables 4, 5 and 6 presented the results of the power estimates of different indirect effects in Combinations 1, 2 and 3 respectively. Even if the sample size was small (*N*=100), the power of the total indirect effects in Combination 1 and Combination 2 reached a level of 1. Additionally, the power of the total indirect effect in Combination 3 was below 0.8 when the sample size was 100. However, each of the mediated pathways showed us different information. Take Combination 1 as an example. The power of total mediating effect in Combination 1 was extremely high at 1.000 in a small sample ($N = 100$), but the power to detect the mediating effect $\alpha_3\beta_{13}$ were low at 0.038 and 0.175 when the sample sizes were 100 and 200 respectively. The power of $\alpha_3\beta_{13}$ in Combination 1 reached at 0.770 when the sample size was increased to 500, but this power was still lower than 0.8. These findings indicated that when the mediating effect was large, a small sample size can acquire adequate power to detect significant mediating effect. In Combination 1, when $\alpha_1\beta_{11} = 0.520$, the power of $\alpha_1\beta_{11}$ reached 1.000 when the sample size was 100, which means that a total sample size of at least 100 was required to reach the power of 1.000 to detect significant mediation. However, the power of $\alpha_1\beta_{11}$ in Combination 2 was only 0.231 when the sample size was 100. When increasing the sample size to 500, the power of $\alpha_1\beta_{11}$ in Combination 2 was increased to 0.882, which was slightly higher than the desired 0.8 power. These findings were consistent with previous research [8].

Table 4. *Power to Detect Significant Mediating Effects in Combination 1*

Combination 1	**Sample Size**		
	N = 100	**N = 200**	**N = 500**
	Power	**Power**	**Power**
Total indirect effect = 0.819	1.000	1.000	1.000
$\alpha_1 = 1.02,\ \beta_{11} = 0.51,\ \alpha_1\beta_{11} = 0.520$	1.000	1.000	1.000
$\alpha_2 = 0.721,\ \beta_{12} = 0.36,\ \alpha_2\beta_{12} = 0.260$	0.918	1.000	1.000
$\alpha_3 = 0.283,\ \beta_{13} = 0.14,\ \alpha_3\beta_{13} = 0.040$	0.038	0.175	0.770

Table 5. *Power to Detect Significant Mediating Effects in Combination 2*

Combination 2	**Sample Size**		
	N = 100	**N = 200**	**N = 500**
	Power	**Power**	**Power**
Total indirect effect = 0.723	0.999	1.000	1.000
$\alpha_1 = 0.283,\ \beta_{11} = 0.36,\ \alpha_1\beta_{11} = 0.102$	0.231	0.497	0.882
$\alpha_2 = 0.721,\ \beta_{12} = 0.14,\ \alpha_2\beta_{12} = 0.101$	0.268	0.612	0.961
$\alpha_3 = 1.02,\ \beta_{13} = 0.51,\ \alpha_3\beta_{13} = 0.520$	1.000	1.000	1.000

Table 6. *Power to Detect Significant Mediating Effects in Combination 3*

Combination 3	Sample Size		
	N = 100	N = 200	N = 500
	Power	Power	Power
Total indirect effect = 0.388	0.665	0.929	1.000
$\alpha_1 = 1.02,\ \beta_{11} = 0.14,\ \alpha_1\beta_{11} = 0.143$	0.268	0.497	0.885
$\alpha_2 = 0.283,\ \beta_{12} = 0.51,\ \alpha_2\beta_{12} = 0.144$	0.257	0.502	0.889
$\alpha_3 = 0.721,\ \beta_{13} = 0.14,\ \alpha_3\beta_{13} = 0.101$	0.233	0.547	0.929

The results of the exact sample sizes reaching 0.8 power to detect significant mediating effects in each of the combinations were presented in Table 7. In Combination 1, the indirect effect $\alpha_3\beta_{13}$ was at 0.8 power when sample size was 524, which indicated that 524 samples were needed to have 80% probability to accept an alternative hypothesis after rejecting a false null hypothesis. Moreover, the power of the indirect effects $\alpha_1\beta_{11}$ and $\alpha_2\beta_{12}$ reached a level of 1.000. These findings confirmed that a smaller sample size is required to reach 0.8 power to detect significant mediating effect if the effect size of the mediating effect is large. However, if the indirect effect was small, a larger sample size is required to detect significant mediating effect.

Muthén and Muthén suggested that several criteria are examined to determine sample size. Firstly, the parameter estimate bias and standard error bias should not exceed 10% for any parameter in the model. Secondly, the standard error bias for the parameter for which power is being assessed should not exceed 5%. Thirdly, the coverage should range between 0.91 and 0.98. If these conditions are met, the sample is chosen to keep power close to 0.8 [25].

Table 7. *Exact Sample Sizes Reaching 0.8 Power to Detect Significant Mediating Effects in the 3 Combinations*

Combination 1 Sample Size (N = 524)	Power	Combination 2 Sample Size (N = 396)	Power	Combination 3 Sample Size (N = 392)	Power
Total indirect effect = 0.819	1.000	Total indirect effect = 0.723	1.000	Total indirect effect = 0.388	0.999
$\alpha_1 = 1.02,\ \beta_{11} = 0.51,\ \alpha_1\beta_{11} = 0.520$	1.000	$\alpha_1 = 0.283,\ \beta_{11} = 0.36,\ \alpha_1\beta_{11} = 0.102$	0.800	$\alpha_1 = 1.02,\ \beta_{11} = 0.14,\ \alpha_1\beta_{11} = 0.143$	0.869
$\alpha_2 = 0.721,\ \beta_{12} = 0.36,\ \alpha_2\beta_{12} = 0.260$	1.000	$\alpha_2 = 0.721,\ \beta_{12} = 0.14,\ \alpha_2\beta_{12} = 0.101$	0.912	$\alpha_2 = 0.283,\ \beta_{12} = 0.51,\ \alpha_2\beta_{12} = 0.144$	0.800
$\alpha_3 = 0.283,\ \beta_{13} = 0.14,\ \alpha_3\beta_{13} = 0.040$	0.800	$\alpha_3 = 1.02,\ \beta_{13} = 0.51,\ \alpha_3\beta_{13} = 0.520$	1.000	$\alpha_3 = 0.721,\ \beta_{13} = 0.14,\ \alpha_3\beta_{13} = 0.101$	0.910

Muthén and Muthén suggested that, to determine the parameter bias, subtract the population value from the parameter estimate average over the replications of the Monte Carlo simulation (such as 10,000 replications in the current study) and divide it by the population value.

To determine the standard error bias, subtract the population standard error from the standard error estimate across the 10,000 replications and then divide it by the population standard error. The coverage gives the proportion that the 95% confidence interval constructed by the 10,000 replications contains the true parameter value [25].

Table 8. *Parameter and Standard Error Biases and Coverage in Combination 1 (N = 524)*

Combination 1 Sample Size (N = 524)	Parameter bias	Standard error bias	Coverage
Total indirect effect = 0.819	0.000611	0.000867	0.950
$\alpha_1 = 1.02,\ \beta_{11} = 0.51,\ \alpha_1\beta_{11} = 0.520$	0.000769	0.007286	0.953
$\alpha_2 = 0.721,\ \beta_{12} = 0.36,\ \alpha_2\beta_{12} = 0.260$	-0.019231	0.005115	0.949
$\alpha_3 = 0.283,\ \beta_{13} = 0.14,\ \alpha_3\beta_{13} = 0.040$	-0.01	-0.000623	0.932

The parameter and standard error biases as well as the coverage rates of the indirect effects in the 3 different combinations over the 10,000 replications were presented in Tables 8, 9 and 10 respectively. Results of parameter and standard error biases were negligible, and all coverage rates ranged from 0.932 to 0.953. These results indicated that the sample sizes chosen in each of the combinations can keep power close to 0.80 to detect significant mediating effects.

Table 9. *Parameter and Standard Error Biases and Coverage in Combination 2 (N = 396)*

Combination 2 Sample Size (N = 396)	Parameter bias	Standard error bias	Coverage
Total indirect effect = 0.723	0.001107	0.001125	0.951
$\alpha_1 = 0.283,\ \beta_{11} = 0.36,\ \alpha_1\beta_{11} = 0.102$	0.011221	-0.000265	0.951
$\alpha_2 = 0.721,\ \beta_{12} = 0.14,\ \alpha_2\beta_{12} = 0.101$	0.000990	-0.003039	0.947
$\alpha_3 = 1.02,\ \beta_{13} = 0.51,\ \alpha_3\beta_{13} = 0.520$	0.001154	0.003149	0.950

Table 10. *Parameter and Standard*

Combination 3 Sample Size (N =	Parameter	Standard	Coverage
Total indirect	-0.000258	0.004525	0.952
$\alpha_1 = 1.02,\ \beta_{11} = 0.14,\ \alpha_1\beta_{11} = 0.143$	-0.000140	0.007018	0.952
$\alpha_2 = 0.283,\ \beta_{12} = 0.51,\ \alpha_2\beta_{12} = 0.144$	-0.001389	0.001894	0.951
$\alpha_3 = 0.721,\ \beta_{13} = 0.14,\ \alpha_3\beta_{13} = 0.101$	0.003960	-0.007160	0.946

3. Discussion

It is flexible and ease of implementation to use Monte Carlo simulation to perform power analyses for multiple mediator models [9]. When performing power analysis via Monte Carlo simulation, a researcher should first select which model to be studied. The population value which is required to be set for each parameter in the model is needed to be selected base on theory research, pilot study or other empirical researches [25]. In the current simulation study, Cohen's guidelines defining a small effect, medium effect and large effect on the R^2 metric were used to generate the coefficient of each path. Furthermore, some covariance algebra was used to determine proper values for the residual variances and transform the desired R^2 effect sizes into unstandardized path coefficients.

In the actual applied settings, researchers should note further that a selected path coefficient should match a residual variance. As an example, the R^2 chosen for path α_1 and path β_{11} were both 26% in Combination 1, and the unstandardized path coefficients of α_1 and β_{11} were solved as 1.02 and 0.51 respectively. Moreover, the mediating effect of X on Y through M_1 was 0.520 in Combination 1, and the residual variance solved for M_1 was 0.74. When fixing the unstandardized path coefficients for α_1 and β_{11} in the population model, the residual variance of M_1 should be fixed constant at the value of 0.74.

Overall, the type I error rates in this simulation study were within the acceptable range, which were close to the 0.05 true value level, or fell within the liberal criterion proposed by Bradley that the type I error rates should range between 0.025 and 0.075. We found that type I error rates of indirect effects were affected by the effect sizes of α_i and β_i, and sample sizes as well. When $\alpha_i = \beta_i = 0$, the type I error rates of all mediating effects in all sample sizes (N = 100, N = 200 and N = 500) in this simulation study were far from the value of 0.05, and did not fulfill the liberal criterion proposed by Bradley.

However, when $\alpha_i = 0$, $\beta_i \neq 0$, or $\alpha_i \neq 0$ and $\beta_i = 0$, the type I error rates were close to 0.05 or felt within 0.025 ~ 0.075. These findings were consistent with previous simulation study [17]. Furthermore, we noticed that the type I error rates of $\alpha_3\beta_{13}$ (small effect) in both Combination 2 and Combination 3 were too conservative whether in small samples or large samples. Accordingly, it is confirmed that the effect size of mediating effect is a determinant factor influencing type I error rate. Additionally, the type I error rates of the indirect effects in the 3 combinations were all elevated with the increase of sample sizes, and were finally close to an alpha of 0.05. Although the type I error rates of the indirect effects of small effect sizes were conservative in small samples, when the sample size was increased, the type I error rates were elevated. Findings in the current study were consistent with previous research [8].

In this simulation study, we found that the requirement of sample size and desired power level were strongly depended on one determinant factor, the effect size of the indirect effect. When the effect size of the indirect effect was small, a larger sample size was required to reach 0.8 power to detect a significant mediating effect. However, when the effect size of the indirect effect was large, only a small sample size was required to reach 0.8 power to detect significant indirect effects. For example, $\alpha_3\beta_{13}$ was set as a small mediating effect in Combination 1, but as a large mediating effect in Combination 2. As a result, when the sample size was 100, the power to detect significant mediating effect for $\alpha_3\beta_{13}$ in Combination 1 was 0.038, whereas the power to detect a significant mediating effect for $\alpha_3\beta_{13}$ in Combination 2 was 1.000. When the sample size was increased to 500, power of $\alpha_3\beta_{13}$ in Combination 1 to detect a significant mediating effect was lower than 0.8. Moreover, it is worthy of noting that although the total indirect effect in Combination 1 reached a satisfying power in every sample size, power of individual mediating effect was still (such as $\alpha_3\beta_{13}$ in Combination 1) lower than 0.8 when the sample size was small, which indicated that the different estimates of power exist in one multiple mediator model when there are different mediated pathways [9].

References

[1] Hayes AF, Preacher KJ. Quantifying and testing indirect effects in simple mediation models when the constituent paths are nonlinear. Multivariate Behavioral Research. 2010;45:627-60.

[2] Albert JM, Nelson S. Generalized causal mediation analysis. Biometrics. 2011;67:1028-38.

[3] Imai K, Keele L, Yamamoto T. Identification, inference and sensitivity analysis for causal mediation effects. Statistical Science. 2010;25:51-71.

[4] MacKinnon DP, Lockwood CM, Hoffman JM, West SG, Sheets V. A comparison of methods to test mediation and other intervening variable effects. Psychological methods. 2002;7:83.

[5] Baron RM, Kenny DA. The moderator–mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. Journal of personality and social psychology. 1986;51:1173.

[6] Alwin DF, Hauser RM. The decomposition of effects in path analysis. American Sociological Review. 1975:37-47.

[7] Freedman LS, Schatzkin A. Sample size for studying intermediate endpoints within intervention trials or observational studies. American Journal of Epidemiology. 1992;136:1148-59.

[8] Fritz MS, MacKinnon DP. Required sample size to detect the mediated effect. Psychological science. 2007;18:233-9.

[9] Thoemmes F, MacKinnon DP, Reiser MR. Power analysis for complex mediational designs using Monte Carlo methods. Structural Equation Modeling. 2010;17:510-34.

[10] Preacher KJ, Hayes AF. Asymptotic and resampling strategies for assessing and comparing indirect effects in multiple mediator models. Behavior research methods. 2008;40:879-91.

[11] MacKinnon DP, Fairchild AJ, Fritz MS. Mediation analysis. Annual review of psychology. 2007;58:593.

[12] Schimmack U. The ironic effect of significant results on the credibility of multiple-study articles. Psychological Methods. 2012;17:551.

[13] Christley R. Power and error: increased risk of false positive results in underpowered studies. Open Epidemiology Journal. 2010;3:16-9.

[14] Altman DG. Statistics and ethics in medical research: III How large a sample? British Medical Journal. 1980;281:1336.

[15] Halpern SD, Karlawish JH, Berlin JA. The continuing unethical conduct of underpowered clinical trials. JAMA: the journal of the American Medical Association. 2002;288:358-62.

[16] Mackinnon DP. Introduction to statistical mediation analysis. New York: Lawrence Erlbaum Associates; 2008.

[17] MacKinnon DP, Lockwood CM, Williams J. Confidence limits for the indirect effect: Distribution of the product and resampling methods. Multivariate behavioral research. 2004;39:99-128.

[18] Williams J, MacKinnon DP. Resampling and distribution of the product methods for testing indirect effects in complex models. Structural Equation Modeling. 2008;15:23-51.

[19] Cohen J. Statistical power analysis for the behavioral sciencies: Routledge; 1988.

[20] Sedlmeier P, Gigerenzer G. Do studies of statistical power have an effect on the power of studies? Psychological Bulletin. 1989;105:309.

[21] Oakes JM. Statistical Power and Sample Size: Some Fundamentals for Clinician Researchers. Essentials of Clinical Research. 2008:261-78.

[22] Fox N, Mathers N. Empowering research: statistical power in general practice research. Family practice. 1997;14:324-9.

[23] Maxwell SE. Sample size and multiple regression analysis. Psychological methods. 2000;5:434.

[24] Aberson CL. Applied power analysis for the behavioral sciences: Routledge; 2011.

[25] Muthén LK, Muthén BO. How to use a Monte Carlo study to decide on sample size and determine power. Structural Equation Modeling. 2002;9:599-620.

[26] Abraham WT, Russell DW. Statistical power analysis in psychological research. Social and Personality Psychology Compass. 2008;2:283-301.

[27] Van Vleet BL. An Investigation of Power Analysis Approaches for Latent Growth Modeling: Arizona State University; 2011.

[28] Oertzen T. Power equivalence in structural equation modelling. British Journal of Mathematical and Statistical Psychology. 2010;63:257-72.

[29] Shrout PE, Bolger N. Mediation in experimental and nonexperimental studies: new procedures and recommendations. Psychological methods. 2002;7:422.

[30] Bradley JV. Robustness? British Journal of Mathematical and Statistical Psychology. 1978;31:144-52.