
Experimental Verification of Theoretical Configuration Mixing in the Energy Levels of Er II Spectra via Isotope Shift Measurements Using a FTS

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Abstract: We report the first detailed investigation on isotope shift, $\Delta\sigma^{166, 170}$ measurements carried out in the spectrum of singly-ionized erbium (Er II/ Er⁺) recorded with a FTS. Isotope shift in 85 spectral lines were determined in the 350- 590 nm wavelength region. The source was accomplished of mixture of highly enriched isotopes of ¹⁶⁶Er:¹⁷⁰Er in 7:10 ratio respectively and the detector was a photomultiplier tube. These investigations have contributed significantly to the understanding of the 92 known energy levels of Er⁺. Level isotope shift, $\Delta T^{166, 170}$ values have been evaluated for 29 even- and 63 odd- parity energy levels for the first time. On the basis of the derived level isotope shifts the configuration mixing was estimated for altogether 92 involved levels and compared those with the theoretically predicted configuration mixings available in the literature and found that both the theoretical and experimental mixings have excellent agreement with each other.

Keywords: Isotope Shift, Configuration Mixing, Singly-Ionized Spectra, Fourier Transform Spectrometer, Hollow Cathode Lamp

1. Introduction

The comprehensive list of the energy levels of the singly ionized erbium atom (Er II) was published by Martin et al. [1] taking aid from the unpublished extension of the Er II spectral analysis carried out by van Kleef TAM et al. [2]. Sugar et al. [3] have derived the ionization potential 11.93 eV (± 0.08 eV) of Er⁺ along with the other rare earth ions *i.e.* from La II to Lu II by means of semi empirical calculations. The analysis of lifetimes of 11 odd- parity levels and measurement of precise transition energies for several spectral lines of Er II were conducted in the reference [4]. Improved values of laboratory transition probabilities for 418 lines of Er II were presented and employed those values for the Er abundance measurement of the sun and five r- process-rich metal-poor stars [5]. Wyart et al. [6] have classified and extended both theoretically and experimentally the Er II spectra to interpret the known energy levels first

parametrically using Cowan code and then the predictions of the unknown energy levels employed for the classification of the experimental hollow cathode FT spectra. Their detailed investigation of level configurations has shown that the levels of 4f¹²6p were strongly mixed with 4f¹¹5d6s and 4f¹¹5d² configurations and provided the configuration mixing in percentages for most of the known even and odd levels. The hyperfine structure for 4f¹²6s and 4f¹²5d configurations of ¹⁶⁷Er in 14 transitions of Er II were carried out with the collinear fast-beam laser and radio-frequency laser double-resonance spectroscopy techniques and reported the hfs A- and B- coupling constants of various Er II levels for the ¹⁶⁷Er isotope [7]. Isotope shift (IS) data in the spectral lines of Er II was very scarce in the literature. Wilets and Bradley [8] have reported IS in 67 spectral lines of Er and they argued that most of these lines belong to Er II spectrum. However only 6 of these lines falling at 4552.13 Å, 4820.75 Å, 5164.77 Å, 5485.93 Å, 6006.80 Å and 6076.44 Å were classified as Er II

lines in NBS Tables [9]; and the remaining 61 lines belong to Er I spectrum as have claimed by [10]. Determination of the intrinsic quadrupole moment of ^{162}Er was carried out in [11] using the IS data of the Er I and Er II lines. Pacheva et al. [12] have published only abstract where no details are given of the Er II lines in which they conducted IS measurements. IS in 9 lines along with the crossed-second-order effects of the IS in the ground configuration $4f^{12}6s$ of Er II were measured [13] using enriched isotopes of ^{166}Er and ^{170}Er and computer- interfaced Fabry- Perot Spectrometer.

The objectives of the present investigation were to obtain the IS data in as many spectral lines of Er II as possible in the first stage because so far IS data in only 9 lines have been reported earlier [13] and in the second stage to evaluate the IS of even- and odd- parity energy levels and use this data to designate the configurations to the known but unassigned levels as all the even- levels between $38400\text{-}43400\text{ cm}^{-1}$ and the odd- levels above 21000 cm^{-1} have tentative configuration assignments and all the known 54 even levels above 43400 cm^{-1} and 144 odd levels between $33000\text{-}45000\text{ cm}^{-1}$ have no configuration designations. The status of classification of these levels [1] has been summarized in Table 1. In the third stage compare the experimentally derived configuration mixing with the theoretically calculated configuration mixing available in the literature [6].

As can be seen from Table 1, about 50 percent of the known even- and 80 percent of the known odd- parity levels are without configuration assignments.

Table 1. Status of configuration assignment to the known energy levels of Er II [1].

Parity	Energy Levels	Configuration status [1]
Even (117 known levels)	12	$4f^{12}6s$
	23	$4f^{12}5d$
	9	$4f^{11}6s6p$
	8	$4f^{11}6s6p?$
	11	$4f^{11}5d6p?$
	54	Unassigned
Odd (243 known levels)	3	$4f^{11}6s^2$
	28	$4f^{11}5d6s$
	5	$4f^{11}5d6s(?)$
	2	$4f^{11}5d^2(?)$
	10	$4f^{12}6p(?)$
	195	Unassigned

2. Experimental Techniques

IS, $\Delta\sigma^{166,170}$ were measured in the 67 lines falling in the 350-590 nm region of the Er II spectra with a Bomem DA8 Fourier Transform Spectrometer (FTS). The light detector was photo multiplier tube (PMT) and the source was liquid nitrogen cooled hollow cathode lamp (HCL). The source consisted of manmade mixture (7:10 ratio) of highly enriched isotopes ^{166}Er (96.3%) and ^{170}Er (98.0%) respectively in the oxide form. The sample weighing about 12- 15 mg was coated on the copper crucible as a thin layer of paste prepared using distilled water. The coated crucible was dried under the infrared lamp and later heated to red hot on the Bunsen flame before inserting it into the HCL. Ne was filled at 2.5 mbar as a buffer gas and the

discharge was run at 45 mA DC between the anode and cathode maintaining 2-3 mm gap. To get an acceptable signal to noise ratio, about 90 minute integration time (~50 scans co-added) was used to record the each set of data. The entrance aperture of the spectrometer adjusted to ~1 mm to give rise to the resolution of 0.02 cm^{-1} .

During the IS measurements we considered the different parameters like intensity, Full width at half maximum (FWHM), the center of gravity of each component. All IS data were converted to $(1.10^{-3}\text{ cm}^{-1}= 30\text{ MHz})$ MHz from wavenumber (cm^{-1}) scale. Er spectra encompass the Ne I and Ne II lines (FWHM 4500 MHz) in addition to Er I and Er II. IS $\Delta\sigma^{166,170}$ data in 660 Er I spectral lines has been already published in [14] in 2015. Er II lines exhibit 2000 MHz FWHM at 350 nm and 1500 MHz at 590 nm. The spectral positions were shifted to $+0.200\text{ cm}^{-1}$ towards shorter wavelength region and -0.05 cm^{-1} shift observed towards the longer wavelength region. The accuracy of the IS measurements for all lines was $(\pm 0.003\text{ cm}^{-1}) \pm 90\text{ MHz}$. IS was said to be positive if the peak representing heavier isotope, ^{170}Er appears on the higher frequency side and it was negative if it appears on the lower frequency side. The IS 0 MHz suggests that the peaks of both the isotopes ^{166}Er and ^{170}Er overlap one above another.

3. Results and Discussions

IS, $\Delta\sigma^{166,170}$ (MHz) data measured in the 85 lines of Er II are listed in Table 2. The magnitude of IS observed presently was in the range of -3842 to +1400MHz. As can be seen from Table 2, 54 transitions observed are from high odd- parity levels to low even- parity levels, whereas the 31 transitions observed are from high even- to low odd- parity levels. 20 lines have exhibited positive IS and 60 lines have indicated negative IS whereas the remaining 5 spectral lines have shown no or 0 IS. The wavelengths of the spectral lines studied presently, their intensities and their relevant energy level classifications, are taken from Meggers et al. [9] and are presented in the column 1. The IS $\Delta\sigma^{166,170}$ (MHz) data of the lines studied were presented in the column 2 of the Table 2. IS in the chosen 7spectral lines at 357.075 nm, 363.354 nm, 369.265 nm, 371.239 nm, 372.952 nm, 390.631 nm, and at 590.208 nm of Er II recorded on FTS were exhibited in Figs. 1-7 respectively. The magnitude of IS as seen in Figures 1 and 4 is measurably large whereas it is too small to measure directly in Figures 2, 3, 5, 6 and 7 and had been extracted using profiles of single isotopes. Energy level classification ($33129\text{-}5132\text{ cm}^{-1}$) listed by Meggers et al. [9] for the line at 357.075 was unfit (see Fig. 1) and conclusively rejected by Wyart et al. [6]. The IS value -2910 MHz presently measured in the line was not agreeing with the classification listed by Meggers et al. [9] hence we also support the rejection of the said classification. Wyart et al. [6] have provided energy level classifications for the three lines compiled as unclassified lines in [9] at 495.360 nm, 502.428 nm and at 521.826 nm. The present IS data derived in these lines support the recent classifications (see Table 2) calculated by Wyart et al. [6].

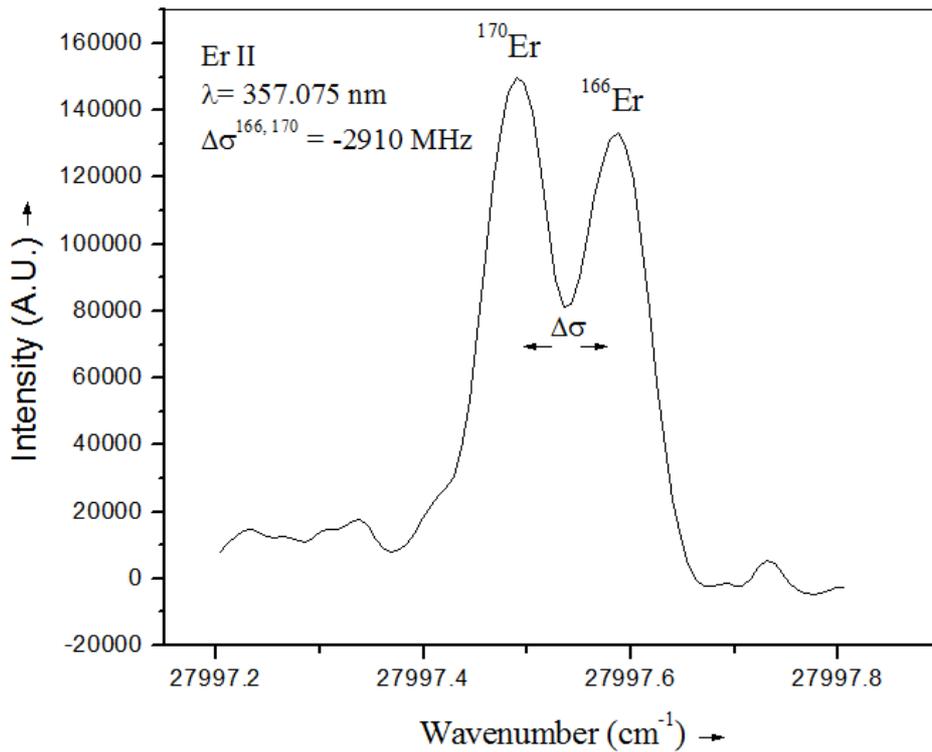


Figure 1. Er II FTS Spectrum, Wavelength, $\lambda = 357.075 \text{ nm}$ [Wavenumber, $\sigma = 27997.304 \text{ cm}^{-1} = 33129.912 \text{ cm}^{-1} (J=9/2) \Delta T^{166,170} (81 \text{ MHz}) - 5132.608 \text{ cm}^{-1} (J=9/2) 4f^{12}6s \Delta T^{166,170} (2991 \text{ MHz})$]. This energy level classification compiled in Meggers et al. [9] was rejected by Wyart et al. [6]. The presently derived $\Delta T^{166,170} (81 \text{ MHz})$ for the level at 33129 cm^{-1} can not be classified under any of the odd configurations listed in the Table 3. Thus experimentally measured IS, $\Delta\sigma^{166,170} = -2910 \text{ MHz}$ data in this line does not suite with the existing classification [9] and supports its rejection. Light Source: Liquid nitrogen cooled HCL, Detector: PMT. a. u. : Arbitrary units.

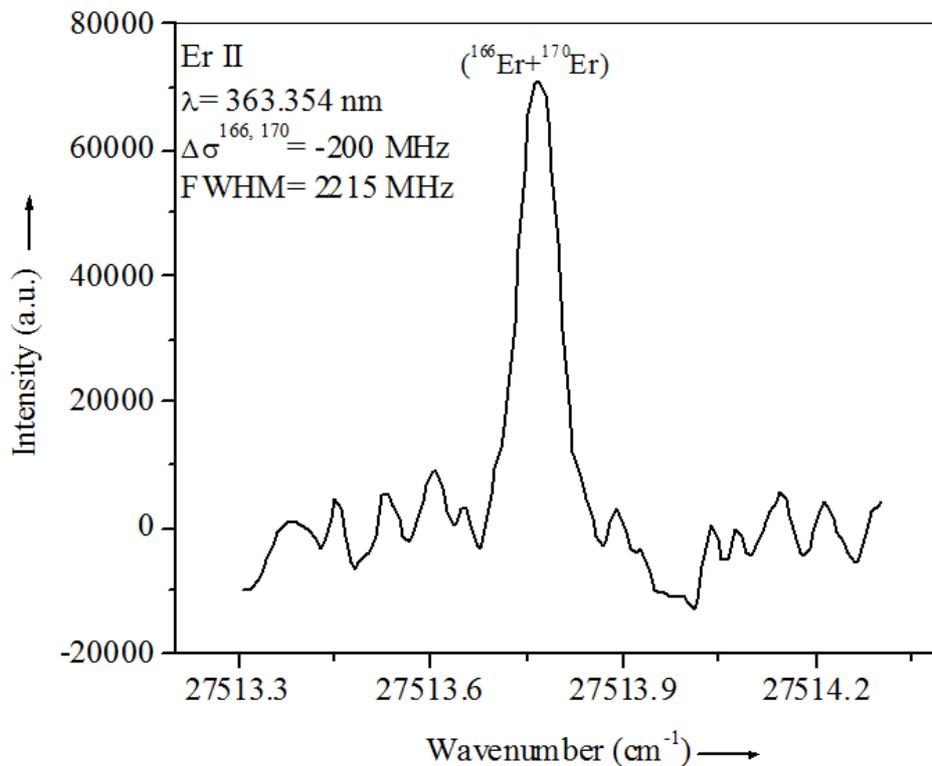


Figure 2. Er II FTS Spectrum, Wavelength, $\lambda = 363.354 \text{ nm}$ [Wavenumber, $\sigma = 27513.555 \text{ cm}^{-1}$, Energy Level Classification, $27513.555 \text{ cm}^{-1} (J=13/2) \Delta T^{166,170} (2798 \text{ MHz}) - 0.00 \text{ cm}^{-1} (J=13/2) 4f^{12}6s \Delta T^{166,170} (3000 \text{ MHz})$]. The extracted IS, $\Delta\sigma^{166,170} = -200 \text{ MHz}$, Light Source: Liquid nitrogen cooled HCL, Detector: PMT.

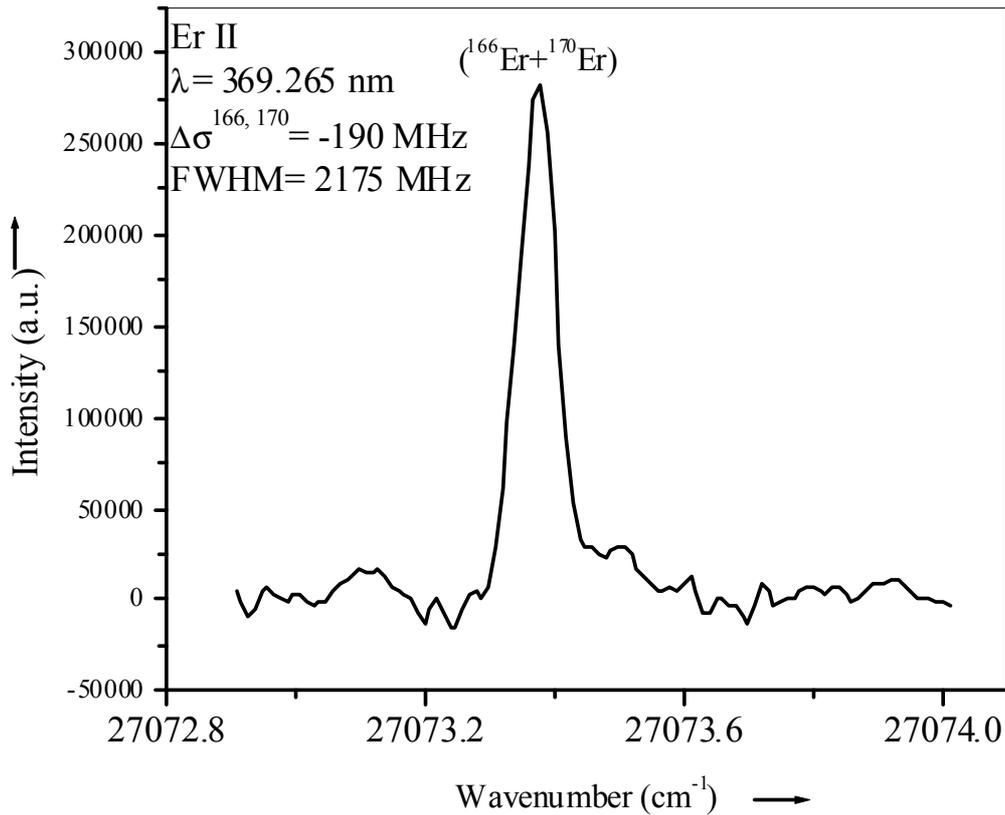


Figure 3. Er II FTS Spectrum, Wavelength, $\lambda = 369.265$ nm [Wavenumber, $\sigma = 27073.11$ cm^{-1} , Energy Level Classification, 27513 cm^{-1} ($J=13/2$) $\Delta T^{166, 170}$ (2798 MHz) – 440 cm^{-1} ($J=11/2$) $4f^{12}6s\Delta T^{166, 170}$ (2990 MHz)]. The extracted IS, $\Delta\sigma^{166, 170} = -190$ MHz, Light Source: Liquid nitrogen cooled HCL, Detector: PMT.

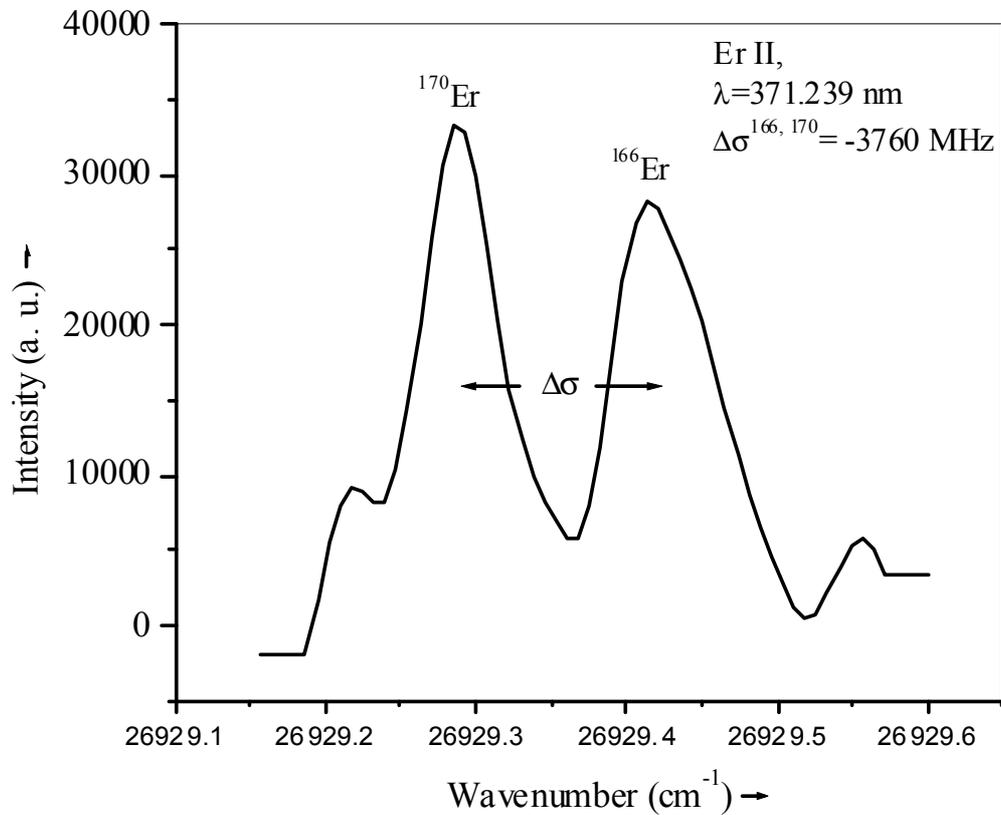


Figure 4. Er II FTS Spectrum, Wavelength, $\lambda = 371.239$ nm [Wavenumber, $\sigma = 26929.246$ cm^{-1} , Energy Level Classification, 33753 cm^{-1} ($J=13/2$) $\Delta T^{166, 170}$ (2671 MHz) $4f^{11}6s6p - 6824$ cm^{-1} ($J=15/2$) $4f^{11}6s^2\Delta T^{166, 170}$ (6430 MHz)]. The experimentally measured IS, $\Delta\sigma^{166, 170} = -3760$ MHz, Light Source: Liquid nitrogen cooled HCL, Detector: PMT.

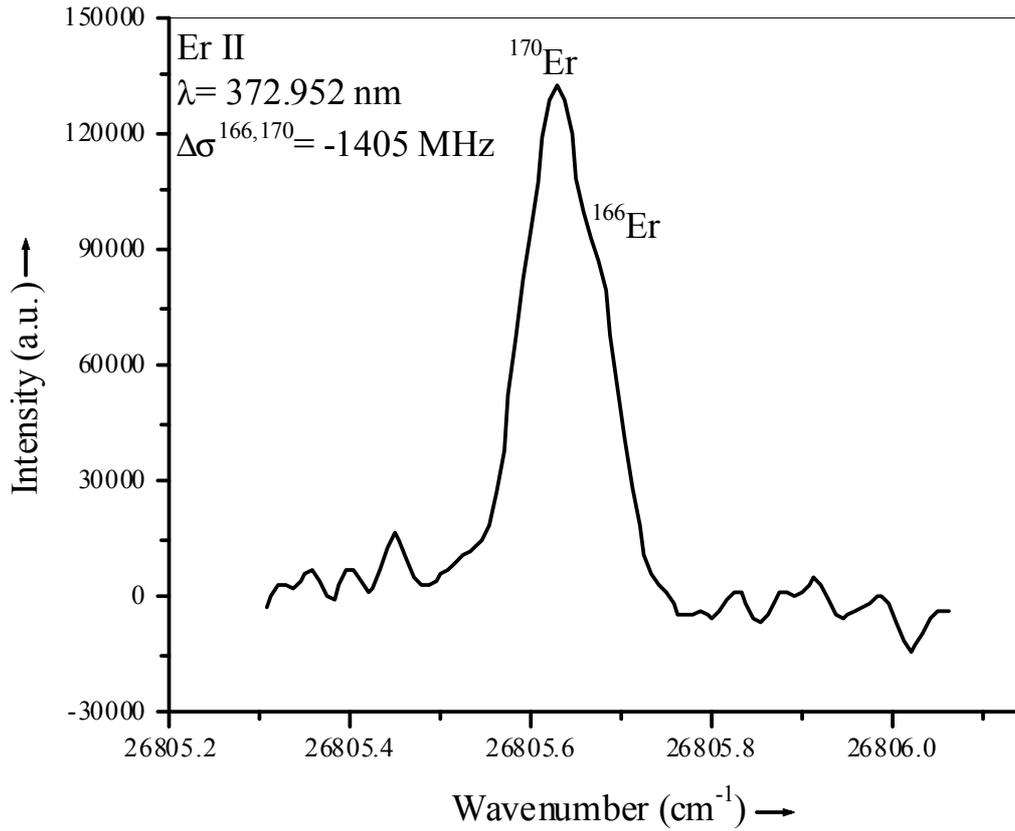


Figure 5. Er II FTS Spectrum, Wavelength, $\lambda = 372.952 \text{ nm}$ [Wavenumber; $\sigma = 26805 \text{ cm}^{-1}$, Energy Level Classification, $26805 \text{ cm}^{-1} (J=11/2) 4T^{166, 170} (1595 \text{ MHz}) - 0.00 \text{ cm}^{-1} (J=13/2) 4f^{12}6s \Delta T^{166, 170} (3000 \text{ MHz})$]. The extracted IS, $\Delta\sigma^{166, 170} = -1405 \text{ MHz}$. Light Source: Liquid nitrogen cooled HCL, Detector: PMT.

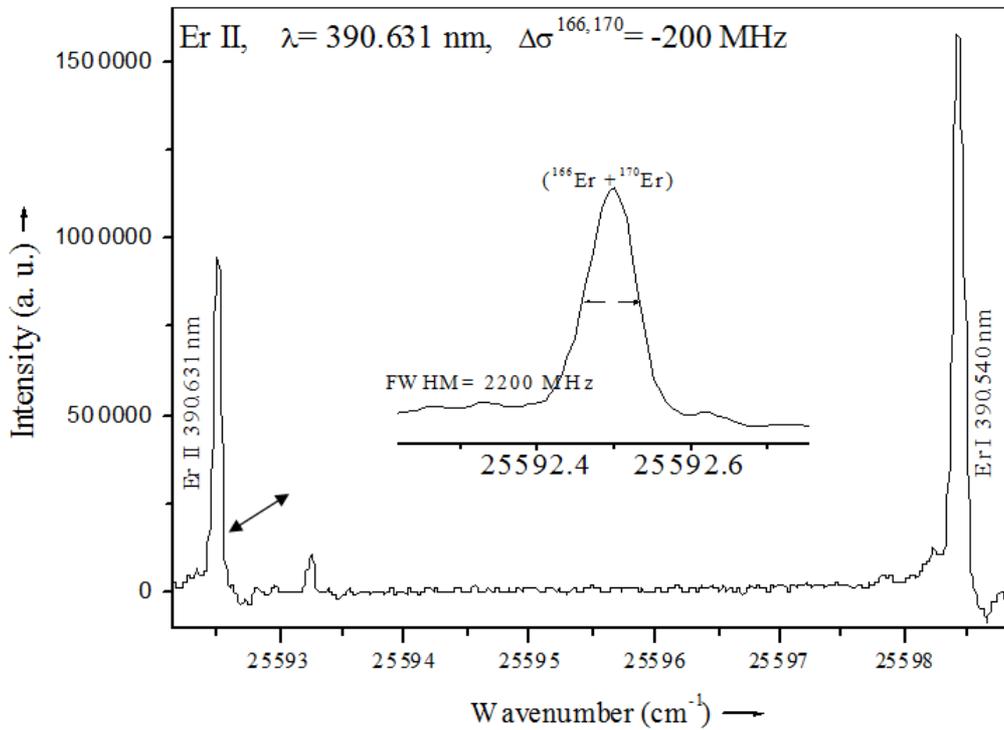


Figure 6. Er II FTS Spectrum, Wavelength, $\lambda = 390.631 \text{ nm}$ [Wavenumber; $\sigma = 25592.343 \text{ cm}^{-1}$, energy level classification $25592 \text{ cm}^{-1} (J=11/2) 4f^{12}6p, \Delta T^{166, 170} (2798 \text{ MHz}) - 0.00 \text{ cm}^{-1} (J=13/2) 4f^{12}6s \Delta T^{166, 170} (3000 \text{ MHz})$]. The extracted IS, $\Delta\sigma^{166, 170} = -200 \text{ MHz}$. Light Source: Liquid nitrogen cooled HCL, Detector: PMT.

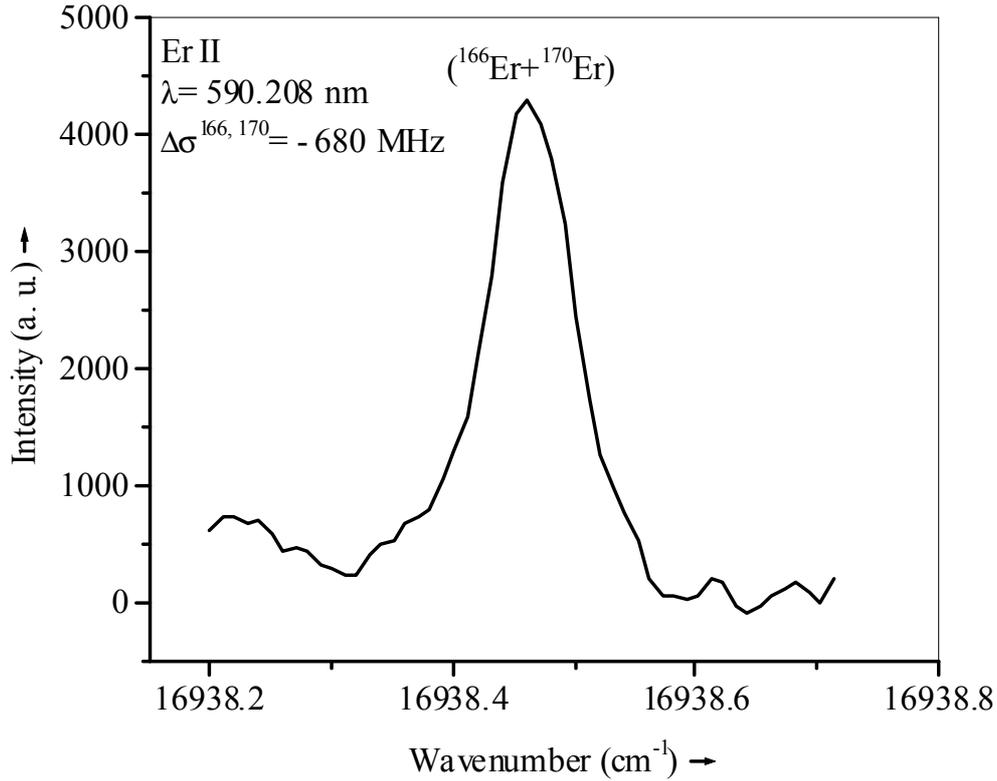


Figure 7. Er II FTS Spectrum, Wavelength, $\lambda = 590.208 \text{ nm}$ [Wavenumber, $\sigma = 16938.483 \text{ cm}^{-1}$, energy level classification $17378.917 \text{ cm}^{-1} (J=13/2) 4f^1 5d6s \Delta T^{166, 170}$ (2311 MHz) – $440.434 \text{ cm}^{-1} (J=11/2) 4f^2 6s \Delta T^{166, 170}$ (2990 MHz)]. The extracted $\Delta\sigma^{166, 170} = -680 \text{ MHz}$. Light Source: Liquid nitrogen cooled HCL, Detector: PMT.

Table 2. List of IS, $\Delta\sigma^{166, 170}$ (MHz) data in the spectral lines of the Er II in the wavelength region of 350-590 nm. a. u.: Arbitrary units. The wavelengths and the energy level classifications are referred from Meggers et al. [9]. A: Wyart et al. [6] have rejected the energy level classification provided (in the parenthesis) by Meggers et al. [9]. B: Energy level classification of this line has been taken from Wyart et al. [6] since this line was unclassified in [9].

λ (nm)	Intensity (a. u.)	Odd (cm^{-1})	LIS, $\Delta T^{166, 170}$ (MHz)	Even (cm^{-1})	LIS, $\Delta T^{166, 170}$ (MHz)	IS, $\Delta\sigma^{166, 170}$ (MHz)
349.910	6700	29011	1071	440	2990	-1920
352.491	610	28361	2132	0	3000	-870
355.990	1000	28082	2051	0	3000	-950
357.075 ^A	920	(33129)	83	(5132)	2991	-2910
358.052	1000	28361	2132	440	2990	-850
359.983	1000	10667	2399	38438	2256	-145
361.656	3100	27642	2159	0	3000	-840
361.892	510	33028	1711	5403	2990	-1280
363.354	1000	27513	2798	0	3000	-200
363.716	270	32619	2268	5132	2991	-725
364.594	900	13027	2400	40447	2251	0
365.287	500	34563	904	7195	2990	-2090
368.270	500	34341	1990	7195	2990	-1000
368.428	380	32267	1902	5132	2991	-1090
369.265	7900	27513	2798	440	2990	-190
369.625	450	34196	2382	7149	2991	-610
369.692	160	16935	2229	43977	2251	0
371.239	520	6824	6430	33753	2671	-3760
372.364	65	39447	1775	12600	2991	-1215
372.952	1300	26805	1595	0	3000	-1405
373.126	450	33988	1854	7195	2990	-1135
373.458	270	26769	2332	0	3000	-670
374.110	340	6824	6430	33547	2682	-3750
374.264	900	31844	1641	5132	2991	-1350
374.571	55	39277	2048	12587	2991	-950
375.054	190	13338	6374	39994	2569	-3805
377.566	230	16935	2229	43413	2256	0
378.447	310	33565	2093	7149	2991	-900
378.684	1800	26399	2338	0	3000	-660
379.706	560	26769	2332	440	2990	-660

λ (nm)	Intensity (a. u.)	Odd (cm ⁻¹)	LIS, $\Delta T^{166,170}$ MHz	Even (cm ⁻¹)	LIS, $\Delta T^{166,170}$ (MHz)	IS, $\Delta \sigma^{166,170}$ (MHz)
383.048	3600	26098	1595	0	3000	-1410
383.926	950	16948	5876	42987	2255	-3620
384.831	80	31381	3020	5403	2990	0
388.061	1500	30894	1781	5132	2991	-1210
388.289	1200	32896	1639	7149	2991	-1350
389.061	400	36738	1208	11042	2991	-1785
389.623	5200	26098	1595	440	2990	-1400
390.276	810	32811	1493	7195	2990	-1500
390.384	50	17378	2311	42987	2255	0
390.631	11000	25592	2798	0	3000	-200
391.242	140	11309	2399	36861	2680	+280
392.188	280	30894	1781	5403	2990	-1210
393.225	810	32619	2268	7195	2990	-725
393.863	2100	25382	2141	0	3000	-860
393.892	85	17842	2188	43223	2289	-100
397.472	1400	25592	2798	440	2990	-190
400.916	280	24935	2184	0	3000	-825
401.557	350	30028	2060	5132	2991	-930
404.624	270	31902	2083	7195	2990	-910
404.949	200	24687	2378	0	3000	-625
405.547	940	29783	1793	5132	2991	-1200
405.978	690	30028	2060	5403	2990	-930
408.124	550	24935	2184	440	2990	-825
409.464	140	24415	2345	0	3000	-655
410.398	60	29492	2089	5132	2991	-900
414.291	550	29263	1446	5132	2991	-1551
427.648	140	12815	2395	36192	2678	+285
430.160	320	23241	2233	0	3000	-770
430.381	60	28361	2132	5132	2991	-860
433.900	50	14280	2400	37321	2677	+275
438.470	300	23241	2233	440	2990	-760
441.961	570	13572	2400	36192	2678	+280
445.924	100	22859	2375	440	2990	-625
450.075	200	14649	2400	36861	2680	+280
467.562	570	10667	2399	32048	2688	+290
467.906	150	12388	2400	33753	2671	+165
473.159	85	15732	2400	36861	2680	+280
475.965	170	21004	2261	0	3000	-740
476.264	60	16643	2375	37634	2682	+310
482.035	190	11309	2399	32048	2688	+290
483.115	85	13060	2408	33753	2671	+265
485.164	30	17063	2798	37669	2626	-175
487.209	150	13027	2400	33547	2682	+280
487.248	30	20517	2362	0	3000	-640
490.008	210	12815	2395	33217	2609	+215
493.411	210	23973	926	44235	2058	+1135
495.360 ^B	45	24053	880	44235	2058	+1180
500.038	90	19992	2175	0	3000	-825
502.428 ^B	45	20319	2368	40217	2677	+310
502.891	120	20319	2368	440	2990	-625
504.205	210	13719	2392	33547	2682	+290
518.890	160	14280	2400	33547	2682	+280
521.826 ^B	30	28016	852	47174	2253	+1400
525.593	140	13027	2400	32048	2688	+290
590.208	27	17378	2311	440	2990	-680

3.1. Electron Configurations and Their Screening Ratios

Transition IS defined as the difference between the energies of upper and lower energy levels (LIS) of two different isotopes. IS in a line consists mainly of mass shifts (MS) and field isotope shifts (FIS) or field shift (FS). MS further divided into the normal mass shift (NMS) and specific mass shifts (SMS). MS dominates in lighter elements whereas FS dominates in the high Z elements with mass

number, $A \geq 100$. FS is observed due to the change in ns and in small extent in np electron densities at the nucleus or simply due to change in size and shape of the nuclei [15]. FS varies according to number of ns electrons present in the configuration and hence helps in identifying the definite configuration if LIS data is available for that energy level. However np and nd electrons screen the ns - electron in the given configuration thus amount of screening is different for different configurations. It has been observed that the

screening ratios of ns - electron densities for different configurations are proportional to the ratios of respective LIS of the pure configurations [16]. The task of identifying the configuration for the particular high lying level becomes theoretically difficult because of the favorable chances of configuration mixing. The configuration mixing takes place between two or more configurations of the close lying energy levels provided these energy levels have same parity and the same J . The LIS, $\Delta T^{166, 170}$ (MHz) were estimated using the well-known ‘‘Sharing Rule’’ (see eq. 1) according to which a state whose wave function (Ψ) results from mixing of ‘ n ’ number of configurations, the LIS, ΔT equals the sum of LIS, ΔT_i of individual configurations, multiplied by square of weight C_i of the configurations in Ψ ,

$$\Delta T = \sum_{i=1}^n C_i^2 \Delta T_i \text{ with } \sum C_i^2 = 1 \quad (1)$$

We evaluated the hypothetical ΔT values for the even and odd energy levels with the aid of various percentage compositions of different configurations. The even levels have configuration mixing of the type ($4f^{12}6s+4f^{12}5d+4f^{11}6s6p+4f^{11}5d6p$) whereas the odd levels have of the type ($4f^{11}6s^2+4f^{11}5d6s+4f^{12}6p+4f^{11}5d^2$) as has been provided in Wyart et al. [6].

The LIS of different even and odd levels of Er II were derived using the transition arrays, the different screening ratios published in reference [16] and the LIS, $\Delta T^{166, 170}$ 4800 MHz (0.160 cm^{-1}) of the ground state level of $4f^{12}6s^2$ configuration of Er I [14]. LIS $\Delta T^{166, 170}$ 3000 MHz (0.100 cm^{-1}) in the ground state level of $4f^{12}6s$ configuration was calculated using the empirical screening ratio provided in [16] $\Delta T(4f^{12}6s^2)/\Delta T(4f^{12}6s) = 1.6$. Some of the screening ratios derived presently for the even and odd configurations of Er II are summarized here;

$$\begin{aligned} \Delta T(4f^{12}6s^2)/\Delta T(4f^{12}6s) &= 1.6, \Delta T(4f^{11}6s6p)/\Delta T(4f^{12}6s) = 0.9, \\ \Delta T(4f^{11}6s5d)/\Delta T(4f^{12}6s) &= 0.8, \Delta T(4f^{12}6s)/\Delta T(4f^{12}6p) = 5.0, \\ \text{and } \Delta T(4f^{11}6s^2)/\Delta T(4f^{12}6s) &= 2.2 \end{aligned}$$

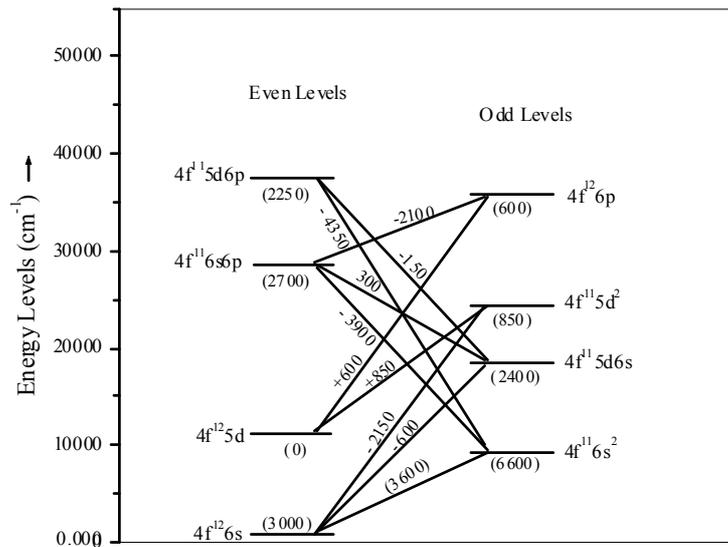


Figure 8. Partial energy level scheme, Horizontal lines represent energy levels, vertical lines represent the transitions, values across the transitions represent IS, $\Delta\sigma^{166,170}$ (MHz), values in the parenthesis represent LIS, $\Delta T^{166,170}$ (MHz) of the pure configuration.

Table 3. Pure Even and odd configurations with their expected LIS, $\Delta T^{166, 170}$ (MHz) of Er II.

Even Configuration	$\Delta T^{166, 170}$ (MHz)	Odd Configuration	$\Delta T^{166, 170}$ (MHz)
$4f^{12}6s^2$	4800	$4f^{11}6s^2$	6600
$4f^{12}6s$	3000	$4f^{11}6s5d$	2400
$4f^{12}5d$	0	$4f^{11}5d^2$	850
$4f^{11}6s6p$	2700	$4f^{12}6p$	600
$4f^{11}5d6p$	2250		

LIS of pure configurations of Er II derived using the above mentioned screening ratios and these were listed in the Table 3. Fig. 4 depicts the partial energy level diagram for the Er II spectrum encompassing the different types of transitions, IS $\Delta\sigma^{166,170}$ (MHz) data, different configurations, and their LIS, $\Delta T^{166, 170}$ (MHz). LIS data was derived with the accuracy of $\pm 0.003 \text{ cm}^{-1}$ ($\pm 90 \text{ MHz}$) accordingly for 29 even and 63 odd levels and presented in Table 4 and 5.

3.1.1. Even Parity Energy Levels and Their Configuration Mixing

Column 1 and 2 in the Table 4 exhibit the previous status of configuration assignment whereas column 3 shows the present experimental LIS, $\Delta T^{166, 170}$ data and the configuration mixing suggested by us. We have derived the hypothetical LIS ΔT values for the energy levels encountered in the present studies using the different possible configuration mixings and listed those along with the experimentally derived LIS ΔT s. All the 29 even parity energy levels exhibit configuration mixings of the type ($4f^{12}6s + 4f^{12}5d + 4f^{11}6s6p + 4f^{11}5d6p$) of the different configurations. As seen in the Table, the experimental LIS values and LIS values derived using ‘Sharing rule’ do not wonder much from each other. Thus the theoretically predicted configuration mixings for all even levels as reported in [6] were confirmed experimentally.

Table 4. The LIS, $\Delta T^{166, 170}$ (MHz) derived in the even- parity levels of Er II. a: The new level referred from [6].

[1]	Configuration Mixing (%) [6]						Present work ← (MHz) → ←		Configuration mixing (%) →			
Level (cm ⁻¹)	configuration	J	4f ¹² 6s	4f ¹² 5d	4f ¹¹ 6s6p	4f ¹¹ 5d6p	ΔT_{FTS}	$\Delta T_{\text{Sharing rule}}$	4f ¹² 6s	4f ¹² 5d	4f ¹¹ 6s6p	4f ¹¹ 5d6p
0.000	4f ¹² 6s	13/2	98.85	0.01	0.01	1.13	3000	2991.195	98.85	0.01	0.01	1.13
440.434	4f ¹² 6s	11/2	98.81	0.02	0.01	1.15	2990	2990.445	98.81	0.02	0.01	1.15
5132.608	4f ¹² 6s	9/2	98.82	0.01	0.02	1.15	2991	2991.015	98.82	0.01	0.02	1.15
5403.688	4f ¹² 6s	7/2	98.80	0.03	0.02	1.15	2990	2990.415	98.80	0.03	0.02	1.15
7149.630	4f ¹² 6s	11/2	98.83	0.02	0.01	1.14	2991	2990.82	98.83	0.02	0.01	1.14
7195.355	4f ¹² 6s	9/2	98.82	0.02	0.01	1.15	2990	2990.745	98.82	0.02	0.01	1.15
10893.936	4f ¹² 6s	7/2	98.81	0.02	0.02	1.15	2991	2990.715	98.81	0.02	0.02	1.15
11042.640	4f ¹² 6s	9/2	98.81	0.03	0.02	1.15	2991	2990.715	98.81	0.03	0.02	1.15
12587.998	4f ¹² 6s	7/2	98.78	0.03	0.03	1.17	2991	2990.535	98.78	0.03	0.03	1.17
12600.093	4f ¹² 6s	5/2	98.79	0.02	0.03	1.16	2991	2990.61	98.79	0.02	0.03	1.16
32048.749	4f ¹¹ 6s6p	15/2	0.00	0.00	97.49	2.51	2688	2688.705	0.00	0.00	97.49	2.51
33217.200	4f ¹¹ 6s6p	17/2	0.06	0.00	95.61	4.39	2682	2682.045	0.06	0.00	95.61	4.39
33547.268	4f ¹¹ 6s6p	15/2	0.00	0.00	95.97	4.03	2682	2681.865	0.00	0.00	95.97	4.03
33753.920	4f ¹¹ 6s6p	13/2	0.01	0.08	93.96	5.96	2671	2671.32	0.01	0.08	93.96	5.96
36192.206	4f ¹¹ 6s6p	19/2	0.00	0.00	95.33	4.67	2678	2678.985	0.00	0.00	95.33	4.67
36861.561	4f ¹¹ 6s6p	17/2	0.00	0.00	95.63	4.37	2680	2680.335	0.00	0.00	95.63	4.37
37321.029	4f ¹¹ 6s6p	15/2	0.00	0.01	94.95	5.04	2677	2677.05	0.00	0.01	94.95	5.04
37634.115	4f ¹¹ 6s6p	13/2	0.00	0.05	96.21	3.73	2682	2681.595	0.00	0.05	96.21	3.73
37669.535	4f ¹¹ 6s6p	11/2	0.01	1.94	93.31	4.74	2626	2626.02	0.01	1.94	93.31	4.74
38438.391	4f ¹¹ 5d6p?	11/2	0.00	0.00	1.37	98.63	2256	2256.165	0.00	0.00	1.37	98.63
39994.031	4f ¹¹ 6s6p?	15/2	0.00	0.00	70.99	29.01	2569	2569.455	0.00	0.00	70.99	29.01
40217.711 ^a	4f ¹¹ 6s6p?	11/2	0.00	0.05	95.13	4.82	2677	2676.96	0.00	0.05	95.13	4.82
40447.910	4f ¹¹ 5d6p?	19/2	0.00	0.00	0.26	99.74	2251	2251.17	0.00	0.00	0.26	99.74
42987.453	4f ¹¹ 5d6p?	11/2	0.00	0.01	1.29	98.68	2255	2255.13	0.00	0.01	1.29	98.68
43223.145	4f ¹¹ 5d6p?	15/2	0.00	0.01	4.31	96.58	2289	2289.42	0.00	0.01	4.31	96.58
43413.717	4f ¹¹ 5d6p?	19/2	0.00	0.00	1.32	98.68	2256	2255.94	0.00	0.00	1.32	98.68
43977.651	4f ¹¹ 5d6p?	19/2	0.00	0.00	0.18	99.82	2251	2251.17	0.00	0.00	0.18	99.82
44235.276	4f ¹¹ 5d6p?	17/2	0.00	0.00	8.54	91.45	2058	2058.71	0.00	0.00	8.54	91.45
47174.273	4f ¹¹ 5d6p?	17/2	0.00	0.01	0.67	99.20	2253	2253.42	0.00	0.01	0.67	99.20

Table 5. The list of the odd parity energy levels of Er II and their LIS, $\Delta T^{166, 170}$ values in MHz. A: This is newly calculated level taken from [6]. B: The unassigned level at 32811.006 cm⁻¹ tabulated in [1] was revised to 32810.980 cm⁻¹ in reference [6].

[1]	Configuration mixing (%) [6]						Present work ← (MHz) → ←		Configuration mixing (%) →			
Level (cm ⁻¹)	Configuration	J	4f ¹¹ 6s ²	4f ¹¹ 5d6s	4f ¹² 6p	4f ¹¹ 5d ²	ΔT_{FTS}	$\Delta T_{\text{Sharing rule}}$	4f ¹¹ 6s ²	4f ¹¹ 5d6s	4f ¹² 6p	4f ¹¹ 5d ²
6824.774	4f ¹¹ 6s ²	15/2	97.04	0.04	0.00	2.91	6430	6430.335	97.04	0.04	0.00	2.91
10667.165	4f ¹¹ 5d6s	13/2	0.00	99.97	0.00	0.02	2399	2399.45	0.00	99.97	0.00	0.02
11309.180	4f ¹¹ 5d6s	15/2	0.01	99.91	0.00	0.08	2399	2400.62	0.01	99.91	0.00	0.08
12388.090	4f ¹¹ 5d6s	11/2	0.01	99.78	0.03	1.18	2400	2405.59	0.01	99.78	0.03	1.18
12815.043	4f ¹¹ 5d6s	19/2	0.00	99.61	0.00	0.39	2395	2393.955	0.00	99.61	0.00	0.39
13027.927	4f ¹¹ 5d6s	17/2	0.00	99.99	0.00	0.01	2400	2399.845	0.00	99.99	0.00	0.01
13060.693	4f ¹¹ 5d6s	13/2	0.79	97.58	0.04	1.60	2408	2407.90	0.79	97.58	0.04	1.60
13338.777	4f ¹¹ 6s ²	13/2	96.26	0.84	0.00	2.90	6376	6397.97	96.26	0.84	0.00	2.90
13572.118	4f ¹¹ 5d6s	21/2	0.00	100.00	0.00	0.00	2400	2400.00	0.00	100.00	0.00	0.00
13719.562	4f ¹¹ 5d6s	17/2	0.00	99.46	0.00	0.54	2392	2391.63	0.00	99.46	0.00	0.54
14280.723	4f ¹¹ 5d6s	15/2	0.00	99.44	0.03	0.53	2400	2391.245	0.00	99.44	0.03	0.53
14649.277	4f ¹¹ 5d6s	19/2	0.00	99.98	0.00	0.02	2400	2399.69	0.00	99.98	0.00	0.02
15732.917	4f ¹¹ 5d6s	17/2	0.00	99.90	0.00	0.10	2400	2398.45	0.00	99.90	0.00	0.10
16643.237	4f ¹¹ 5d6s	13/2	0.00	97.89	0.98	1.13	2375	2364.845	0.00	97.89	0.98	1.13
16935.811	4f ¹¹ 5d6s	19/2	0.00	88.99	0.00	11.01	2229	2229.345	0.00	88.99	0.00	11.01
16948.197	4f ¹¹ 6s ²	11/2	84.30	12.86	0.08	2.75	5876	5896.295	84.30	12.86	0.08	2.75
17063.709	4f ¹¹ 5d6s	11/2	10.74	86.03	0.35	2.88	2798	2800.14	10.74	86.03	0.35	2.88
17378.917	4f ¹¹ 5d6s	13/2	0.02	94.26	0.51	5.21	2311	2310.905	0.02	94.26	0.51	5.21
17842.664	4f ¹¹ 5d6s	17/2	0.00	86.34	0.00	13.66	2188	2188.27	0.00	86.34	0.00	13.66
19992.895	4f ¹¹ 5d6s	11/2	0.20	89.45	1.60	8.74	2175	2243.89	0.20	89.45	1.60	8.74
20319.934	4f ¹¹ 5d6s	13/2	0.00	98.11	1.04	0.85	2368	2368.105	0.00	98.11	1.04	0.85
20517.717	-	15/2	0.01	97.68	0.88	1.43	2362	2362.415	0.01	97.68	0.88	1.43
21004.060	4f ¹¹ 5d6s	13/2	0.00	91.70	4.00	4.30	2261	2261.35	0.00	91.70	4.00	4.30
22859.510	4f ¹¹ 5d6s?	13/2	0.00	98.49	0.42	1.09	2375	2375.545	0.00	98.49	0.42	1.09
23240.649	-	11/2	0.06	89.74	4.55	5.65	2233	2233.045	0.06	89.74	4.55	5.65

[1] Level (cm ⁻¹)	Configuration	J	Configuration mixing (%) [6]				Present work ← (MHz) → ← Configuration mixing (%) →						
			4f ¹¹ 6s ²	4f ¹¹ 5d6s	4f ¹² 6p	4f ¹¹ 5d ²	ΔT _{FTS}	ΔT _{Sharing rule}	4f ¹¹ 6s ²	4f ¹¹ 5d6s	4f ¹² 6p	4f ¹¹ 5d ²	
23973.877	-	15/2	0.00	4.93	0.05	95.01	926	926.205	0.00	4.93	0.05	95.01	
24053.520	-	17/2	0.00	1.92	0.00	98.08	880	879.76	0.00	1.92	0.00	98.08	
24415.265	-	11/2	0.01	95.97	0.75	3.27	2345	2336.235	0.01	95.97	0.75	3.27	
24687.523	-	11/2	0.02	98.67	1.13	0.18	2378	2377.71	0.02	98.67	1.13	0.18	
24935.855	-	13/2	0.01	86.44	2.63	10.91	2184	2183.735	0.01	86.44	2.63	10.91	
25382.379	-	11/2	0.37	84.37	12.55	2.71	2141	2147.635	0.37	84.37	12.55	2.71	
25592.343	4f ¹² 6p?	11/2	0.92	40.35	46.15	12.58	2798	2800.14	0.92	40.35	46.15	12.58	
26098.957	4f ¹² 6p?	13/2	0.01	54.98	43.06	1.95	1595	1595.115	0.01	54.98	43.06	1.95	
26399.775	-	11/2	2.89	86.63	8.02	2.46	2338	2338.89	2.89	86.63	8.02	2.46	
26769.141	-	13/2	0.00	95.96	2.29	1.75	2332	2331.655	0.00	95.96	2.29	1.75	
26805.448	-	11/2	0.16	6.38	9.23	84.23	1595	1595.115	0.16	6.38	9.23	84.23	
27513.555	-	13/2	0.00	54.95	40.56	4.48	2798	2800.14	10.74	86.03	0.35	2.88	
27642.658	-	11/2	0.26	85.25	11.07	3.42	2159	2158.65	0.26	85.25	11.07	3.42	
28016.137 ^A	-	19/2	0.00	0.12	0.00	99.88	852	851.86	0.00	0.12	0.00	99.88	
28082.701	-	13/2	0.00	78.65	7.19	14.16	2051	2051.10	0.00	78.65	7.19	14.16	
28361.286	-	11/2	0.31	83.08	9.27	7.35	2132	2132.475	0.31	83.08	9.27	7.35	
29011.015	4f ¹² 6p?	9/2	0.01	24.21	61.99	13.80	1071	1070.94	0.01	24.21	61.99	13.80	
29263.402	-	7/2	0.10	40.58	15.66	43.66	1446	1445.59	0.10	40.58	15.66	43.66	
29492.329	4f ¹² 6p?	11/2	0.10	81.70	13.35	4.86	2089	2082.87	0.10	81.70	13.35	4.86	
29783.733	-	9/2	0.26	62.86	18.47	18.41	1793	1793.105	0.26	62.86	18.47	18.41	
30028.618	-	9/2	0.12	79.79	13.32	6.76	2060	2060.20	0.12	79.79	13.31	6.76	
30894.447	4f ¹² 6p?	7/2	0.02	64.74	29.44	5.80	1781	1781.02	0.02	64.74	29.44	5.80	
31381.779	-	9/2	19.85	67.39	5.25	7.51	3020	3022.795	19.85	67.39	5.25	7.51	
31844.124	-	9/2	0.92	53.52	36.10	9.46	1641	1642.21	0.92	53.52	36.10	9.46	
31902.682	-	7/2	0.01	81.26	10.74	7.99	2083	2083.255	0.01	81.26	10.74	7.99	
32267.246	-	9/2	0.06	69.91	13.89	16.14	1902	1902.33	0.06	69.91	13.89	16.14	
32618.753	-	11/2	3.15	81.47	9.87	5.50	2268	2269.15	3.15	81.47	9.87	5.50	
32811.006 ^B	-	11/2	0.01	45.34	24.01	30.63	1493	1493.475	0.01	45.34	24.01	30.63	
32896.371	4f ¹² 6p?	9/2	0.41	54.03	26.06	19.50	1639	1645.89	0.41	54.03	26.06	19.50	
33028.394	-	9/2	1.03	57.60	29.03	12.35	1711	1729.536	1.03	57.60	29.03	12.35	
33565.895	-	9/2	0.09	81.79	11.79	6.33	2093	2093.445	0.09	81.79	11.79	6.33	
33988.301	-	11/2	2.47	59.18	22.01	16.33	1854	1854.205	2.47	59.18	22.01	16.33	
34196.355	-	9/2	23.25	15.36	14.76	46.62	2382	2387.91	23.25	15.36	14.76	46.62	
34341.611	-	11/2	0.11	77.15	13.14	9.60	1990	2019.30	0.11	77.15	13.14	9.60	
34563.257	-	11/2	0.19	4.65	6.92	88.25	905	915.785	0.19	4.65	6.92	88.25	
36738.247	-	7/2	0.12	29.61	43.32	26.95	1208	1207.555	0.12	29.61	43.32	26.95	
39277.605	-	7/2	0.50	77.96	15.44	6.10	2048	2048.53	0.50	77.96	15.44	6.10	
39447.909	-	5/2	3.76	50.43	28.89	16.92	1775	1775.62	3.76	50.43	28.89	16.92	

3.1.2. Odd Parity Energy Levels and Their Configuration Mixing

All the 63 odd levels involved in the present experiment have configuration mixing of the type (4f¹¹6s²+4f¹¹5d6s+4f¹²6p+4f¹¹5d²) [6]. A multiplet ⁴F^o consisted of 3 low odd levels of 4f¹¹6s² configuration lying at 6824 cm⁻¹, 13338 cm⁻¹ and 16948 cm⁻¹ [1] listed in Table 5 exhibit LIS 6430, 6376 and 5896 MHz and thus confirmed having dominant 96% 4f¹¹6s² configuration as quoted in the reference [6]. 11 levels of 4f¹¹5d6s configuration lying at 11309 – 20319 cm⁻¹ and one level at 22859 cm⁻¹ tentatively assigned to 4f¹¹5d6s configuration [1] have been confirmed on the basis of LIS value ~ 2400 MHz. The LIS 1071 MHz observed in the level at 29011 cm⁻¹ confirms the tentative 4f¹²6p configuration. The 4 levels falling at 26098, 29492, 30894, and 32896 cm⁻¹ were previously assigned tentatively to 4f¹²6p configuration [1] exhibit LIS 1595, 2089, 1781 and 1639 MHz respectively and found belonging to the dominant 4f¹¹5d6s configuration [6]. 7 unassigned levels falling between 23973- 34563 cm⁻¹ listed in Table 5 can be assigned to dominant 4f¹¹5d² configuration. 24 unassigned levels including the level at 32811.008 cm⁻¹ (in reference [6] it was

revised to 32810.980 cm⁻¹) falling in the energy range of 20517- 39447 cm⁻¹ [1] have average LIS 2000 MHz suggests that these levels should belong to the dominant 4f¹¹5d6s configuration [6].

4. Conclusion

The high resolution Er II spectra recorded using the mixture of highly enriched ¹⁶⁶Er and ¹⁷⁰Er isotopes (7:10 ratio) in the liquid nitrogen cooled HCL. This first detailed investigation of IS, Δσ^{166, 170} in the 85 spectral lines of Er⁺ were conducted using a FTS. The measurements have contributed significantly to the acquaintance of the known even and odd energy levels of Er⁺. The main features of this work could be summarized as, the present IS data in 85 lines has enabled us to evaluate LIS, ΔT^{166, 170} values for 29 even and 63 odd parity energy levels for the first time. The theoretically predicted configuration mixings found in the excellent agreement with the experimentally derived mixings. 4 odd levels tentatively assigned to 4f¹²6p configuration were revised to 4f¹¹5d6s configuration. 7 unassigned levels assigned to dominant 4f¹¹5d²

configuration. 9 even levels of $4f^{11}6s6p$ and 19 odd levels of $4f^{11}5d6s$ configuration were confirmed whereas 24 unassigned odd levels designated dominantly to the $4f^{11}5d6s$ configuration.

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