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Smarandachely Adjacent-Vertex-Distinguishing Proper Edge Chromatic Number of $C_m \vee K_n$

Shunqin Liu

School of Information & Technology, Xiamen University Tan Kah Kee College, Zhangzhou, China

Email address:
pytxlx@163.com

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Abstract: According to different conditions, researchers have defined a great deal of coloring problems and the corresponding chromatic numbers. Such as, adjacent-vertex-distinguishing total chromatic number, adjacent-vertex-distinguishing proper edge chromatic number, smarandachely-adjacent-vertex-distinguishing proper edge chromatic number, smarandachely-adjacent-vertex-distinguishing proper total chromatic number. And we focus on the smarandachely adjacent-vertex-distinguishing proper edge chromatic number in this paper, study the smarandachely adjacent-vertex-distinguishing proper edge chromatic number of joint graph $C_m \vee K_n$.

Keywords: Graph Theory, Joint Graph, Smarandachely Adjacent-Vertex-Distinguishing Proper Edge Chromatic Number

1. Introduction

Coloring problem in graph theory, is one of the most famous NP-complete problems. Four color conjecture which is one of the world's three major mathematical conjecture says that each map can be used only four colors to dye, and no two adjacent areas dyed the same color. In the spring of 1976, with the help of the computer, the four color conjecture was proved. The conjecture finally became a theorem. The significance of graph coloring theory is much more than that. Known to all, coloring problems can solve many problems such as scheduling problem, time tabling, transportation, arrangement, circuit design and storage problems.

In recent years, more and more coloring problems was put forward by experts of graph theory, such as proper-adjacent-vertex-distinguishing edge coloring, proper -adjacent-vertex-distinguishing total coloring, smarandachely adjacent-vertex-distinguishing proper edge coloring.

2. Smarandachely Adjacent-Vertex-Distinguishing Proper Edge Coloring

Definition 1[1] A k-proper edge coloring of a graph G is a mapping f from $E(G)$ to $\{1,2,\ldots, k\}$ that satisfies the condition described as below:

For $\forall e_i, e_j \in E(G)$, $e_i \neq e_j$, if e_i, e_j have a common end vertex, then $f(e_i) \neq f(e_j)$.

The number $\min\{k \mid G$ has a k-proper edge coloring of graph $G\}$ is called the proper edge chromatic number of G, denoted by $\chi(G)$. If $f(e_i)=l$, then we call the number l to be the color of edge e_i.

Definition 2[1] A k-proper edge coloring f is called a k-proper-adjacent-vertex-distinguishing proper edge coloring, short for k-AVDPEC when f satisfies condition described as below:

Denote $C(u)=\{f(uv) \mid v\in V(G) \wedge uv\in E(G)\}$ for every vertex $u\in V(G)$, if for $\forall u,v\in V(G), uv\in E(G)$, we have $C(u)\neq C(v)$.

The number $\min\{k \mid G$ has a k-proper-adjacent-vertex-distinguishing proper edge coloring$\}$ is called the adjacent –vertex-distinguishing proper edge chromatic number and denoted by $\chi'_a(G)$. Set $C(u)$ is called the color set of the vertex.

Definition 3[1] A k-proper edge coloring f is called a smarandachely adjacent- vertex-distinguishing proper edge coloring, short for k-SA when f satisfies conditions described as below:

Denote $C(u)=\{f(uv)\mid v\in V(G)\wedge uv\in E(G)\}$ for every vertex $u\in V(G)$, if $\forall u,v\in V(G), uv\in E(G)$, we have $C(u)\not\subset C(v)$ and $C(v)\not\subset C(u)$ mean well.

The number $\min\{k\mid G\ has\ a\ k-SA\}$ is called the smarandachely adjacent-vertex-distinguishing proper edge chromatic number of G, denoted by $\chi'_{sa}(G)$.

It's obviously of below conclusion: when G have a vertex who's degree equal to 1, then G have no k-SA for all nature number k.

The paper use C_m to denote the graph of circle with m vertices and K_n to denote the complete graph with n vertices, use $C_m\vee K_n$ to denote the joint graph of C_m and K_n. We denote the vertex sets and the edge set of the graphs such that:

$V(C_m)=\{u_1,u_2,\cdots,u_m\}$, $E(C_m)=\{(u_{i-1}u_i)\mid 2\le i\le m\}\cup\{(u_m u_1)\}$, $V(K_n)=\{v_1,v_2,\cdots,v_n\}$.

$d(u)$ is the degree of the vertex u, Δ is the maximum degree of the graph discussed, S is the universal set of the color used, that $S=\bigcup_{u\in V(G)} C(u)$, $\overline{C}(u)$ is the complement of $C(u)$.

Lemma 1 [1] If G denote the graph have no one degree vertex, then

1) $\chi'_{sa}(G)\ge\chi'_a(G)$, if G is a regular graph then $\chi'_{sa}(G)=\chi'_a(G)$.

2) $\chi'_{sa}(G)\ge\Delta+1$.

Lemma 2 [1] If K_n is the complete graph with vertices n, $n\ge 3$, then

$$\chi'_a(K_n)=\begin{cases} n, & n\ is\ odd. \\ n+1, & n\ is\ even\end{cases}.$$

Lemma 3[2] If K_n is the complete graph with vertices n, $n\ge 4$ and n is even, then $\chi(K_n)=n-1$.

3. Smarandachely Adjacent-Vertex-Distinguishing Proper Edge Coloring of $C_m \vee K_n$

Theorem 1 If $m\ge 3$, $n\ge 3$, m,n are both even, then $C_m\vee K_n$ have no $m+n-SA$.

Proof Suppose that $C_m\vee K_n$ have a $m+n-SA$, then $S=\{1,2,\cdots,m+n\}$. Be aware of the facts that:

$d(v_1)=d(v_2)=\cdots=d(v_n)=m+n-1$

Then $|C(v_1)|=|C(v_2)|=\cdots=|C(v_n)|=m+n-1$

$$|\overline{C}(v_1)|=|\overline{C}(v_2)|=\cdots=|\overline{C}(v_n)|=1 \quad (1)$$

$\forall i\ne j,\ 1\le i\le n,\ 1\le j\le n$, v_i is adjacent to v_j, so $C(v_i)\ne C(v_j)$, so

$$\forall i\ne j\ \ 1\le i\le n,\ 1\le j\le n,\ \overline{C}(v_i)\ne\overline{C}(v_j) \quad (2)$$

Inferred from (1) and (2), then each vertex $v_i (1\le i\le n)$ has one different color from each other.

Then we may as well suppose that $i\notin\overline{C}(v_i)$, for $\forall 1\le i\le n$.

Then it must have the result such as $\{1,2,\cdots,n\}\subset C(u_j)$, $\forall 1\le j\le m$ (otherwise, if there exists a color $k, 1\le k\le n$, and exists a vertex u_j satisfies $k\notin C(u_j)$, then we can deduce the result such as $C(u_j)\subset C(v_i), \forall 1\le i\le n$, but the vertex u_j is adjacent to the vertex v_i, this result is in contradiction with the definition of $k-SA$). Now we consider all vertices of the whole graph $C_m\vee K_n$ who satisfy $f(u)=1$, according to previous discussion, except vertex v_1, all the remaining $m+n-1$ vertices of the graph $C_m\vee K_n$ satisfied the condition $f(u)=1$, that is to say, except the vertex v_1, the remaining $m+n-1$ vertices form a matching, this result is in contradiction with the fact such as that $m+n-1$ is odd.

So, $C_m\vee K_n$ have no $m+n-SA$.

Theorem 2 If $n\ge 6$, n is even, then

$$\chi'_{sa}(C_{n-2}\vee K_n)=2n-1.$$

Proof Be aware of the fact that the maximum degree of graph $C_{n-2}\vee K_n$ is $\Delta=2n-3$, by the 2) of Lemma 1, we get the result such as $\chi'_{sa}(C_{n-2}\vee K_n)\ge 2n-2$, also because of the fact that n is even, then by theorem 1, the graph $C_{n-2}\vee K_n$ have no $(2n-2)-SA$, so we get the result such as $\chi'_{sa}(C_{n-2}\vee K_n)\ge 2n-1$.

For the subgraph K_n, by lemma 3, there is a $n-1$-proper edge coloring on K_n, that is to say, there is a mapping g from $E(K_n)$ to $\{1,2,\ldots,\ n-1\}$ satisfied that $\forall e_i,e_j\in E(K_n)$, $e_i\ne e_j$, if e_i,e_j have a common end vertex, then $g(e_i)\ne g(e_j)$.

Now we define a mapping f from $E(C_{n-2}\vee K_n)$ to $\{1,2,\ldots,\ n-1,n,n+1,n+2,\cdots,2n-1\}$ described as below:

If $e\in K_n$, then $f(e)=g(e)$, then $\{1,2,\ldots,\ n-1\}\subset C(v_i)$, for $\forall 1\le i\le n$,

$$f(u_1v_j)=n+j-1,\ \ 1\le j\le n,$$

$$f(u_2v_j)=n+j,\ \ 1\le j\le n-1,\ \ f(u_2v_n)=n,$$

$$f(u_3v_j)=n+j+1,\ \ 1\le j\le n-2,$$

$f(u_3v_{n-1})=n,\quad f(u_3v_n)=n+1$,

$f(u_4v_j)=n+j+2,\quad 1\le j\le n-3$,

$f(u_4v_{n-2})=n,\quad f(u_4v_{n-1})=n+1$,

$f(u_4v_n)=n+2$,

$\cdots$

$f(u_{n-2}v_1)=2n-3,\quad f(u_{n-2}v_2)=2n-2$,

$f(u_{n-2}v_3)=2n-1$,

$f(u_{n-2}v_j)=n+j-4,\quad 4\le j\le n$,

$f(u_{i-1}u_i)=i,\quad 2\le i\le n-2$, $f(u_{n-2}u_1)=1$.

By the definition of f, we get the $C(u)$ and $\overline{C}(u)$ of every vertex of the graph such as below:

$C(v_1)=\{1,2,3,\cdots,n-1\}\cup\{n,n+1,n+2,\cdots,2n-3\}$
$\overline{C}(v_1)=\{2n-2,2n-1\}$, $\overline{C}(v_2)=\{n,2n-1\}$,

$\overline{C}(v_3)=\{n,n+1\}$, $\overline{C}(v_4)=\{n+1,n+2\}$,

$\cdots$

$\overline{C}(v_n)=\{2n-3,2n-2\}$,

$C(u_1)=\{n,n+1,n+2,\cdots,2n-1\}\cup\{1,2\}$

$\overline{C}(u_1)=\{3,4,\cdots,n-1\}=\{1,2,\cdots,n-1\}\setminus\{1,2\}$,

$\overline{C}(u_2)=\{1,2,\cdots,n-1\}\setminus\{2,3\}$,

$\cdots$

$\overline{C}(u_i)=\{1,2,3,\cdots,n-1\}\setminus\{i,i+1\}$, $\forall 2\le i\le n-3$,

$\overline{C}(u_{n-2})=\{1,2,\cdots,n-1\}\setminus\{n-2,1\}$.

We can see that

$\overline{C}(u_i)\not\subset\overline{C}(v_j),\quad 1\le i\le n-2,\quad 1\le j\le n$,

$\overline{C}(v_j)\not\subset\overline{C}(u_i),\quad 1\le i\le n-2,\quad 1\le j\le n$,

$\overline{C}(u_i)\not\subset\overline{C}(u_j),\quad 1\le i,j\le n-2, |i-j|=1$,

$\overline{C}(u_1)\not\subset\overline{C}(u_{n-2}),\quad \overline{C}(u_{n-2})\not\subset\overline{C}(u_1)$,

$\overline{C}(v_i)\not\subset\overline{C}(v_j),\quad 1\le i\le n,\quad 1\le j\le n,\quad i\ne j$.

So the given f is a $(2n-1)-SA$ for $C_{n-2}\vee K_n$. That is to say

$$\chi'_{sa}(C_{n-2}\vee K_n)=2n-1.$$

Example 1 For $n=6$, then $\chi'_{sa}(C_4\vee K_6)=11$.

In fact, for the subgraph K_6 of $C_4\vee K_6$, we defined a 5-proper edge coloring g such that:

$g(v_1v_2)=g(v_3v_6)=g(v_4v_5)=1$,

$g(v_1v_3)=g(v_2v_4)=g(v_5v_6)=2$,

$g(v_1v_4)=g(v_3v_5)=g(v_2v_6)=3$,

$g(v_1v_5)=g(v_4v_6)=g(v_2v_3)=4$,

$g(v_1v_6)=g(v_2v_5)=g(v_3v_4)=5$.

Obviously, every adjacent edge have different colors, so g is a 5-proper edge coloring of K_6.

Then, for graph $C_4\vee K_6$, we define the mapping f from $E(C_4\vee K_6)$ to $\{1,2,,3,\ldots,10,11\}$ described as below:

For edge v_iv_j, $f(v_iv_j)=g(v_iv_j),\quad 1\le i\ne j\le 6$,

$f(u_1v_j)=n+j-1,\quad 1\le j\le 6$,

$f(u_2v_j)=n+j,\quad 1\le j\le 5,\quad f(u_2v_6)=6$,

$f(u_3v_j)=n+j+1,\quad 1\le j\le 4$,

$f(u_3v_5)=6,\quad f(u_3v_6)=7$,

$f(u_4v_j)=n+j+2,\quad 1\le j\le 3$,

$f(u_4v_4)=6,\quad f(u_4v_5)=7, f(u_4v_6)=8$,

$f(u_{i-1}u_i)=i,\quad 2\le i\le 4$, $f(u_{n-2}u_1)=1$.

We can see that the $C(u)$ and $\overline{C}(u)$ of every vertex for graph $C_4\vee K_6$ are described as below:

$C(v_1)=\{1,2,3,4,5\}\cup\{6,7,8,9\}$,

$\overline{C}(v_1)=\{10,11\}$,

$\overline{C}(v_2)=\{6,11\}$, $\overline{C}(v_3)=\{6,7\}$,

$\overline{C}(v_4)=\{7,8\}$, $\overline{C}(v_5)=\{8,9\}$, $\overline{C}(v_6)=\{9,10\}$,

$\overline{C}(u_1)=\{3,4,5\}$, $\overline{C}(u_2)=\{1,4,5\}$,

$\overline{C}(u_3)=\{1,2,5\}$, $\overline{C}(u_4)=\{2,3,5\}$.

We can see that the color set of the adjacent vertices meet the requirements of definition[3], so f is a $11-SA$ for $C_4\vee K_6$, then $\chi'_{sa}(C_4\vee K_6)=11$.

Theorem 3 If $n\ge 4$, n is even, then

$$\chi'_{sa}(C_{n-1} \vee K_n) = 2n-1 .$$

Proof Because of the fact that the maximum degree of the graph $C_{n-1} \vee K_n$ is $\Delta = 2n-2$, by the 2) of Lemma 1, we get the result such as $\chi'_{sa}(C_{n-1} \vee K_n) \ge 2n-1$.

For the subgraph K_n of the graph $C_{n-1} \vee K_n$, by lemma 3, there is a $n-1$-proper edge coloring on K_n, that is to say, there is a mapping g from $E(K_n)$ to $\{1,2,\ldots, n-1\}$ that is satisfied the conditions such as:

$\forall e_i, e_j \in E(K_n)$, $e_i \ne e_j$, if e_i, e_j have a common end vertex, then $g(e_i) \ne g(e_j)$.

Now we define a mapping f from $E(C_{n-1} \vee K_n)$ to $\{1,2,\ldots, n-1, n, n+1, n+2, \cdots, 2n-1\}$ as below:

If $e \in K_n$, then $f(e) = g(e)$, then $\{1,2,\ldots, n-1\} \subset C(v_i)$, $\forall 1 \le i \le n$,

$$f(u_1v_j) = n+j-1, \quad 1 \le j \le n,$$

$$f(u_2v_j) = n+j, \quad 1 \le j \le n-1, \quad f(u_2v_n) = n,$$

$$f(u_3v_j) = n+j+1, \quad 1 \le j \le n-2,$$

$$f(u_3v_{n-1}) = n, \quad f(u_3v_n) = n+1,$$

$$\cdots$$

$$f(u_{n-1}v_1) = 2n-2, \quad f(u_{n-1}v_2) = 2n-1,$$

$$f(u_{n-1}v_j) = n+j-3, \quad 3 \le j \le n,$$

$$f(u_{i-1}u_i) = i, \quad 2 \le i \le n-1 \quad f(u_{n-1}u_1) = 1, \quad 2 \le i \le n-1.$$

By the definition of f, we get that the $C(u)$ and $\overline{C}(u)$ of every vertex as below:

$$C(v_1) = \{1,2,\cdots,n-1\} \cup \{n, n+1, \cdots, 2n-2\},$$

$$\overline{C}(v_1) = \{2n-1\}, \overline{C}(v_2) = \{n\},$$

$$\overline{C}(v_j) = \{n+j-2\}, \quad 2 \le j \le n,$$

$$C(u_1) = \{n, n+1, n+2, \cdots, 2n-1\} \cup \{1,2\},$$

$$\overline{C}(u_1) = \{1,2,\cdots,n-1\} \setminus \{1,2\},$$

$$\overline{C}(u_2) = \{1,2,\cdots,n-1\} \setminus \{2,3\},$$

$$\cdots$$

$$\overline{C}(u_i) = \{1,2,3,\cdots,n-1\} \setminus \{i, i+1\},$$

$$\forall 2 \le i \le n-2,$$

$$\overline{C}(u_{n-1}) = \{1,2,3,\cdots,n-1\} \setminus \{n-1, 1\}.$$

We can see that

$$\overline{C}(u_i) \not\subset \overline{C}(v_j), \quad 1 \le i \le n-1, \quad 1 \le j \le n,$$

$$\overline{C}(v_j) \not\subset \overline{C}(u_i), \quad 1 \le i \le n-1, \quad 1 \le j \le n,$$

$$\overline{C}(u_i) \not\subset \overline{C}(u_j), \quad 1 \le i \ne j \le n-1, | \quad |i-j| = 1,$$

$$\overline{C}(u_1) \not\subset \overline{C}(u_{n-1}), \quad \overline{C}(u_{n-1}) \not\subset \overline{C}(u_1),$$

$$\overline{C}(v_i) \not\subset \overline{C}(v_j), \quad 1 \le i \le n, \quad 1 \le j \le n, \quad i \ne j.$$

That is to say, all the vertices have color sets that are not contained in other color set of the adjacent vertices. So the given f is a $(2n-1)-SA$ for $C_{n-1} \vee K_n$.

So $\chi'_{sa}(C_{n-1} \vee K_n) = 2n-1$.

Example 2 For $n = 4$, then $\chi'_{sa}(C_3 \vee K_4) = 7$.

In fact, we defined the mapping f from $E(C_3 \vee K_4)$ to $\{1,2,3,4,5,6,7\}$ as below:

$$f(v_1v_2) = f(v_3v_4) = f(u_1u_3) = 1,$$

$$f(v_1v_2) = f(v_2v_4) = f(u_1u_2) = 2,$$

$$f(v_1v_4) = f(v_2v_3) = f(u_2u_3) = 3,$$

$$f(u_1v_1) = 4, \quad f(u_1v_2) = 5, \quad f(u_1v_3) = 6, \quad f(u_1v_4) = 7,$$

$$f(u_2v_1) = 5, \quad f(u_2v_2) = 6,$$

$$f(u_2v_3) = 7, \quad f(u_2v_4) = 4,$$

$$f(u_3v_1) = 6, \quad f(u_3v_2) = 7,$$

$$f(u_3v_3) = 4, \quad f(u_3v_4) = 5.$$

We can see that

$$\overline{C}(v_1) = \{6,7\}, \overline{C}(v_2) = \{7,4\},$$

$$\overline{C}(v_3) = \{4,5\}, \overline{C}(v_4) = \{5,6\},$$

$$\overline{C}(u_1) = \{3\}, \overline{C}(u_2) = \{1\}, \overline{C}(u_3) = \{2\}.$$

Obviously, f is a $7-SA$ for $C_3 \vee K_4$.

So $\chi'_{sa}(C_3 \vee K_4) = 7$.

4. Conclusion

Coloring problem is a classical difficult problem of graph theory. Smarandachely adjacent- vertex-distinguishing proper edge coloring was first put forward by Zhang Zhong-fu in 2008. A lot of problems need to be solved urgently, such as finding out the smarandachely adjacent-vertex-distinguishing proper edge chromatic number, such as how smarandachely adjacent-vertex-distinguishing proper edge chromatic number changes when the vertices n grows.

In the paper, we deduce the smarandachely adjacent-vertex -distinguishing proper edge chromatic number of the joint graph $C_m \vee K_n$ by the methods of combination analysis and reduction to absurdity, also the method of apagoge.

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