Applied and Computational Mathematics
2014; 3(1): 38-42
Published online March 10, 2014 (http://www.sciencepublishinggroup.com/j/acm)
doi: 10.11648/j.acm.20140301.16

An easy computable approximate solution for a squeezing flow between two infinite plates by using of perturbation method

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To cite this article:

U. Filobello-Nino, H. Vazquez-Leal, A. Perez-Sesma, J. Cervantes-Perez, V. M. Jimenez-Fernandez, L. Hernandez-Martinez, D. Pereyra-Diaz, R. Castaneda-Sheissa,J. Sanchez-Orea, C. Hoyos-Reyes, S. F. Hernandez-Machuca, J. Huerta-Chua, J. L. Rocha-Fernandez, A. D. Contreras-Hernandez, J. M. Mendez-Perez. An Easy Computable Approximate Solution for a Squeezing Flow between Two Infinite Plates by using of Perturbation Method. *Applied and Computational Mathematics.* Vol. 3, No. 1, 2014, pp. 38-42.
doi: 10.11648/j.acm.20140301.16

Abstract: This article proposes Perturbation Method (PM) to find an approximate solution for the problem of an axis symmetric Newtonian fluid squeezed between two large parallel plates. After comparing figures between approximate and exact solutions, we will see that the proposed solutions besides of handy, are highly accurate and therefore that PM is efficient.

Keywords: Mixed Boundary Conditions, Nonlinear Differential Equation, Perturbation Method, Approximate Solutions

1. Introduction

Although the studies of squeezing flows have its origins in the 19th century, at present time, it is an issue of considerable importance due to its practical applications in different areas such as physical, biophysical, chemical engineering, and food industry, also are relevant in liquid metal lubrication theory, polymer processing, compression and injection molding, among many others.

The purpose of this job is the search for an approximate solution for the nonlinear problem, which describes a viscous, incompressible fluid, squeezed between two infinite parallel plates, separated instantaneously a distance *2l*so that the plates are moving towards each other with a certain velocity, say *W*(see Fig. 1).

As mentioned in [1] these fluids are of paramount importance in hydrodynamic lubrication theory. Thus, [2] and [3] analysed isothermal compressible squeeze films neglecting inertial effects; while [4] found an explicit solution, taking into account these effects. Also, some numerical solutions to these problems have been found, such as those provided in [5] and [6]. Additionally, [7] and [8] extended the previous investigations for the case of flow between rotating parallel plates.

The perturbation method (PM) is a widely known and established method; it is among the pioneer techniques to approach various kinds of nonlinear problems. This procedure was originated by S. D. Poisson and extended by J. H. Poincare. Although the method appeared in the early 19th century, the application of a perturbation procedure to solve nonlinear differential equations was performed later on that century. The most significant efforts were focused on

celestial mechanics, fluid mechanics, and aerodynamics [9,10].

In a broad sense, it is possible to express a nonlinear differential equation in terms of one linear part and other nonlinear. The nonlinear part is considered as a small perturbation through a small parameter (the perturbation parameter). The assumption that the nonlinear part is small compared to the linear is considered as a disadvantage of the method. There are other modern alternatives to find approximate solutions to the differential equations that describe some nonlinear problems such as those based on: variational approaches [11-14], tanh method [15], exp-function [16-18], Adomian's decomposition method [19-24], parameter expansion [24], homotopy perturbation method [14,24,26-28,29-47], homotopy analysis method [48], homotopy asymptotic method [26],perturbation method [49,50], and (G'/G)-expansion method [51,52] among many others. Also, a few exact solutions to nonlinear differential equations have been reported occasionally [53].

Although the PM method provides in general, better results for small perturbation parameters $\varepsilon << 1$, we will see that our approximations, besides to be handy, have a good accuracy, even for relatively large values of the perturbation parameter [49,50].

This paper is organized as follows. In Section 2, we introduce the basic idea of the PM method. For Section 3, we provide the basic equation for the problem of the axisymmetric Newtonian fluid above mentioned. Section 4 discusses the main results obtained. Finally, a brief conclusion is given in Section 5.

2. Basic Idea of Perturbation Method

Let the differential equation of one dimensional nonlinear system be in the form

$$L(x)+\varepsilon N(x)=0, \tag{1}$$

where we assume that x is a function of one variable $x=x(t)$, $L(x)$ is a linear operator which, in general, contains derivatives in terms of t; $N(x)$ is a nonlinear operator, and ε is a small parameter.

Considering the nonlinear term in (1) to be a small perturbation and assuming that the solution for (1) can be written as a power series in the small parameter ε,

$$x(t)=x_0+\varepsilon x_1(t)+\varepsilon^2 x_2(t)+\cdots. \tag{2}$$

Substituting (2) into (1) and equating terms having identical powers of ε, we obtain a number of differential equations that can be integrated, recursively, to find the values for the functions:$x_0(t)$, $x_1(t)$, $x_2(t)$, ...

3. Approximate Solution for a Nonlinear Problem of Fluids

The objective of this section is employ PM, to find an analytical approximate solution for the nonlinear problem which describes an axisymmetric Newtonian fluid squeezed between two large parallel plates given by

$$d^4y(x)/dx^4+\varepsilon y(x)d^3y(x)/dx^3=0,$$

$$0\le x\le 1,\ y(0)=0,\ y''(0)=0,\ y(1)=1, y'(1)=0, \tag{3}$$

where, ε is a positive parameter of the fluid, related to its density and with instantaneous separation distance $2l$(see Fig. 1).

It is possible to find a handy solution by applying the LT-HPM method. Identifying terms

$$L(y)=y^{(4)}(x), \tag{4}$$

$$N(y)=y(x)y'''(x), \tag{5}$$

whereprime denotes differentiation with respect to x.

Identifyingε with PM parameter, we assume a solution for (3) in the form

$$y(x)=y_0(x)+\varepsilon y_1(x)+\varepsilon^2 y_2(x)+\varepsilon^3 y_3(x)+\varepsilon^4 y_4(x)+\cdots. \tag{6}$$

(see (2))

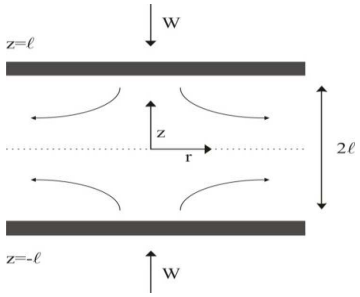

Figure 1. *Shows an axisymmetric fluid, squeezed between two infinite parallel plates.*

On comparing the coefficients of like powers of ε it can be solved for $y_0(x)$, $y_1(x)$, $y_2(x)$, $y_3(x)$,…, and so on. Later, we will see that a very good handy result is obtained by keeping a second order approximation.

$$\varepsilon 0)\ y0(4)=0,$$
$$y0(0)=0,\ y0''(0)=0,\ y0(1)=1,\ y0'(1)=0, \tag{7}$$

$$\varepsilon 1)\ y1(4)+y0y0'''=0,$$
$$y1(0)=0,\ y1''(0)=0,\ y1(1)=0,\ y1'(1)=0, \tag{8}$$

$$\varepsilon 2)\ y2(4)+y0y1'''+y1y0'''=0,$$
$$y2(0)=0, y2''(0)=0, y2(1)=0, y2'(1)=0, \tag{9}$$

Thus, the results obtained are

$$y0(x)=(3/2)x-(1/2)x3, \tag{10}$$

$$y1(x)=(19/560)x-(117/1680)x3+(3/80)x5-(1/560)x7, \tag{11}$$

$$y2(x)=-(137/10780)x-(443/517440)x3+$$
$$(17/2800)x5-(177/39200)x7+$$
$$(1/1680)x^9-(3/123200)x^{11}, \tag{12}$$

By substituting (10) thru (12) into (6) we obtain a second order approximation for the solution of (3).

Considering as cases study, the values of parameters ε=1 and ε=3; we obtain the following handy approximate solutions

$$y(x)=(859/560)x-(295199/517440)x^3-(247/39200)x^7+(61/1400)x^5-(3/123200)x^{11}-(1/1680)x^9-137/107800, \quad (13)$$

$$y(x)=(897/560)x-(24721/34496)x^3-(1803/39200)x^7+(117/700)x^5-(27/123200)x^{11}+(3/560)x^9-1233/107800. \quad (14)$$

As a matter of fact, we will see that (13) and (14) are also, highly accurate.

4. Discussion

The fact that the PM depends on a parameter, which is assumed to be small, suggests that the method is for a limited use. In this work, PM method has been applied, successfully to the problem of finding approximate solutions for nonlinear differential equation with mixed boundary conditions that describes an axisymmetric Newtonian fluid squeezed between two large parallel plates.

Fig. 2 and Fig. 3 shows the comparison between the approximate solutions (13) and (14) for differential equation (3) with a built-in numerical routine for BVP from Maple 15. It can be noticed that figures are in good agreement, showing the accuracy of proposed solutions. This proves the efficiency of PM method in this case, despite of the fact that it was only considered the second-order approximation to the equation to be solved. Therefore, accuracy can be increased considering higher order approximations.

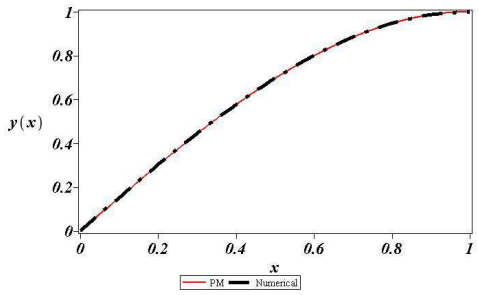

Figure 2. *Approximation for (3) and numerical comparison when ε =1.*

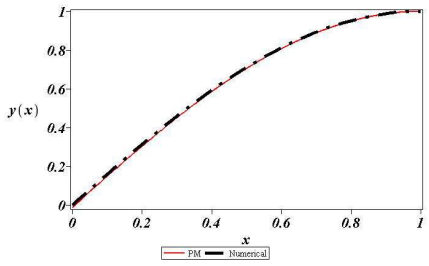

Figure 3. *Approximation for (3) and numerical comparison when ε =3.*

It is well known that PM method provides, in general, better results for small perturbation parameters ε << *1* (see (1)) and when are included the most number of terms from (2). To be precise, ε is a parameter of smallness, which measure how greater is the contribution of linear term *L(x)* than the one from *N(x)*in (1). Fig. 2 and Fig. 3 shows a noticeable fact, (13) and (14)turn out to be a good approximation of (3), although perturbation parameters ε=*1* and ε=3 are, indeed, large.

Finally, our approximate solutions do not depend on any adjustment parameter; thus they are, in principle, general expressions for the proposed problem.

5. Conclusions

In this study, PM was presented to construct analytical approximate solutions for nonlinear differential equations in the form of rapidly convergent series. In order to prove the versatility of this method, we proposed as an example the approximate solution for the nonlinear differential equation that describes a viscous, incompressible fluid, squeezed between two infinite parallel plates with mixed boundary conditions, obtaining acceptable results. The success of PM for this case, where we employed large values for perturbation parameter, must be considered as a real possibility to apply it in other nonlinear problems, instead of using other sophisticated and difficult methods. From Fig. 2 and Fig. 3, it is deduced that the proposed solutions have good precision.

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