Efficient Pricing of European Options with Stochastic Interest Rate Using Fourier Transform Method

Michael Chimezie Anyanwu, Roseline Ngozi Okereke

Department of Mathematics, Michael Okpara University of Agriculture, Umudike, Abia State, Nigeria

Email address:
manyanwu71@yahoo.com (M. C. Anyanwu), okerekeroseline@gmail.com (R. N. Okereke)

Abstract: The inclusion of stochastic interest rate is an essential element of any realistic option pricing method. Therefore, the purpose of this paper is to incorporate interest rates in the Fourier transform method for pricing European options in exponential Levy models. With the assumption of stochastic independence between the underlying log asset price and the stochastic interest rate, we obtain a pricing of pure discount bond available in the market. Our method of valuation is to apply eigenfunction expansion to the variable that describes the evolution of the interest rate, and Fourier transform to the variable that describes the log asset price.

Keywords: Fourier Transform, Eigenfunction Expansion, Stochastic Interest Rate, Levy Process, Simple Harmonic Oscillator

1. Introduction

The first acceptable contribution to the field of quantitative modelling of derivative securities is given by Black and Scholes [1], in which the price of a European option on an underlying asset with price process given by a geometric Brownian motion was given in close-form. However, in most exponential Levy models and stochastic volatility models, analytic formula for the option price does not exist. Hence, alternative methods are sought. One of such approaches is the Fourier transform and inversion methods, since it has been observed that when characteristic function of the density function is mapped to the option payoff in Fourier space, option prices are easily calculated much easier for some of the much complex processes. The idea behind Fourier transform is to take an integral of the payoff function over the probability distribution function, where the probability distribution function is obtained by inverting the corresponding Fourier transform.

In mathematical finance, Fourier transform method was first used in Stein and Stein [3] volatility model, to obtain the distribution of the underlying asset price. This was followed by Heston [2], where through characteristic function, obtains an analytical formula for the valuation of European options when the volatility of the underlying asset price is stochastic.

Following these two papers, quite a lot have being written on the application of Fourier transform and inversion methods in the valuation of more complex contingent claims. For example, Carr and Madan [4] introduces fast Fourier transform algorithm for the numerical valuation of the Fourier integral encountered in option pricing. Since then, fast Fourier transform algorithm has become an efficient mathematical tool used in mathematical finance in the valuation problems, e.g. see [10, 7, 8, 12, 13]. A more general pricing framework was developed in Lewis [9] by separating the underlying asset price from the option payoff through the use of Plancherel-Parseval theorem. Through his method, a variety of pricing formula for different types of contingent claims can be obtained. For other applications of Fourier transform methods to option pricing, see [5, 6, 8] and bibliographies therein.

All the existing Fourier transform methods in option pricing known in literature make a simplifying assumption that the interest rate in the market remain constant throughout the life of the option. This assumption is not generally true, especially when pricing options of long maturity, as interest rates are bound to change, and this level of change is not known before time (that is, it is stochastic). Therefore, the contribution of this paper to the existing literature on Fourier transform method for option pricing is to incorporate

stochastic interest in pricing European style contingent claim under the assumption that the underlying log asset price X_t is an exponential Levy process. Therefore, to price the European option, we assume that the interest rate, r, is a stochastic function of the Markov process $Y_t: r = r(Y_t)$, when state space is $D = \mathbb{R}$.

The method of valuation we adopt here is the joint application of the Fourier transform and eigenfunction expansion. To the best of our knowledge, this is the first time the two methods are being applied together. The idea is that we use the Fourier transform for the variable X_t that models the log-asset of the firm, and apply the eigenfunction expansion with respect to the variable Y_t that describes the evolution of interest rate.

The study of eigenfunction expansion of linear operators was motivated by the seminal work of Mckean [14], where the spectral representation for the transition density of one dimensional diffusion process is obtained. Since then, eigenfunction expansion has been shown to be an important methodology for derivative pricing. For example, Gorovoi and Linetsky [20] models the Japanese interest rate using eigenfunction expansion method, to obtain analytical solutions for zero coupon bonds and bonds options under the Vasicek [19] and CIR [18] processes for the shadow rate. Lewis [27] uses eigenfunction expansion to price generalized European style options on stocks that pay constant dividends, and finds an estimate for the yield curve associated with the Merton [26] economy. Davydov and Linetsky [16] develops eigenfunction expansion approach for valuing options on scalar diffusion processes. Boyarchenko and Levendorskii [17] applies eigenfunction expansion to multifactor models of interest rate. For more applications of eigenfunction expansion method to derivative pricing, see in applying eigenfunction expansion, the problem of derivative pricing is reduced to that of solving a single eigenvalue-eigenfunction equation. Once the equation is solved, the approximate value of the derivative can be computed. In this paper, we do not do eigenfunction expansions with the infinitesimal generator of the pricing semigroup, rather we do expansion with infinitesimal generator $L_0 = L_y - r(y)$, L_y is the infinitesimal generator of the process Y_t. This ultimately leads to solving an infinite system of disjoint first order ordinary differential equation for the bond price, which is easy to solve. In this case, we obtain an infinite series that gives the bond price.

The remaining of this article is organized as follows: in the remaining part of section one, we discuss the preliminaries, which include Fourier transform and characteristic function, Levy process and its theory and eigenfunction expansion. In section two, our main method for pricing European option with stochastic interest rate is presented. This is the joint application of eigenfunction expansion and Fourier transform. In section three, we process our numerical result, discuss the result and conclude the article.

1.1. Fourier Transform and Characteristic Function

Let $f(x)$ be a piecewise continuous real function which satisfies the condition $\int_{-\infty}^{\infty} |f(x)| dx < \infty$. The Fourier transform of $f(x)$ defined for any complex variable u is given by

$$f(u) = \int_{-\infty}^{\infty} e^{-iux} f(x) dx \tag{1}$$

This transform is known to exist and is analytic for all u in the strip $S = \{u \in \mathbb{C}: a < u < b, provided \int_{-\infty}^{\infty} e^{-ax} | f(x) | dx < \infty \text{ and } \int_{-\infty}^{\infty} e^{-bx} | f(x) | dx < \infty$. The inverse of the transform through which the function $f(x)$ can be recovered is

$$f(x) = \frac{1}{2\pi} \int_{2\omega-\infty}^{2\omega+\infty} e^{iux} f(u) du, \ a < \omega < b. \tag{2}$$

1.2. Brief on Levy Processes

Levy processes are becoming very popular in mathematical finance because they have been discovered to describe the observed reality of financial markets more accurately than models with Brownian motion such as geometric Brownian motion of Black and Scholes [1]. In real world, it is known that asset price paths do exhibit jump or spikes, and these have to be taken into account in order to manage risks efficiently. Moreover, the empirical distribution of asset returns exhibits fat tails and skewness. These show that the distribution of asset returns are far from being normal, as predicted in Brownian motion models. Levy processes are known to possess the appropriate properties that accurately describe all these properties of asset price and its returns in the real world and risk neutral world.

By definition, a Levy process X_t is stochastic processes with stationary and independent increments, and it is continuous in probability. A general 1-dimensional Levy process can be represented as

$$X_t = \gamma t + \omega_t + N_t + lim_{\epsilon \to 0} M_t^\epsilon \tag{3}$$

Where ω_t is a 1-dimensional Brownian motion, $\gamma \in \mathbb{R}$, N_t is a compound Poisson process which includes the jumps of X_t, and M_t^ϵ is a compensated compound Poisson process which includes the jumps of X_t with $\epsilon < | \Delta X_t | \leq 1$. The characteristic function of the distribution of X_t can be represented in the form

$$E[e^{iX_t}] = e^{-t\psi()}, \ \in \mathbb{R}, t \geq 0$$

The function ψ is called the characteristic exponent for X. the general form of characteristic exponent of any Levy process can be deduced from the Levy. Khintchine formula

$$\psi(\eta) = \frac{\sigma^2}{2} \eta^2 - ib\eta + \int_{\mathbb{R}\setminus 0} (1 + i\eta x 1_{[-1,1]}(x) - e^{ix\eta}) F(dx) \tag{4}$$

where $\sigma \geq 0, b \in \mathbb{R}, and F$ is a measure on $\mathbb{R}\setminus 0$ satisfying

$$\int_{\mathbb{R}\setminus 0} \{|x|^2, 1\} F(dx) < \infty$$

The triple (b, σ^2, F) is called the Levy triple for X_t. The proof of this Levy – Khintchine formula can be found in Cont and Tankov [21]. The known exponential Levy models in financial modeling literature correspond to assigning different values to $\sigma^2 and F(dx)$. For example, the case

where $F(dx)$ is a finite measure and $\sigma \neq 0$, gives rise to jump-diffusion Levy processes, e.g. Merton [22] and Kou [23] models. The case where $F(dx)$ an infinite measure, and $\sigma = 0$, gives rise to pure jump model of infinite activity, e.g. Variance Gamma (VG) model of Madan, *et al* [24], Normal Inverse Gaussian model of Barndorff-Nielson [25], etc. As an example in this article, we use a VG process whose density of jumps is given by

$$F(dx) = c_+ e^{\lambda_- x} x^{-1} 1_{(0,+\infty)}(x)dx + c_- e^{\lambda_+ x} x^{-1} 1_{(-\infty,0)}(x)dx$$

Here, $1_{(a.b)}$ is an indicator function of the interval (a, b). The coefficients c+ (respectively, c-) gives the intensity of upwards jumps (respectively, downward jumps). The characteristic exponent $\psi(\eta)$ of the VG process can be obtained using (4):

$$\psi(\eta) = c[\ln(-\lambda_- - i\eta) - \ln(-\lambda_-) + \ln(\lambda_+ - i\eta) - \ln(\lambda_+)]$$

1.3. Eigenfunction Expansion for the Bond Price

Consider a pure discount bond that pays the owner a certain unit amount of currency at maturity date T. The price $B(t,T,y)$ of this bond at time $t < T$ is given by

$$B(t,T,y) = E^{Q,y}\left[e^{-\int_t^T y(s)ds}\right], y \epsilon D_0 \quad (5)$$

where D_0 is the interior of D, and Q is the risk-neutral measure chosen for pricing. In the CIR (1985) model, the variable Y_t satisfies the stochastic differential equation

$$dy_t = (\alpha - \beta y)dt + \sigma\sqrt{y}dw_t \quad (6)$$

Where $\propto \geq 0 \beta, \sigma > 0$ and w_t is one dimensional Brownian motion. In (6), we assume that the feller condition, $2\alpha > \sigma^2$ is satisfied to ensure that the point 0 is not attained by the process Y_t. Applying Feymann-Kac theorem to $B(t,T,y)$, we obtain the backward Kolmogorov equation

$$\left(\partial_t + L_y - r(y)\right)B(t,T,y) = 0 \quad (7)$$

Subject to $B(T,T,y) = 1$, where $L_y = (\alpha - \beta y)\partial_y + \frac{\sigma^2 y}{2}\partial_y^2$ is the infinitesimal generator of the process Y_t. We assume that $L_0 = L_y - r(y)$ is diagonalizable so that we can expand $1_D(y)$ into the series

$$1_D = \sum_{n=0}^{\infty} c_n g_n(y) \quad (8)$$

where g_n are the orthogonal eigenfunctions of L_0 and c_n are the coefficients of expansion given by

$$c_n = \frac{\langle 1_D, g_n\rangle L^2(m,D)}{\langle g_n, g_n\rangle L^2(m,D)}, \quad (9)$$

m is the measure on D chosen so that the infinitesimal generator L_0 becomes unbounded in the Hilbert space $L^2(m,D)$, with discrete spectrum, and

$$\langle 1_D, g_n\rangle = \int_D 1D(y)g_n(y)m(y)dy$$

Is the inner product in $L^2(m,D)$. We can now expand the bond price $B(t,T,y)$ in the series

$$B(t,T,y) = \sum_{n=0}^{\infty} c_n B_n(\tau)g_n(y) \quad (10)$$

where $\tau = T - t$. If we substitute (10) into (7), with $\partial_t = -\partial_\tau$, we get $-\sum_{n=0}^{\infty} c_n g_n(y)(\partial_\tau + \lambda_n)B_n(\tau) = 0$, n= 0,1,2… Hence

$$(\partial_\tau + \lambda_n)B_n(\tau) = 0 \quad (11)$$

Subject to $B_n(0) = 1$. Then, (11) is an infinite system of disjoint scalar first order ordinary differential equations for $B_n(\tau)$. This is solved to get $B_n(\tau) = e^{-\lambda_n \tau}$ for each n. therefore, the eigenfunction expansion (10) for the bond price becomes

$$B(t,T,y) = \sum_{n=0}^{\infty} c_n e^{-\lambda_n \tau}g_n(y) \quad (12)$$

Hence, if the constant of expansion C_n converges fast, and if the eigenvalues and eigenfunctions are known, then with moderately few terms, the series (12) can be truncated appropriately to find the approximation to the bond price. Suppose the risk-free interest rate is quadratic function of the stochastic variable $y = r(y) = \frac{1}{2}a_2 y^2 + a_1 y + a_0, a_2 \neq 0$, then L_0 can be transformed to the operator of the simple Harmonic oscillator whose eigenvalues and eigenfunctions are well known. The transformed infinitesimal generator becomes $\tilde{L}_0 = -\kappa H - \tilde{a}_0$, where κ and $\tilde{a}_0$ are constants to be determined in the transformation, $H = \frac{1}{2}(x^2 - \partial_x^2)$ is the operator of the simple harmonic oscillator.

The normalized eigenfunctions of H can be generally written as

$$\psi_j(x) = \hat{H}_j(x)e^{-(x)^2/2}$$

With $\|\psi_j\|^2 = 2^j j! \sqrt{\pi}, \forall j = 0,1,...,$ and the corresponding eigenvalues are

$$\lambda_j = j + \frac{1}{2}, \quad j = 0,1,2,\ldots\ldots$$

Where $\hat{H}_j(x) = j! \sum_{k=0}^{\lfloor j/2\rfloor} \frac{(-1)^k (2x)^{j-2k}}{k!(j-2k)}$ are the Hermite polynomials, and $\lfloor x \rfloor = floor(x)$ means the largest integer which is not greater than x. it is verifiable that $\psi_0(x) = \left(\frac{1}{\pi}\right)^{\frac{1}{4}} e^{-\frac{x^2}{2}}$ is the initial eigenfunction that corresponds to the initial eigenvalue $\lambda_0 = \frac{1}{2}$.

We use the following theorem from [17] which we state without proof.

Theorem
1. The eigenfunctions of the transformed infinitesimal generator $\tilde{L}_0$ and that of H are the same.
2. The eigenvalues of $\tilde{L}_0$ are $k\lambda_j - \tilde{a}_0$, where λ_j are the eigenvalues of H for j = 0,1,2,…

With these, the eigenfunction expansion for the bond price becomes

$$B(0,T,y) = \sum_{k=0}^{\infty} C_k \psi_k(y) e^{-T\lambda_k} \quad (13)$$

The detail of this transformation can be found in [17]

The graph below shows how the bond prices from the eigenfunction expansion method approximates closely, the bond prices from the analytical solution.
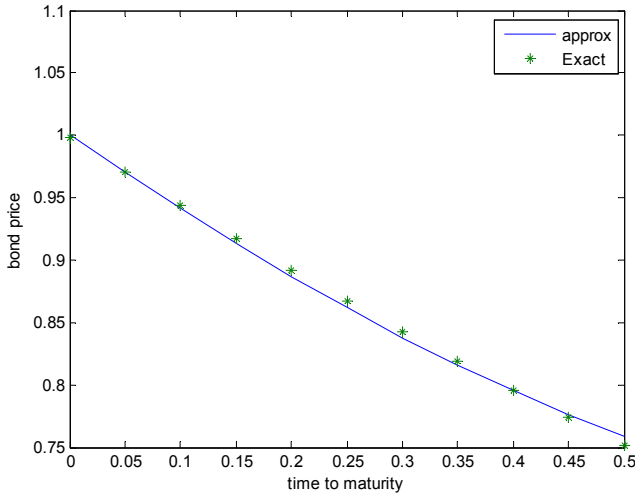

Figure 1. *Approximate and Exact Bond Price for T = .5 years.*

2. Pricing European Option with Stochastic Interest Rate

We begin by choosing a risk neutral measure Q, suitable for pricing options on the underlying stock or index X_t with stochastic interest rate $r(Y_t)$, where X_t is a Levy process of exponential type (λ_-, λ_+). In the presence of stochastic interest rate, the time 0 < T price of a European option with payoff $H(X_T, Y_T)$ and expiry date T can be written as

$$V(t, T, y) = E^{Q,x,y}[H(X_T, Y_T) \mid X_0 = x, Y_0 = y] \quad (14)$$

Where $\quad H(X_T, Y_T) = e^{-\int_0^T y(s)ds} H(X_T).$ Since X_t and Y_t are independent, we can write (14) to be

$$V(t, T, y) = E^{Q,y}[H(Y_T) \mid Y_0 = y] E^{Q,x}[H(X_T) \mid X_0 = x] \quad (15)$$

Where $H(Y_T) = E^{Q,y}\left[e^{-\int_0^T y(s)ds}\right]$ is the risk neutral value of a pure discount bond that pays one unit of currency at maturity date T.

Define $B(0, T, y) := E^{Q,y}\left[e^{-\int_0^T y(s)ds}\right]$. Then, the value of the option can be written as

$$V(0, T, y) = B(0, T, y) E^{Q,x}[H(X_T)|X_0 = x] \quad (16)$$

Suppose there exists some $\omega \in (\lambda_-, \lambda_+)$ such that $H(x) := e^{\omega x}H(x)$ is integrable, we can decompose H into Fourier integral to get

$$H(x) = \frac{1}{2\pi} \int_{im\xi = \omega} e^{ix\xi} \bar{H}(\xi)d\xi, \quad (17)$$

With

$$\bar{H}(\xi) = \int_{\mathbb{R}} e^{-ix\xi} H(x)dx \quad (18)$$

Putting (16) into the pricing formula (14), we get

$$V(0, T, y) = B(0, T, y)E^{Q,x}\left[\frac{1}{2\pi} \int_{im\xi = \omega} e^{iX_T\xi} \bar{H}(\xi)d\xi\right] \quad (19)$$

We can apply Fubinis theorem to change the order of taking expectation and integration to obtain

$$V(0, T, y) = B(0, T, y)\frac{1}{2\pi} \int_{im\xi = \omega} e^{ix\xi} E^{Q,x}\left[e^{i(X_T - X_0)\xi} \bar{H}(\xi)d\xi\right] \quad (20)$$

Using the characteristic function for the distribution of X, we can write (20) as

$$V(0, T, y) = B(0, T, y)\frac{1}{2\pi} \int_{im\xi = \omega} e^{ix\xi} e^{-\psi(\xi)T} \bar{H}(\xi)d\xi \quad (21)$$

Where $\psi(\xi)$ is the so-called characteristic exponent for the Levy process X. the payoff for European call and put options with strike price K are $(e^x - K)$ and $(K - e^x)$ respectively. Hence, using (18), we have $\bar{H}(\xi) = \frac{-Ke^{-i\xi \ln k}}{\xi(i+\xi)}$ for call option and $\bar{H}(\xi) = \frac{-Ke^{-i\xi \ln k}}{\xi(i+\xi)}$ for put option. The major task in applying Fourier transform to pricing European option is the evaluation of the integral in (21). To do this, we truncate the integral and discretize the integrand. This involves choosing a convenient grid $\xi_j = i\omega_+ + j\zeta$, to obtain

$$I = \frac{\zeta}{2\pi}\sum_{j\in\mathbb{Z}} e^{ix\xi_j - \psi(\xi_j)T} \bar{H}(\xi_j) \quad (22)$$

where $j \in \mathbb{Z}, \omega_+ > 0$, and ζ is the grid size

3. Numerical Example

In this section, we implement the method discussed in We consider this paper. We experiment with the following parameters: For the Levy process, we use $\mu = 0.45, \sigma = 0$ (since we are considering pure jump process), $\lambda_1 = -3, \lambda_2 = 2, c = 1.5$. For the CIR interest rate process, we use $\alpha = 1.2, \beta = 1, \sigma = 1.6428, a_2 = 1, a_1 = 0.9, a_0 = 0.0005$. We experiment with different stock prices with fixed strike price, K = 100 for both European call and put options. The result of our calculations are presented in Table 1 and Table 2 for call and put options respectively. In this tables, N represent the number of simulations used in (22).

Table 1. *Call option values for K = 100, T = 1.*

S/N	20	30	40	50	60	70	80	90
120	1.3781	1.3745	1.3753	1.3750	1.3751	1.3751	1.3751	1.3751
150	6.2955	6.3002	6. 2996	6. 2996	6. 2996	6. 2996	6. 2996	6. 2996
180	11.3756	11.3751	11.3758	11.3759	11.3758	11.3758	11.3758	11.3758
200	14.7766	14.7728	14.7727	14.7728	14.7729	14.7729	14.7729	14.7729

***Table 2.** Put option values for K = 100, T = 1.*

S/N	20	30	40	50	60	70	80	90
20	2.9252	2.9255	2.9255	2.9255	2.9255	2.9255	2.9255	2.9255
40	5.2309	5.2310	5.2312	5.2311	5.2312	5.2312	5.2312	5.2312
60	5.6639	5.6667	5.6672	5.6672	5.6672	5.6672	5.6672	5.6672
80	4.2085	4.2084	4.2042	4.2042	4.2043	4.2043	4.2043	4.2043

4. Summary

Pricing European options with short maturity date on a given stock, may not require change in interest rate. But when the maturity date is long, it is necessary to consider the possibility of stochastic change in the level of interest rate in the market. In this article, we have succeeded in incorporating stochastic interest rate in the Fourier transform method for valuing European-style contingent claim on a stock driven by an exponential Levy process. Moreover, the tables above reveals that the method of simplified trapezoid rule adopted here for evaluating the Fourier integral converges fast, thereby allowing the option value to be computed fast.

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